$f = \cos(2x)$

$f' = -2\sin(2x) \qquad f'(0) = 0$

$f'' = -4\cos(2x) \qquad f''(0) = -4$

$f^{(3)} = 8\sin(2x) \qquad f^{(3)}(0) = 0$

$f^{(4)} = 16\cos(2x) \qquad f^{(4)}(0) = 16$

$$T_0^4 f = f(0) + f'(0)(x) + \frac{f''(0)}{2}x^2 + \frac{f^{(3)}(0)}{6}x^3 + \frac{f^{(4)}(0)}{24}x^4$$

$$T_0^4 \cos(2x) = 1 - 2x^2 + \frac{2}{3}x^4$$

2)

$$f = \sin^2 x$$
$$f' = 2 \sin x \cos x = \sin(2x)$$
$$f'' = 2\cos(2x)$$
$$f^{(3)} = -4\sin(2x)$$
$$f^{(4)} = -8\cos(2x)$$
$$f^{(5)} = 16\sin(2x)$$
$$f^{(6)} = 32\cos(2x)$$
$$f^{(7)} = -64\sin(2x)$$

Chyba:

$$\frac{|f^{(7)}(c)|\left(\frac{\pi}{4}\right)^7}{7!}$$

$$c \in \left(0, \frac{\pi}{4}\right)$$

3) $D_f = \mathbb{R} - \{2\}$

$$f = \frac{x-1}{x-2} = 1 + \frac{1}{x-2}$$

$$f' = -\frac{1}{(x-2)^2} < 0 \text{ na } D_f$$

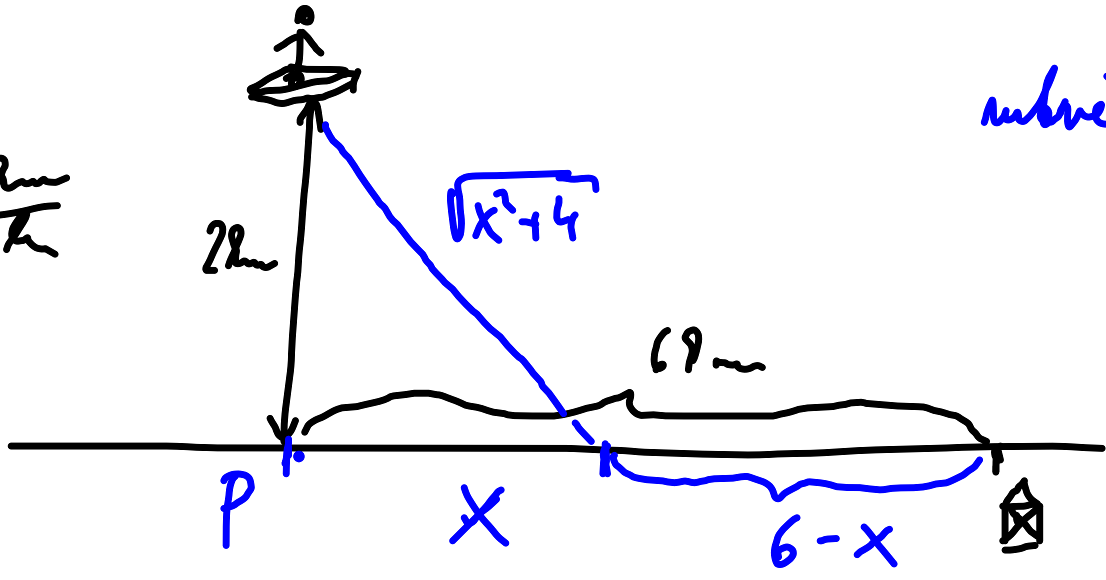
2) $f = \operatorname{arctan}\left(\frac{x-1}{x}\right) = \operatorname{arctan}\left(1 - \frac{1}{x}\right) \qquad D_f : \mathbb{R} - \{0\}$

$$f' = \frac{1}{1+\left(1-\frac{1}{x}\right)^2} \cdot \left(\frac{1}{x^2}\right) > 0 \text{ na } D_f$$

