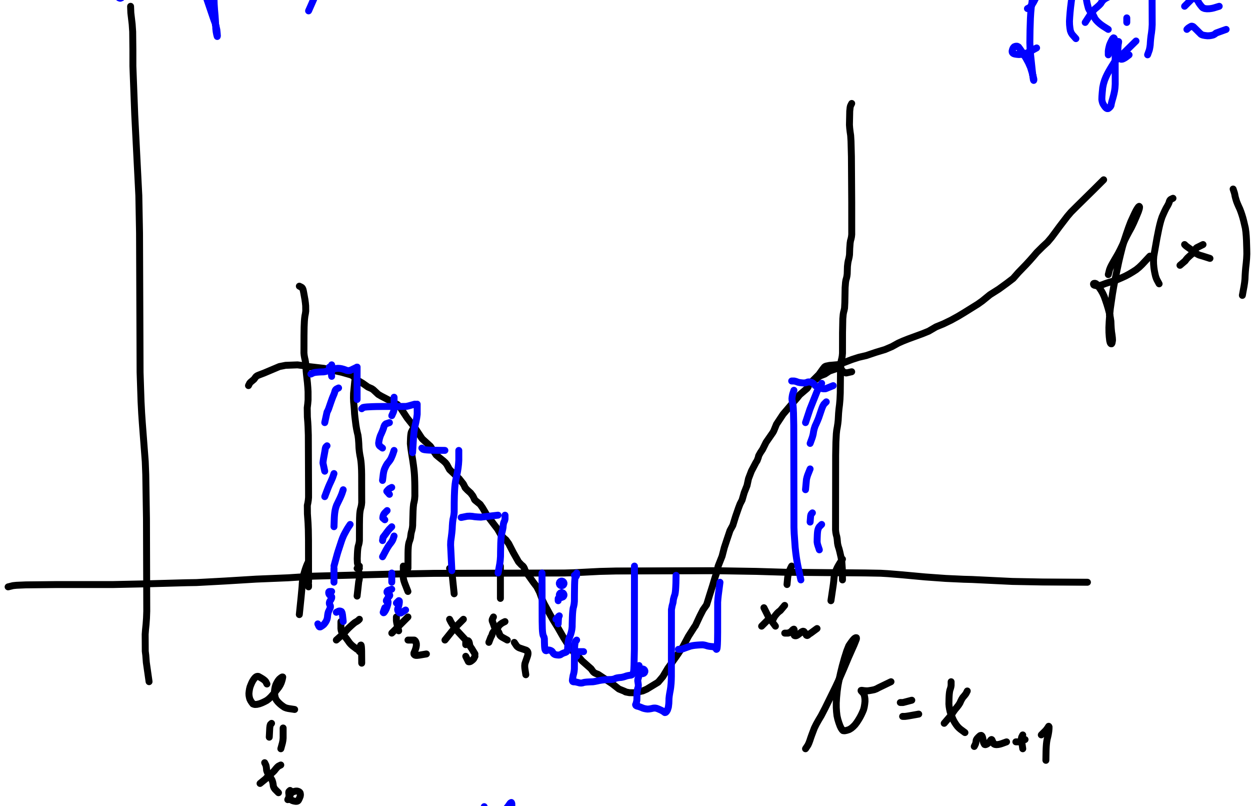
$$F'(x) = f(x)$$

$$f(x_j) \approx \frac{F(x_{j+1}) - F(x_j)}{x_{j+1} - x_j}$$



$$\sum_{i=1}^{n+1} f(x_i)(x_{i+1} - x_i) \approx \sum_{i=0}^{n+1} \frac{F(x_{i+1}) - F(x_i)}{x_{i+1} - x_i}(x_{i+1} - x_i)$$

$$= F(x_{n+1}) - F(x_0) = F(b) - F(a)$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

(určitý integrál)

Věta. Dvě dif. fce na int. (a,b) se liší o konstantu, právě když mají stejné derivace

