$$\lim_{x\to 0} \frac{\operatorname{arctan} x}{x}$$

substituce $x = \operatorname{tg} y$

$$\lim_{y\to 0} \frac{\operatorname{arctg}(\operatorname{tg} y)}{\operatorname{tg} y} = \lim_{y\to 0} \frac{y}{\operatorname{tg} y} = \lim_{y\to 0} \cos y \cdot \frac{y}{\sin y}$$

$$= \lim_{y\to 0} \cos y \cdot \lim_{y\to 0} \frac{y}{\sin y} = 1 \cdot 1 = 1$$

$$\lim_{x\to\infty} \left(x - \sqrt{x}\ln x\right) = \lim_{x\to\infty} \sqrt{x}\,(\underbrace{\sqrt{x} - \ln x}_{>1 \text{ pro } x>1})$$

$x>1$: $\boxed{\sqrt{x} > \ln x}$

$$e^{x^{\frac{1}{2}}} > x$$

pro $x \in \mathbb{N}$ indukcí:

$x = 1$

$$\frac{e^{\sqrt{x+1}}}{e^{\sqrt{x}}} \;?\!> \frac{x+1}{x}$$

pro $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\arccos(\sin x) = \arccos\left(\cos\left(\frac{\pi}{2} - x\right)\right) = \frac{\pi}{2} - x$$

$$\left(\arccos(\sin x)\right)' = -1$$

$$-\frac{\cos x}{\sqrt{1-\sin^2 x}} = -\frac{\cos x}{|\cos x|} = \begin{cases} -1 & \text{pro } x \in \left(-\frac{\pi}{2}+2k\pi, \frac{\pi}{2}+2k\pi\right), k \in \mathbb{Z} \\ 1 & \text{pro } x \in \left(\frac{\pi}{2}+2k\pi, \frac{3\pi}{2}+2k\pi\right), k \in \mathbb{Z} \end{cases}$$

3. Změny ve zadání

Uvažujme $\sum_{n=1}^{\infty} a_n x^n$ (mocninnou řadu)

poloměr konvergence:

odm. krit.: $\lim_{n\to\infty} \sqrt[n]{|a_n x^n|} < 1$, $\lim_{n\to\infty} \sqrt[n]{|a_n|}\,|x| < 1$, $|x| < \frac{1}{\lim \sqrt[n]{|a_n|}}$

podílové krit.: $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}x\right| < 1 \iff |x| < \frac{1}{\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|}$

$$\lim_{x\to 0}\frac{1-\cos(x)}{x^2} = \lim_{x\to 0}\frac{\sin(x)}{2x} = \lim_{x\to 0}\frac{\cos(x)}{2} = \frac{1}{2}$$

$$\lim_{x\to 0}\frac{\operatorname{arctg}(x)}{x} = \lim_{x\to 0}\frac{\frac{1}{1+x^2}}{1} = 1$$

$$\lim_{x\to\infty} x - \sqrt{x}\ln x = \lim_{x\to\infty}\sqrt{x}\left(\sqrt{x}-\ln x\right)$$

$$f(x)-g(x) = \frac{\frac{1}{g(x)}-\frac{1}{f(x)}}{\frac{1}{f(x)g(x)}}$$

$$\lim_{x\to\infty}\left(\sqrt{x}-\ln x\right) = \lim_{x\to\infty}\frac{\frac{1}{\ln x}-\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}\ln x}} = \lim_{x\to\infty}\frac{-\frac{1}{x}\cdot\frac{1}{\ln^2 x}+\frac{1}{2}x^{-\frac{3}{2}}}{-\frac{\frac{1}{2}\frac{\ln x}{\sqrt{x}}+\frac{1}{\sqrt{x}}}{x\ln^2 x}}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x}\left(x^{-\frac{1}{2}} - \frac{1}{\ln^2 x}\right)}{-x^{-\frac{3}{2}}\left(\frac{1}{2\ln x} + \frac{1}{\ln^2 x}\right)} \quad \ldots.$$

$$\lim_{x \to 0_+} \sin(x) \cdot \ln(x) = \lim_{x \to 0_+} \frac{\ln(x)}{\frac{1}{\sin(x)}} =$$

$$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}} = \lim_{x \to 0} -\frac{\sin^2 x}{x \cos x} =$$

$$= \lim_{x \to 0} \left(-\frac{\sin x}{x}\right) \cdot \frac{\sin x}{\cos x} = 0$$

$$\lim_{x\to\infty} \sqrt[x]{\ln(x)} = \lim_{x\to\infty} \left[\ln(x)\right]^{\frac{1}{x}} = \lim_{x\to\infty} e^{\frac{1}{x}\ln(\ln(x))} =$$

$$= e^{\lim\limits_{x\to\infty} \frac{1}{x}\ln(\ln(x))} = e^{0} = 1$$

$$\lim_{x\to\infty} \frac{1}{x}\ln(\ln(x)) = \lim_{x\to\infty} \frac{\ln(\ln(x))}{x} = \lim_{x\to\infty} \frac{\frac{1}{\ln(x)}\cdot\frac{1}{x}}{1} =$$

$$= 0$$

$f(x) = \cos^2 x - \sin^2 x = \cos 2x$

$f'(x) = -2 \sin 2x$ $\qquad f'(0) = 0$

$f''(x) = -4 \cos 2x$ $\qquad f''(0) = -4$

$f^{(3)}(x) = +8 \sin 2x$ $\qquad f^{(3)}(0) = 0$

$f^{(4)}(x) = 16 \cos 2x$ $\qquad f^{(4)}(0) = 16$

$$T_{x_0}^{2} f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2 + \frac{1}{3!} f^{(3)}(x_0)(x - x_0)^3 + \dots$$

$$T_0^4(\cos 2x) = 1 - 2x^2 + \frac{2}{3}x^4$$

$$T^1_{\frac{\pi}{2}}[\cos(2x)] = \cos(\pi) + f'(\pi)(x-\tfrac{\pi}{2}) + \tfrac{1}{2} f''(\pi)(x-\tfrac{\pi}{2})^2 \cdot$$

$$= -1 + 2(x-\tfrac{\pi}{2})^2 = 2x^2 - (2\pi)x + \left(\tfrac{\pi^2}{2} - 1\right)$$

Určete extrémy funkce

$$f(x) = \frac{(x+2)(x-1)}{(x+1)}$$

$$D_f = \mathbb{R} - \{-1\}$$

Zkoumáme, kdy je $f'(x) = 0$

$$f'(x) = \frac{[(x+2)+(x-1)](x+1) - (x+2)(x-1)}{(x+1)^2}$$

$f'(x) = 0 \Leftrightarrow$

$$(x+2)(x-1) - (x+2)(x+1) - (x^2-1) = 0$$

$$x^2+x-2 - x^2-3x-2-x^2+1 = 0$$

$$-x^2-2x-3=0$$

$$x^2+2x+3=0 \qquad x^2+2x+3>1$$

funkce nemá extrém