Dané funkce extrémy nemají.

zřejmě jde o minimum

$v_1 = 4\ \frac{km}{h}$



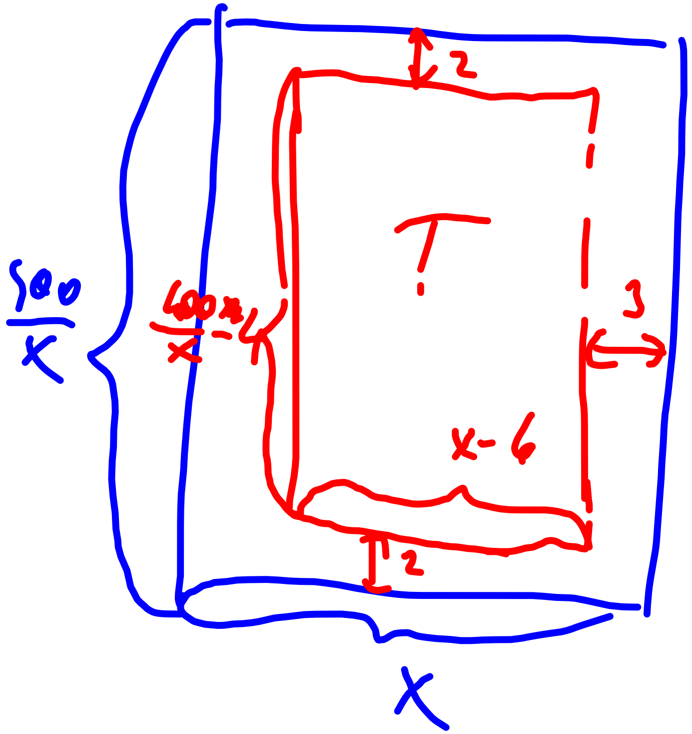
$v_2 = 6\ \frac{km}{h}$

Celkový čas na cestu: $T = \frac{\sqrt{x^2+4}}{4} + \frac{6-x}{6}$

$$T' = \frac{1}{4}\frac{x}{\sqrt{x^2+4}} - \frac{1}{6} = 0$$

$$3x = 2\sqrt{x^2+4} \iff 9x^2 = 4x^2 + 16$$

$$5x^2 = 16 \Rightarrow x = \frac{4}{\sqrt{5}}$$



$S = 500\,cm^2$

Obsah tiskové plochy:

$$T = (x-6)\left(\frac{500}{x}-4\right) =$$

$$= 400 - 4x - \frac{2400}{x} + 24$$

$$T' = -4 + \frac{2400}{x^2} = 0$$

$$x^2 = 600$$

$$x = 10\sqrt{6}$$

1, Určíme definiční obor

$$D_f = \mathbb{R} \setminus \{1\}$$

Nulové body: nejsou

Fce je klesající na $(1, \infty)$

rostoucí na $(-\infty, +1)$

Určíme derivaci:

$$f = \frac{x^2+1}{x-1} = \frac{x^2-1+2}{x-1} = x+1+\frac{2}{x-1}$$

$$f' = 1 - \frac{2}{(x-1)^2}$$

$$x = 1 \pm \sqrt{2}$$

Určíme kritické body

$$f' = 0 \iff \frac{2}{(x-1)^2} = 1 \iff (x-1)^2 = 2$$

Určíme intervaly monotonie

	$(1-\sqrt{2})$		1		$(1+\sqrt{2})$	
f'	$+$		$-$	$-$		$+$
f	$\nearrow$		$\searrow$	$\searrow$		$\nearrow$

$\Rightarrow$ v bodě $(1-\sqrt{2})$ má fce maximum
$(1+\sqrt{2})$ má fce minimum

Na základě znaménka druhé derivace určíme intervaly konvexnosti a konkávnosti

$$f''(x) = \left(1 - \frac{2}{(x-1)^2}\right)' = +\frac{4}{(x-1)^3}$$

na intervalu $(1,\infty)$ konvexní

		1	
f''	<0		>0
f	$\cap$		$\cup$

f je na intervalu $(-\infty,1)$ konkávní.