Dk. „$\Rightarrow$" $F(x)$ a $G(x)$ se liší o konstantu

$$F(x) = G(x) + c \Rightarrow F'(x) = G'(x)$$

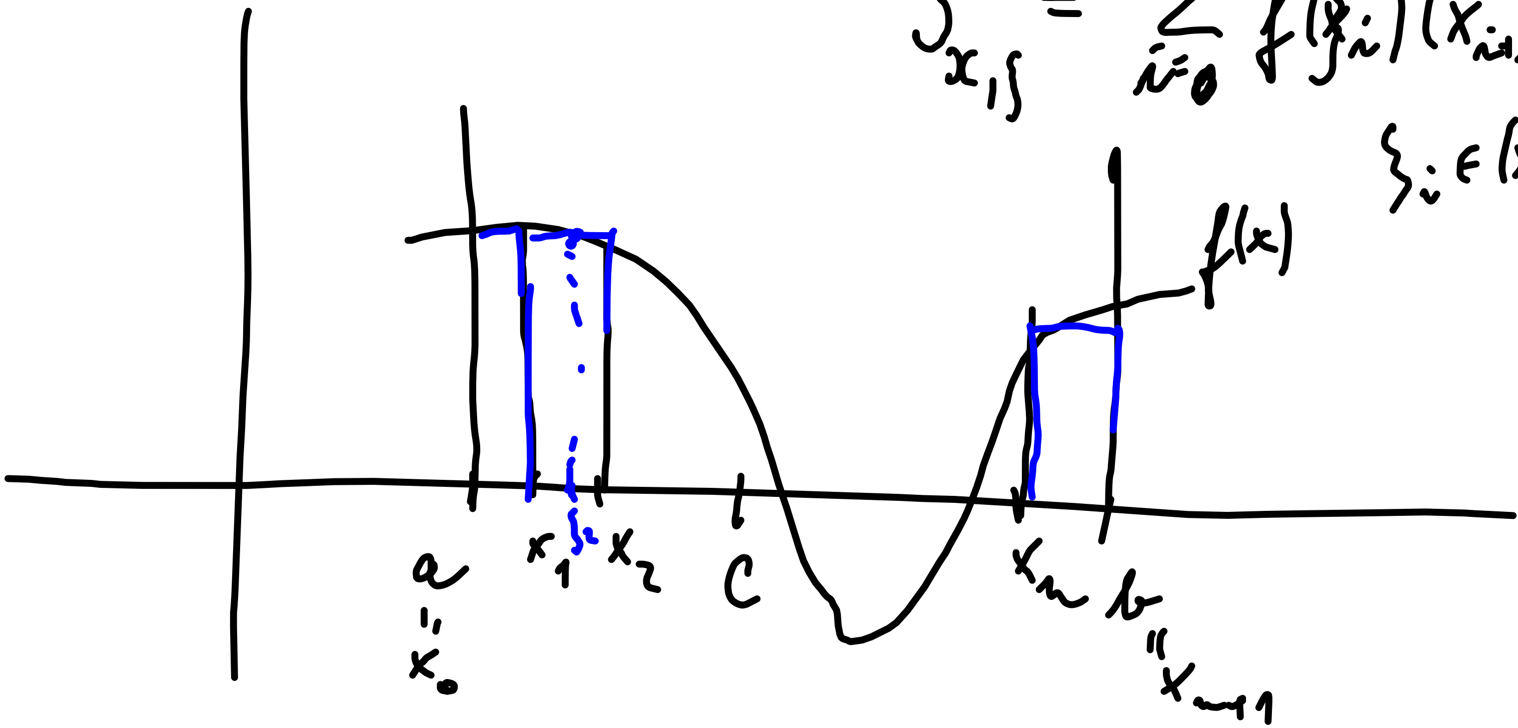
„$\Leftarrow$" Nechť $F'(x) = G'(x) = f(x)$. Uvažme

$$(F-G)(x) = F(x) - G(x) = F(a) - G(a) + (f(c) - f(c))(x-a) =$$
$$= F(a) - G(a)$$

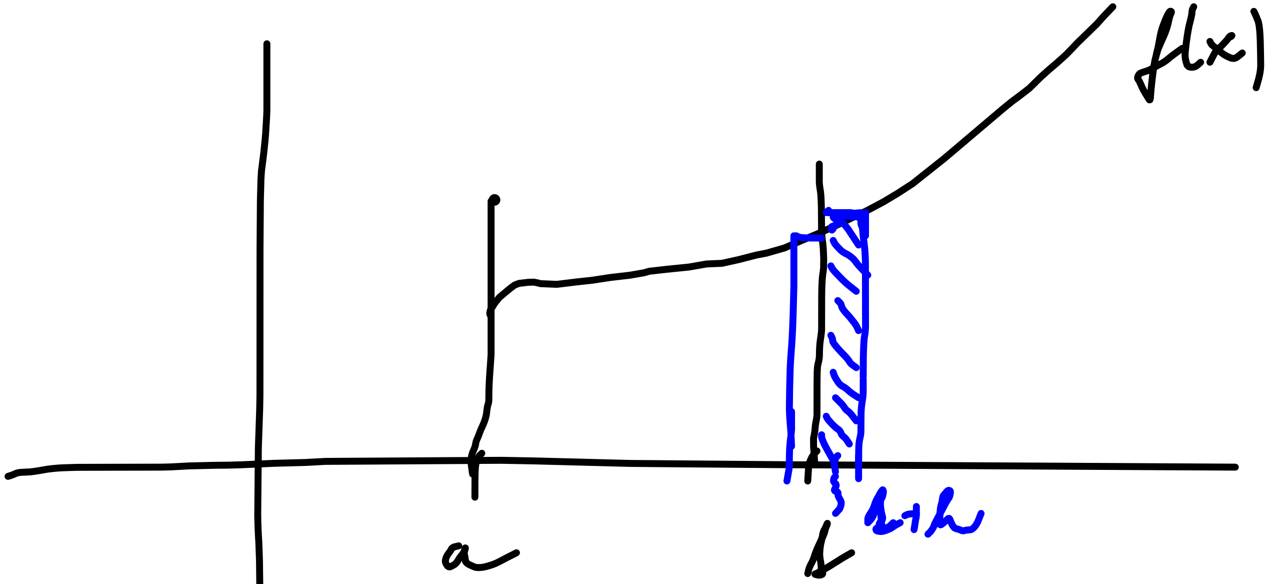
$$F(t) = \int f(x)\,dx + C$$

$$S_{x,\xi} = \sum_{i=0}^{n} f(\xi_i)(x_{i+1} - x_i)$$

$$\xi_i \in (x_i, x_{i+1})$$



$$F(t) = \int_a^t f(x)\,dx$$



$$F'(t) = \lim_{h \to 0} \frac{F(t+h) - F(t)}{h} = f(\xi).$$

$$\int F'(x)\,G(x)\,dx = F(x)\,G(x) - \int F(x)\,G'(x)$$

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = \int -\frac{1}{t}\,dt =$$

$$\left[\begin{array}{l} t = \cos(x) \\ dt = -\sin(x)\,dx \end{array}\right]$$

$$= -\int \frac{1}{t}\,dt = -\ln(t) = -\ln(\cos(x)).$$

$$\int \sin^2 x\,dx = -\cos x \sin x - \int -\cos^2 x\,dx \quad \left[\int u'v = uv - \int uv'\right]$$

$$\left[\begin{array}{ll} u' = \sin x & u = -\cos x \\ v = \sin x & v' = \cos x \end{array}\right]$$

$$= -\cos x \sin x + \int \cos^2 x\,dx =$$

$$= -\cos x \sin x + \int 1 - \sin^2 x\,dx =$$

$$= x + c - \cos x \sin x - \int \sin^2 x \, dx$$

$$\Rightarrow \int \sin^2 x = \frac{1}{2}\left[x - \sin x \cos x\right] + c$$

Jiný postup:

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\int \sin^3 x \, dx = \int \frac{1-\cos 2x}{2} \, dx = \frac{1}{2}\int (1-\cos 2x)\, dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + c$$

$$\int \sin^3 x \, dx = \int \sin x \sin^2 x \, dx = \int \sin x (1-\cos^2 x)\, dx =$$

$$\begin{cases} t = \cos x \\ dt = -\sin x \, dx \end{cases}$$

$$= -\int (1-t^2)\, dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c$$

$$\int \arcsin(x)\,dx = \qquad x\cdot\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx =$$

$$\left[\begin{matrix} u' = 1 & u = x \\ v = \arcsin x & v' = \frac{1}{\sqrt{1-x^2}} \end{matrix}\right] \qquad \left[\begin{matrix} t = x^2 \\ dt = 2x\,dx \end{matrix}\right]$$

$$= x\cdot\arcsin x - \frac{1}{2}\int \frac{dt}{\sqrt{1-t}} = x\cdot\arcsin x + \sqrt{1-t} + c =$$

$$= x\arcsin x + \sqrt{1-x^2} + c$$

$$\int \sqrt{1-x^2}\,dx = \int \sqrt{1-\sin^2 t}\,\cos t\,dt = \int \cos^2 t\,dt =$$

$x = \sin t$

$dx = \cos t\,dt$

$$= \int \frac{1+\cos 2t}{2}\,dt = \frac{1}{2}\left[t + \frac{\sin 2t}{2}\right] + C =$$

$$= \frac{1}{2}\arcsin \cancel{x} + \frac{1}{4}\sin\left(2\arcsin x\right) =$$

$=$

$\frac{1}{2}\arcsin x + \frac{1}{2}\cancel{x}\cdot\arcsin(\cos x) + C =$

$\arcsin(\cos(x)) = \arcsin\left(\sin\left(\frac{\pi}{2}-x\right)\right) = \frac{\pi}{2}-x$

$\frac{1}{2}\arcsin x + \frac{1}{2}x\left(\left(\frac{\pi}{2}-x\right)\right) + C = \frac{1}{2}\arcsin x + \frac{\pi}{4}x - \frac{1}{2}x^2 + C$

$\int x^2 \ln x \, dx = \frac{x^3}{3}\cdot\ln(x) - \int \frac{x^2}{3}dx =$

$\begin{bmatrix} u' = x^2 & u = \frac{x^3}{3} \\ v = \ln x & v' = \frac{1}{x} \end{bmatrix}$

$= \frac{x^3}{3}\ln(x) - \frac{x^3}{9} + C$

$\int x^3 e^{2x} dx = \frac{1}{2}x^3 e^{2x} - \frac{3}{2}\int x^2 e^{2x} dx =$

$\begin{bmatrix} u' = e^{2x} & u = \frac{1}{2}e^{2x} \\ v = x^3 & v' = 3x^2 \end{bmatrix}$ $\begin{bmatrix} u' = e^{2x} & u = \frac{1}{2}e^{2x} \\ v = x^2 & v' = 2x \end{bmatrix}$

$= \frac{1}{2}x^3 e^{2x} - \frac{3}{4}x^2 e^{2x} + \int x e^{2x} =$

$\begin{bmatrix} u' = e^{2x} & u = \frac{1}{2}e^{2x} \\ v = x & v' = 1 \end{bmatrix}$

$$\frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{4}\int e^{2x} =$$

$$= \frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x}$$