$$\int x\sqrt{1+x^2}\,dx = \frac{1}{2}\int \sqrt{1+t}\,dt =$$

$$\begin{bmatrix} t = x^2 \\ dt = 2x\,dx \end{bmatrix}$$

$$= \frac{1}{3}(1+t)^{\frac{3}{2}} = \frac{1}{3}(1+x^2)^{\frac{3}{2}}$$

$$\int x^2 \ln(x)\,dx = \frac{1}{3}x^3\ln x - \int \frac{x^2}{3}\,dx =$$

$$\begin{bmatrix} u = \ln(x) & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{bmatrix}$$

$$= \frac{1}{3}x^3\ln x - \frac{x^3}{9} =$$

$$= \frac{1}{9}x^3(3\ln x - 1)$$

$$\underline{\int e^{2x}\cos(2x)\,dx} = \frac{1}{2}e^{2x}\sin(2x) - \int e^{2x}\sin(2x)\,dx =$$

$$\left[\begin{array}{ll} u = e^{2x} & u' = 2e^{2x} \\ v' = \cos(2x) & v = \frac{1}{2}\sin(2x) \end{array}\right] \qquad \left[\begin{array}{ll} u = e^{2x} & u' = 2e^{2x} \\ v' = \sin 2x & v = -\frac{1}{2}\cos(2x) \end{array}\right.$$

$$= \frac{1}{2}e^{2x}\sin(2x) + \frac{1}{2}e^{2x}\cos(2x) - \underline{\int e^{2x}\cos(2x)\,dx}$$

$$\Rightarrow \int e^{2x}\cos(2x)\,dx = \frac{1}{4}\left(e^{2x}\sin(2x) + e^{2x}\cos(2x)\right)$$

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^2 t \cos t}{\cos t}\,dt =$$

$$\left[\begin{aligned} x &= \sin t \\ dx &= \cos t\,dt \end{aligned}\right] \Rightarrow \left[\begin{aligned} &t = \arcsin x \\ &x = \sin t \Rightarrow \cos t = \sqrt{1-\sin^2 t} = \sqrt{1-x^2} \end{aligned}\right]$$

$$= \int \sin^2 t\,dt = -\sin t \cos t + \int \cos^2 t\,dt =$$

$$\left[\begin{aligned} u &= \sin t & u' &= \cos t \\ v' &= \sin t & v &= -\cos t \end{aligned}\right]$$

$$= -\sin t \cos t + \int (1-\sin^2 t)\,dt \Rightarrow$$

$$\Rightarrow \int \sin^2 t\,dt = \frac{1}{2}\left(t - \sin t \cos t\right) \Rightarrow$$

$$\Rightarrow \int \frac{x^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\left(\arcsin x - x\sqrt{1-x^2}\right)$$

$$\int \sqrt{3-x^2}\,dx = \sqrt{3}\int \sqrt{3-3\sin^2 t}\,\cos t\,dt =$$

$$\left[\begin{array}{l} x = \sqrt{3}\sin t \\ dx = \sqrt{3}\cos t\,dt \end{array}\right]$$

$$= 3\int \cos^2 t\,dt = \ldots$$

$$\int \frac{2x+1}{4x^3-8x^2-11x-3}\,dx$$

a) rozložíme jmenovatele $4x^3-8x^2-11x-3$ na součin ireducibilních polynomů nad $\mathbb{R}$ (tj. lineárních a kvadr. polynomů se zápornými diskrimin.)

	4	−8	−11	−3
3	4	4	1	0

$$4x^3 - 8x^2 - 11x - 3 = (x-3)(4x^2+4x+1) = (x-3)(2x+1)^2$$

b) Rozložíme $\dfrac{2x+1}{4x^3-8x^2-11x-3}$ na parciální zlomky:

$$\frac{2x+1}{4x^3-8x^2-11x-3} = \frac{A}{(x-3)} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

$$2x+1 = A(2x+1)^2 + B(2x+1)(x-3) + C(x-3)$$

$x=3$: $\quad 7 = A \cdot 49 \Rightarrow A = \frac{1}{7}$, $x = -\frac{1}{2} \Rightarrow C = 0$

$$\frac{2x+1}{4x^3-8x^2-11x-3} = \frac{1}{(2x+1)(x-3)} = \frac{A}{(x-3)} + \frac{B}{(2x+1)}$$

$$1 = A(2x+1) + B(x-3)$$

$x = -\frac{1}{2}$: $\quad 1 = B \cdot \left(-\frac{7}{2}\right) \Rightarrow B = -\frac{2}{7}$, $x=3$: $A = \frac{1}{7}$

$$\frac{2x+1}{4x^3-8x^2-11x-3} = \frac{1}{(x-3)(2x+1)} = \frac{1}{7(x-3)} + \frac{2}{7(2x+1)}$$

$$\int \frac{1}{(x-3)(2x+1)}\,dx = \int \left(\frac{1}{7}\frac{1}{(x-3)} - \frac{2}{7}\frac{1}{(2x+1)} \right) dx =$$

$$= \frac{1}{7}\ln(x-3) - \frac{2}{7}\ln(2x+1)$$

$$\int \frac{2x+1}{2x^3-6x^2+x-3}\,dx$$

a) Rozložíme jmenovatele

	2	−6	1	−3
3	2	0	1	0

$$2x^3-6x^2+x-3 = (x-3)(2x^2+1)$$

$$\frac{2x+1}{2x^3-6x^2+x-3} = \frac{A}{(x-3)} + \frac{Bx+C}{2x^2+1}$$

$$2x+1 = A(2x^2+1) + (Bx+C)(x-3) = x^2(2A+B) + (-3B+C)x + (A-3C)$$

$$x=3:\quad 7 = 19A \Rightarrow A = \frac{7}{19}$$

$$x^2:\quad 0 = 2A+B \Rightarrow B = -2A = -\frac{14}{19}$$

$$x^0:\quad 1 = A-3C \Rightarrow C = \frac{1}{3}(A-1) = \frac{1}{3}\left(-\frac{12}{19}\right) = -\frac{4}{19}$$

$$\int \frac{2x+1}{2x^3-6x^2+x-3}\,dx = \frac{7}{19}\int \frac{1\,dx}{x-3} + \int \frac{\frac{14}{19}x - \frac{5}{19}}{2x^2+1}\,dx =$$

$$= \frac{7}{19}\ln(x-3) + \frac{2}{19}\int \frac{7x\,dx}{2x^2+1} + \frac{2}{19}\int \frac{2\,dx}{2x^2+1} =$$

$$= \frac{7}{19}\ln(x-3) + \frac{7}{38}\ln(2x^2+1) + \frac{2\sqrt{2}}{19}\,\mathrm{arctg}(\sqrt{2}x)$$

$$\int \frac{7x}{2x^2+1}\,dx = \frac{7}{4}\int \frac{4x}{2x^2+1}\,dx = \frac{7}{4}\int \frac{1}{t}\,dt = \frac{7}{4}\ln(2x^2+1)$$

$$\begin{bmatrix} t = 2x^2+1 \\ dt = 4x\,dx \end{bmatrix}$$

$$\int \frac{1}{2x^2+1}\,dx = \int \frac{1}{(\sqrt{2}x)^2+1}\,dx = \frac{1}{\sqrt{2}}\int \frac{1}{t^2+1}\,dt =$$

$$t = \sqrt{2}x$$
$$dt = \sqrt{2}\,dx$$

$$= \frac{1}{\sqrt{2}}\,\mathrm{arctg}(\sqrt{2}x)$$

$$\int \frac{1}{x^2+x+3}\,dx = \int \frac{1\,dx}{\left(x+\frac{1}{2}\right)^2+\frac{11}{4}} = \frac{4}{11}\int \frac{1}{\frac{4}{11}\left(x+\frac{1}{2}\right)^2+1}\,dx =$$

$$= \frac{4}{11}\int \frac{1}{\left(\frac{2}{\sqrt{11}}\left(x+\frac{1}{2}\right)\right)^2+1}\,dx = \frac{2}{\sqrt{11}}\int \frac{1}{t^2+1}\,dt =$$

$$\left[\begin{array}{l} t = \frac{2x+1}{\sqrt{11}} \\ dt = \frac{2}{\sqrt{11}}\,dx \\ dx = \frac{\sqrt{11}}{2}\,dt \end{array}\right]$$

$$= \frac{2}{\sqrt{11}}\,\mathrm{arctg}\left(\frac{2x+1}{\sqrt{11}}\right)$$

$$\int_0^2 \frac{dx}{\sqrt{2-x}} = \lim_{\delta \to 2^-} \int_0^\delta \frac{dx}{\sqrt{2-x}} = \Big[-2\sqrt{2-x}\,\Big]_0^\delta =$$

$$= +2\sqrt{2}$$

$$\int_0^4 \frac{dx}{\sqrt{|2-x|}} = \int_0^2 \frac{dx}{\sqrt{|2-x|}} + \int_2^4 \frac{dx}{\sqrt{|2-x|}} = 4\sqrt{2}$$

$$= 2\sqrt{2} + \left[\int_2^4 \frac{dx}{\sqrt{x-2}} = -\int_2^0 \frac{dt}{\sqrt{2-t}} = \int_0^2 \frac{dt}{\sqrt{2-t}} = 2\sqrt{2}\right]$$

subst. $t = 4-x$

$dt = -dx$

$$\int_0^1 x^2 \ln x = \lim_{\delta \to 0} \int_\delta^1 x^2 \ln x \, dx = \lim_{\delta \to 0} \left[\frac{1}{9} x^3 (3 \ln x - 1)\right]_\delta^1$$

$$= \lim_{\delta \to 0} \left[-\frac{1}{9} - \frac{1}{9}\delta^3 (3 \ln \delta - 1)\right] =$$

$$= -\frac{1}{9} - \lim_{\delta \to 0} \left(\frac{1}{3}\delta^3 \ln \delta - \frac{1}{9}\delta^3\right) = -\frac{1}{9}$$

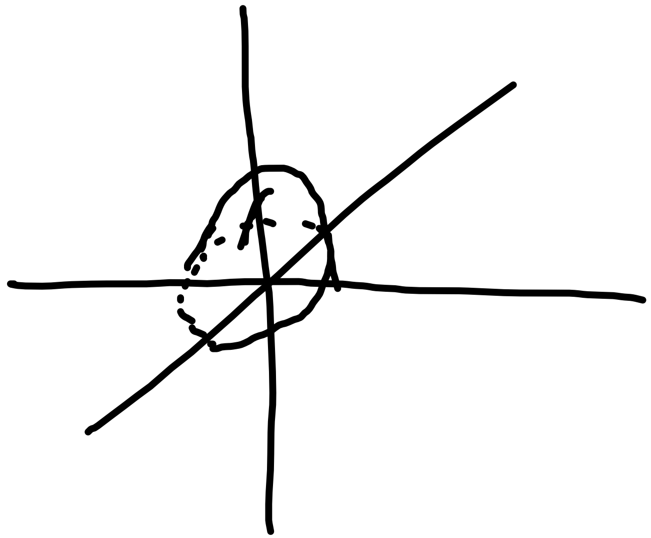
$$\int_0^\infty e^x \, dx = \lim_{\delta \to \infty} \int_0^\delta e^x \, dx = \lim_{\delta \to \infty} e^\delta - e^0 = \infty$$

$$\int_{-\infty}^0 e^x$$

$$\int_{-\infty}^{0} e^x\,dx = \lim_{\delta \to -\infty} \int_{\delta}^{0} e^x\,dx = \lim_{\delta \to -\infty} \left(e^0 - e^\delta\right) = 1$$

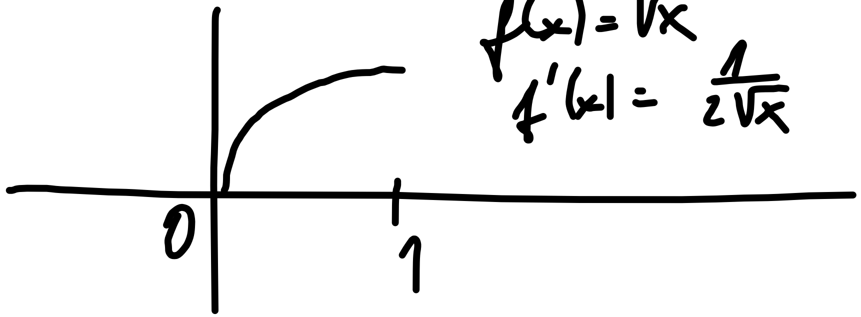
$$\int_{0}^{\infty} \frac{1}{x^2-3x-4}\,dx$$ není definován:

$$\int_{0}^{4} \frac{1}{x^2-3x-4} = -\infty \qquad \int_{4}^{\infty} \frac{1}{x^2-3x-4} = \infty$$



$$x^2+y^2 = 1-z$$

Paraboloid vznikne rotací



$$f(x)=\sqrt{x}$$
$$f'(x)=\frac{1}{2\sqrt{x}}$$

$$V = \pi\int_0^1 x\,dx = \frac{\pi}{2}$$

$$P = 2\pi\int_0^1 \sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx =$$
$$= 2\pi\int_0^1 \sqrt{x}\sqrt{\frac{4x+1}{4x}} =$$
$$= \pi\int_0^1 \sqrt{4x+1} = \frac{\pi}{6}\Big[(4x+1)^{\frac{3}{2}}\Big]_0^1 =$$
$$= \frac{\pi}{6}\left(5^{\frac{3}{2}}-1\right)$$

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot \ln(n)} \qquad \sum \frac{1}{n^{k}}$$

$k > 1$

$$\int_{1}^{\infty} \frac{1}{x \cdot \ln(x)} \, dx > \infty$$