Hledáme asymptoty:

a) bez směrnice:

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+} \dfrac{x^2+1}{x-1} = \infty \\[2ex] \lim\limits_{x\to 1^-} \dfrac{x^2+1}{x-1} = -\infty \end{array}\right\}$$

přímka $x = 1$ je asymptota bez směrnice

graf fce má asymptotu se směrnicí $y = x+1$

b) se směrnicí

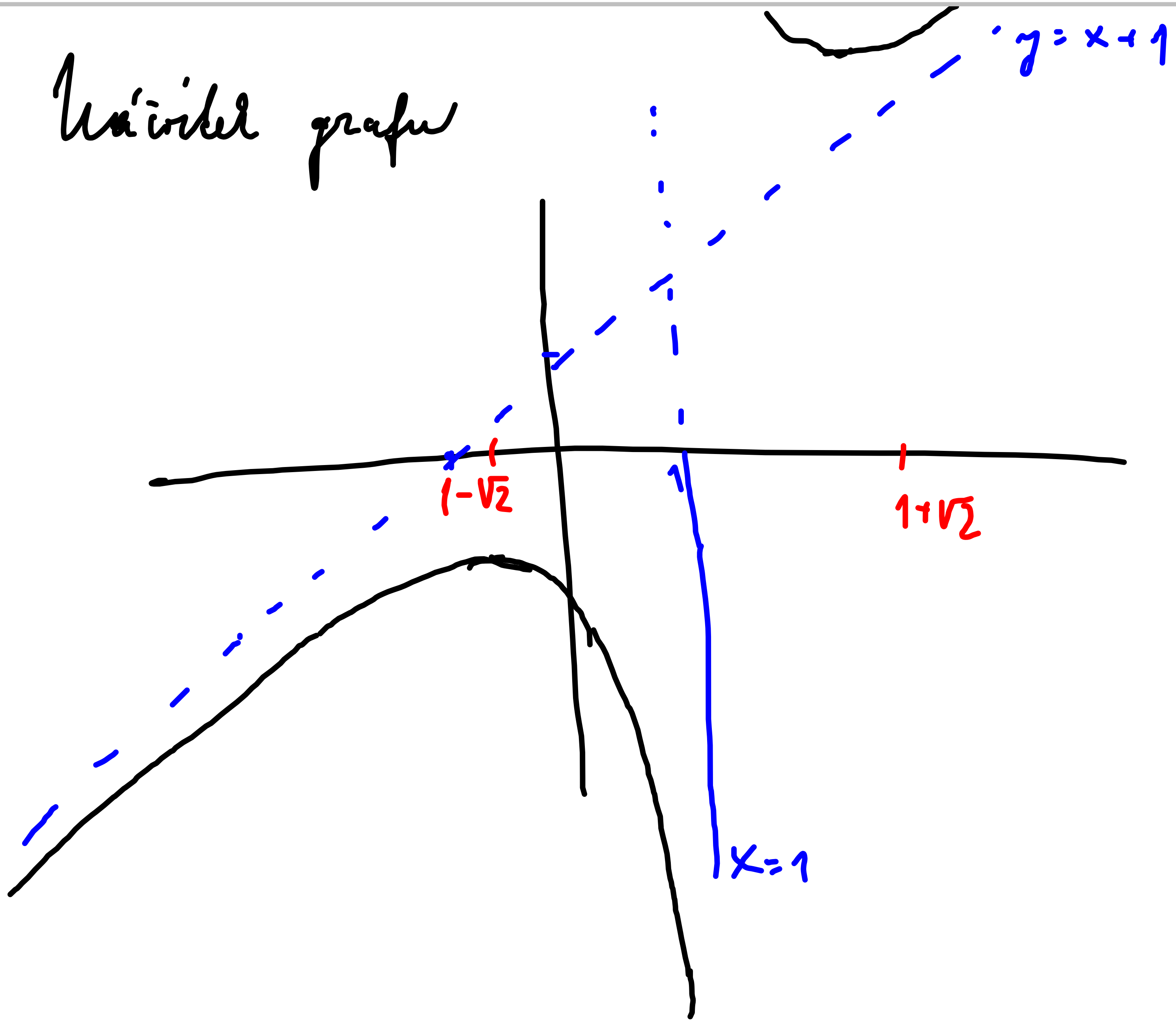
Hledejme směrnici ve tvaru $y = kx + q$. Pak

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2+1}{x(x-1)} = 1 = \lim_{x\to -\infty} \frac{x^2+1}{x^2-x}$$

$$q = \lim_{x\to\infty} \left(f(x) - kx\right) = \lim_{x\to\infty} \left(\frac{x^2+1}{x-1} - x\right) =$$

$$= \lim_{x\to\infty} \left(\frac{x^2+1-x^2+x}{x-1}\right) = \lim_{x\to\infty} \left(\frac{x+1}{x-1}\right) = 1$$

Náčrtek grafu



$$D_f = (-\infty, -\sqrt{2}) \cup \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle \cup (\sqrt{2}, \infty)$$

Vyzkoumáme znaménka $\dfrac{4x^4-1}{x^2-2}$:

$$4x^4 = 1$$
$$x^4 = \frac{1}{4}$$
$$x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

$-\sqrt{2}$ $\quad -\frac{\sqrt{2}}{2}$ $\quad \frac{\sqrt{2}}{2}$ $\quad \sqrt{2}$

$+ \quad - \quad + \quad - \quad +$

$$f' = \frac{1}{2}\,\frac{\dfrac{16x^3\cdot(x^2-2) - 8x^5 + 2x}{(x^2-2)^2}}{\sqrt{\dfrac{4x^4-1}{x^2-2}}} = \frac{1}{2}\,\frac{\dfrac{8x^5 - 32x^3 + 2x}{(x^2-2)^2}}{\sqrt{\dfrac{4x^4-1}{x^2-2}}} = 0$$

$$\Leftrightarrow \; 8x^5 - 32x^3 + 2x = 0$$

$$x(8x^4 - 32x^2 + 2) = 0 \Rightarrow x = 0 \;\vee$$

$$4x^4 - 16x^2 + 1 = 0 \quad \text{subst. } t = x^2$$

$$4t^2 - 16t + 1 = 0, \quad t_{1,2} = \frac{16 \pm \sqrt{256-16}}{8} = 2 \pm \frac{1}{2}\sqrt{15}$$

$$x_{1,2} = \pm\sqrt{2 - \frac{\sqrt{15}}{2}}$$

$$x_{3,4} = \pm\sqrt{2 + \frac{\sqrt{15}}{2}}$$

Vyznačíme znaménka 1. derivace:

	$-\sqrt{2+\frac{\sqrt5}{2}}$		$-\sqrt2$		$-\frac{\sqrt2}{2}$		$-\sqrt{2-\frac{\sqrt5}{2}}$		0		$\sqrt{2-\frac{\sqrt5}{2}}$		$\frac{\sqrt2}{2}$		$\sqrt2$		$\sqrt{2+\frac{\sqrt5}{2}}$
f':		$-$		$+$		$+$		$-$		$+$		$-$		$-$		$+$	
f:		$\searrow$		$\nearrow$		$\nearrow$		$\searrow$		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$	

$\Rightarrow$ fce f má maxima v bodech $\mp\sqrt{2-\frac{\sqrt5}{2}}$

minima v bodech 0, $\pm\sqrt{2+\frac{\sqrt5}{2}}$

náčrt: