$\sqrt{2} = a$

$2 = a^2 \stackrel{?}{=} p^2/q^2 \quad \sim \quad a^2 q^2 = p^2$

$a^2 q^2 = 2 \cdot q^2$

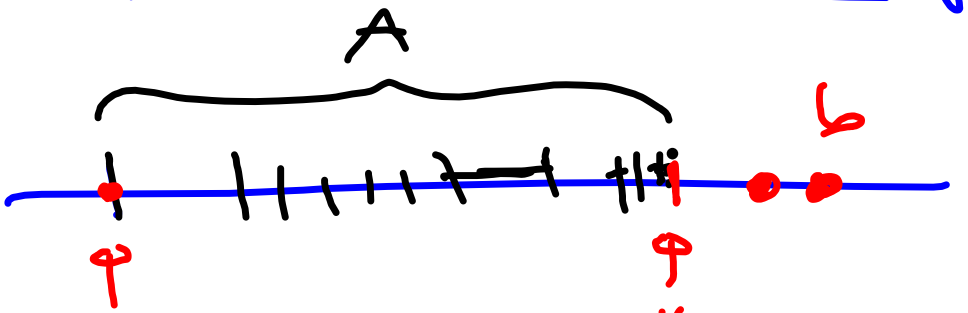
spor

uspořádání !

NENÍ na $\mathbb{C}$ (úplné)

	(R1) – (R9)	(R10) – (R12)	R13
$\mathbb{Q}$	✓	✓	—
$\mathbb{R}$	✓	✓	✓
$\mathbb{C}$	✓	—	—

Supremum × Infimum



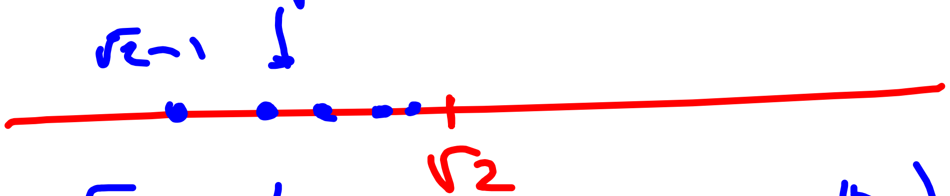
$x \geq b \Rightarrow x \geq a \quad \forall a \in A$

infimum

„nejmenší horní závora" = supremum

Pro $\mathbb{Q}$ neplatí, že každá omezená $A \subset \mathbb{Q}$ má supremum!

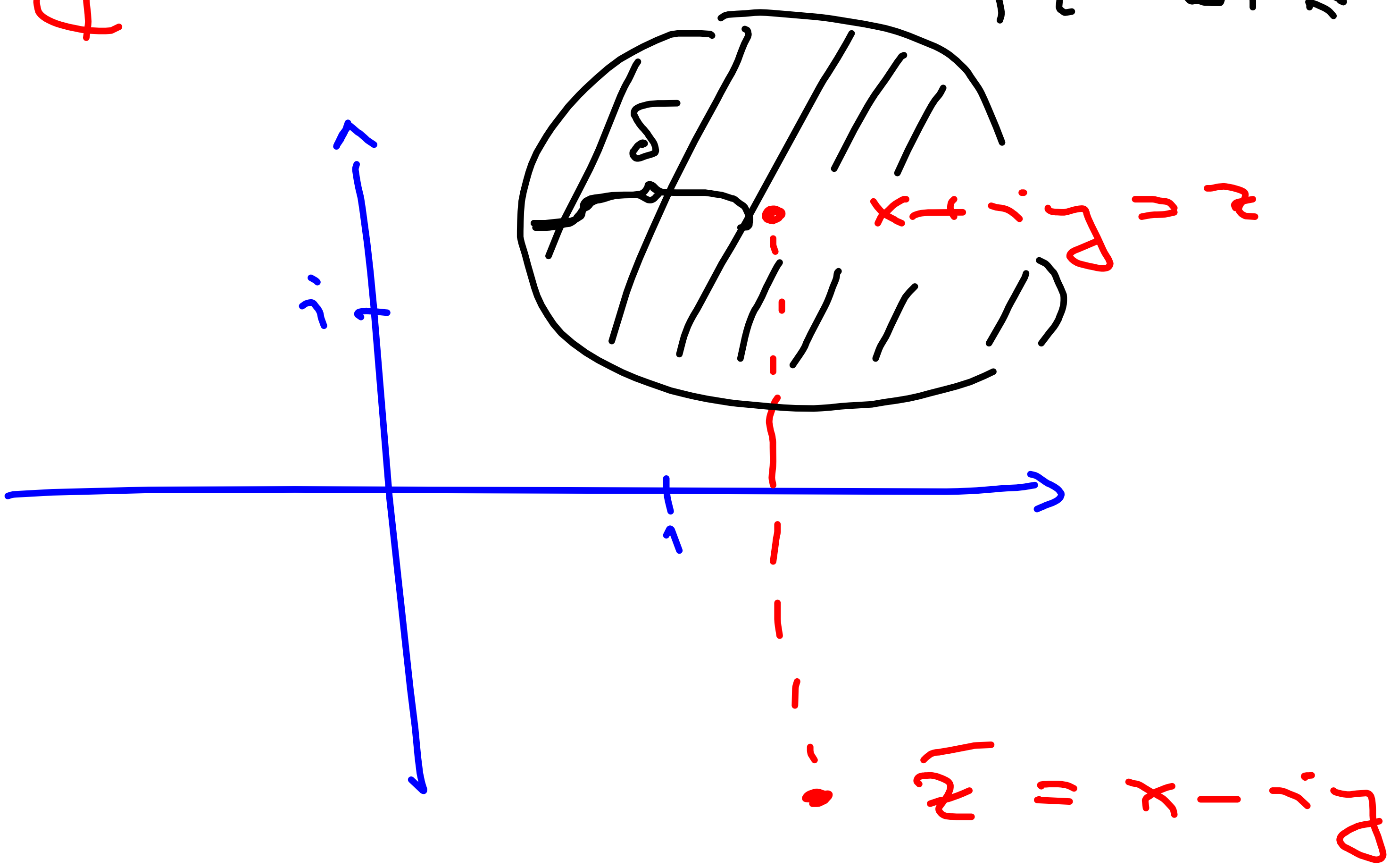
aproximace z $\mathbb{Q}$



$\sqrt{2} - 1/n \qquad n \in \mathbb{N} \setminus \{0\}$

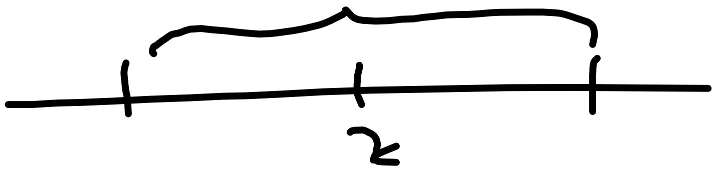
– $\mathbb{C}$

$|z - w| \leq \delta$

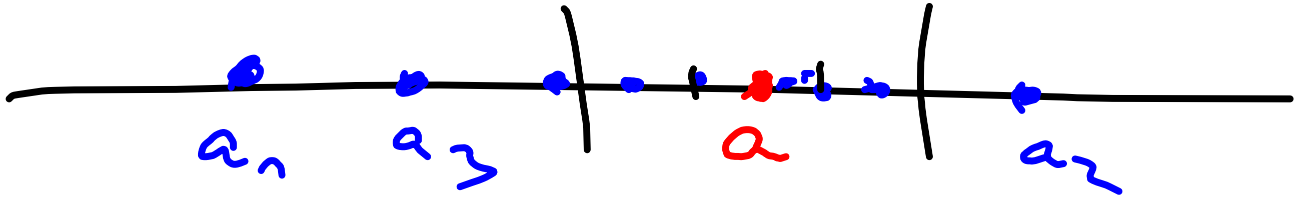


$\Rightarrow$ $|z|$ absolutní hodnota

– $\mathbb{R}$



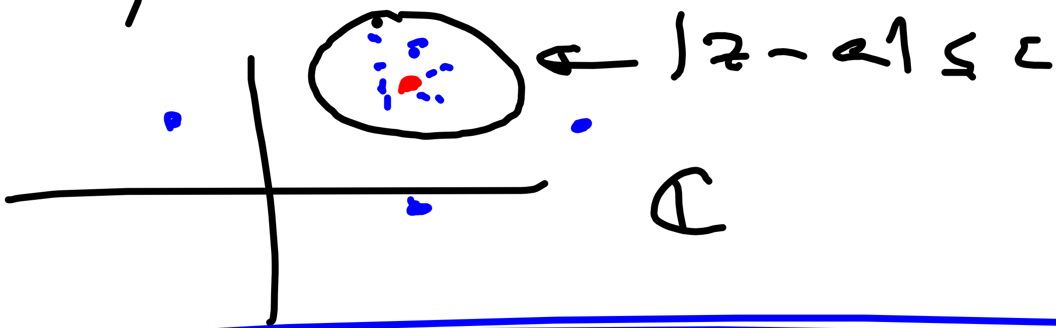
$|z - w| \leq \delta$



$\varepsilon > 0$

$\varepsilon \in \mathbb{R}$

$\forall \varepsilon$, $\forall a_i$ až na konečně mnoho "uvnitř"

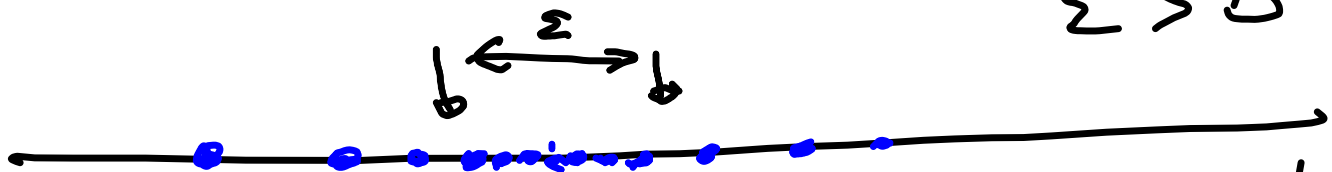


KONVERGENCE

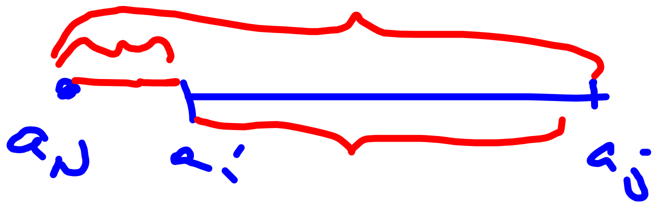
Cauchyovská posloupnost:

$\varepsilon \in \mathbb{R}$

$\varepsilon > 0$



$$|a_i - a_j| < \varepsilon \quad \forall\, i,j \geq n_\varepsilon \in \mathbb{N}$$



Hromadné body:

$a_1, a_2, \dots \in A \subset \mathbb{Q}, \mathbb{R}, \mathbb{C}$

a_i konvergují k $a \in \mathbb{Q}, \mathbb{C}, \mathbb{R}$

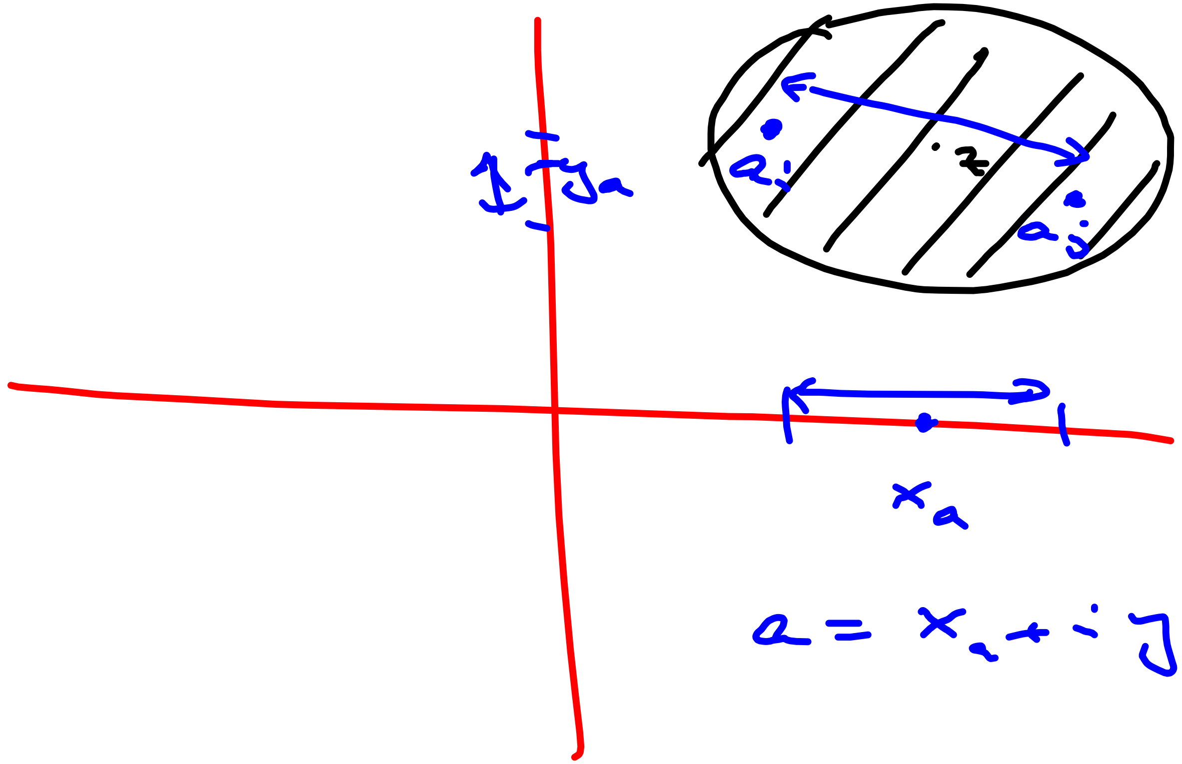
$\Rightarrow$ a hromadný bod množiny A

např. $A = \{\sqrt{2} - 1/n,\ n \in \mathbb{N} - \{0\}\}$

$\sqrt{2}$ je hromadný bod A v $\mathbb{R}$

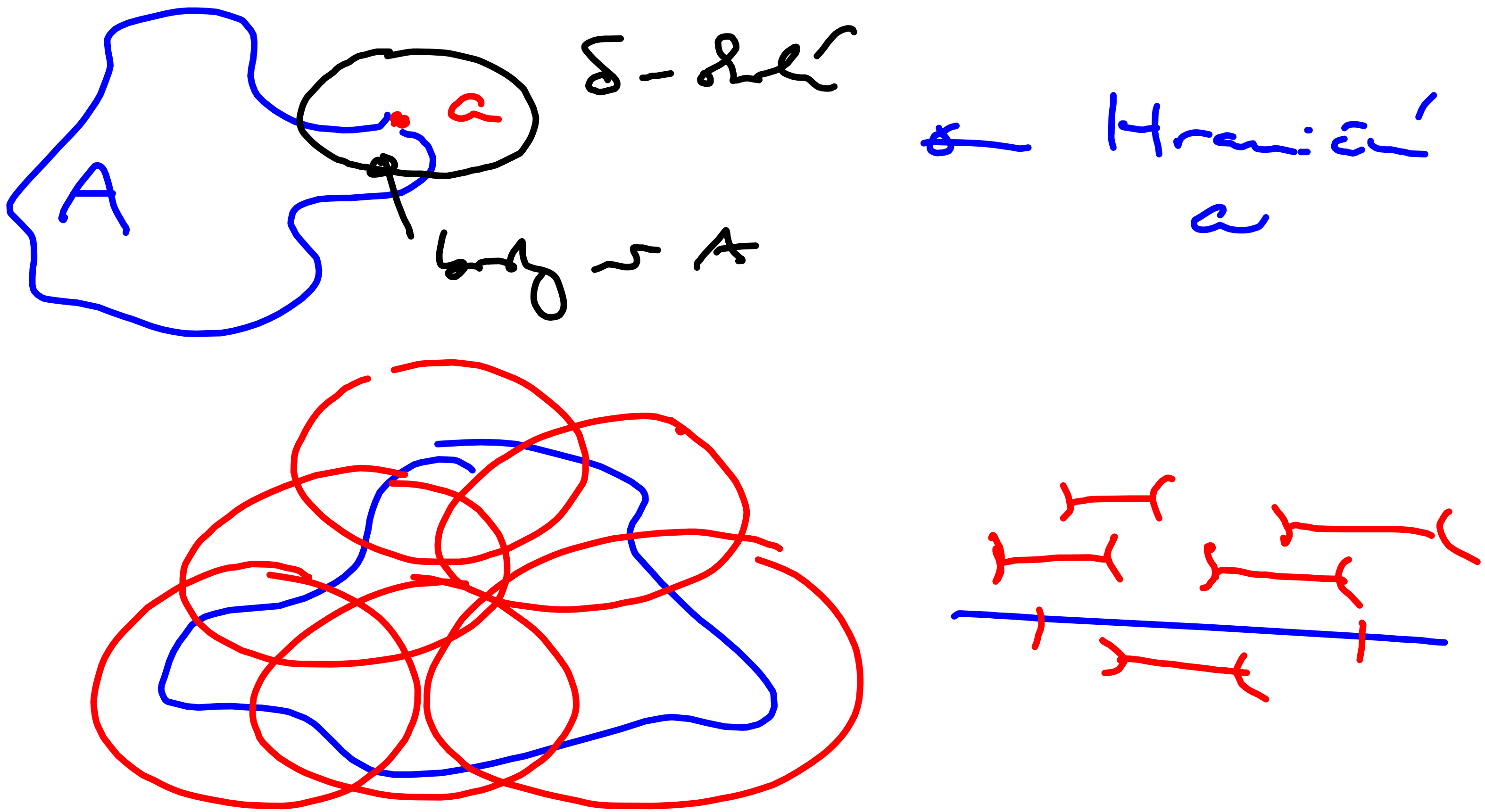
v $\mathbb{C}$

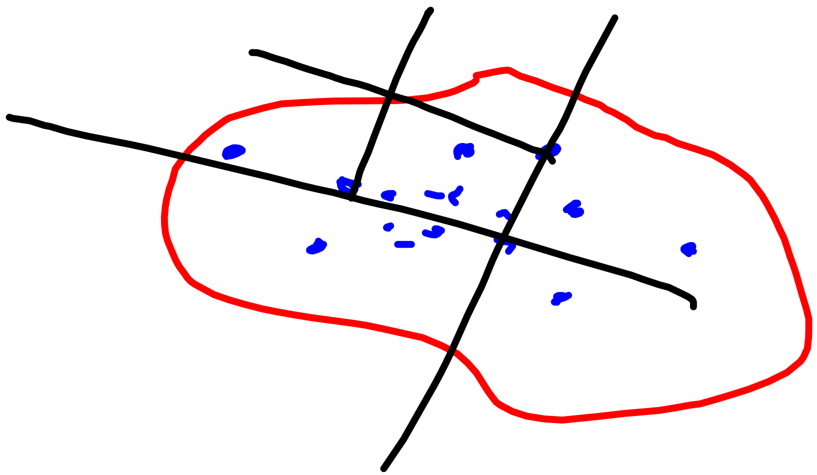
$|z-u| \le 2$

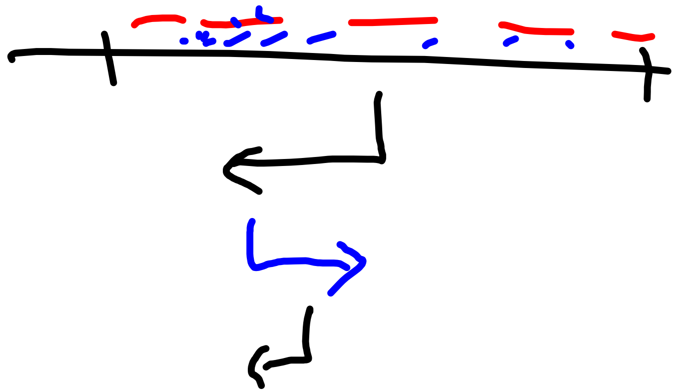


$a = x_a + i y_a \leftarrow$ limita

posloupnost v $\mathbb{C}$ konverguje $\Longleftrightarrow$ když re a_i a im a_i konvergují v $\mathbb{R}$.


A
a
δ-okolí
body z A
← Hraniční
a





$\to$ vybrané posl.

Cauchyovské

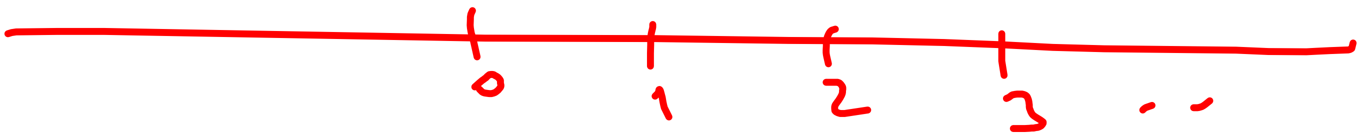
$\Rightarrow$ limita v $\mathbb{R}$

$A \subset \mathbb{R}$ $f: A \to \begin{cases} \mathbb{R} \\ \mathbb{C} \end{cases}$

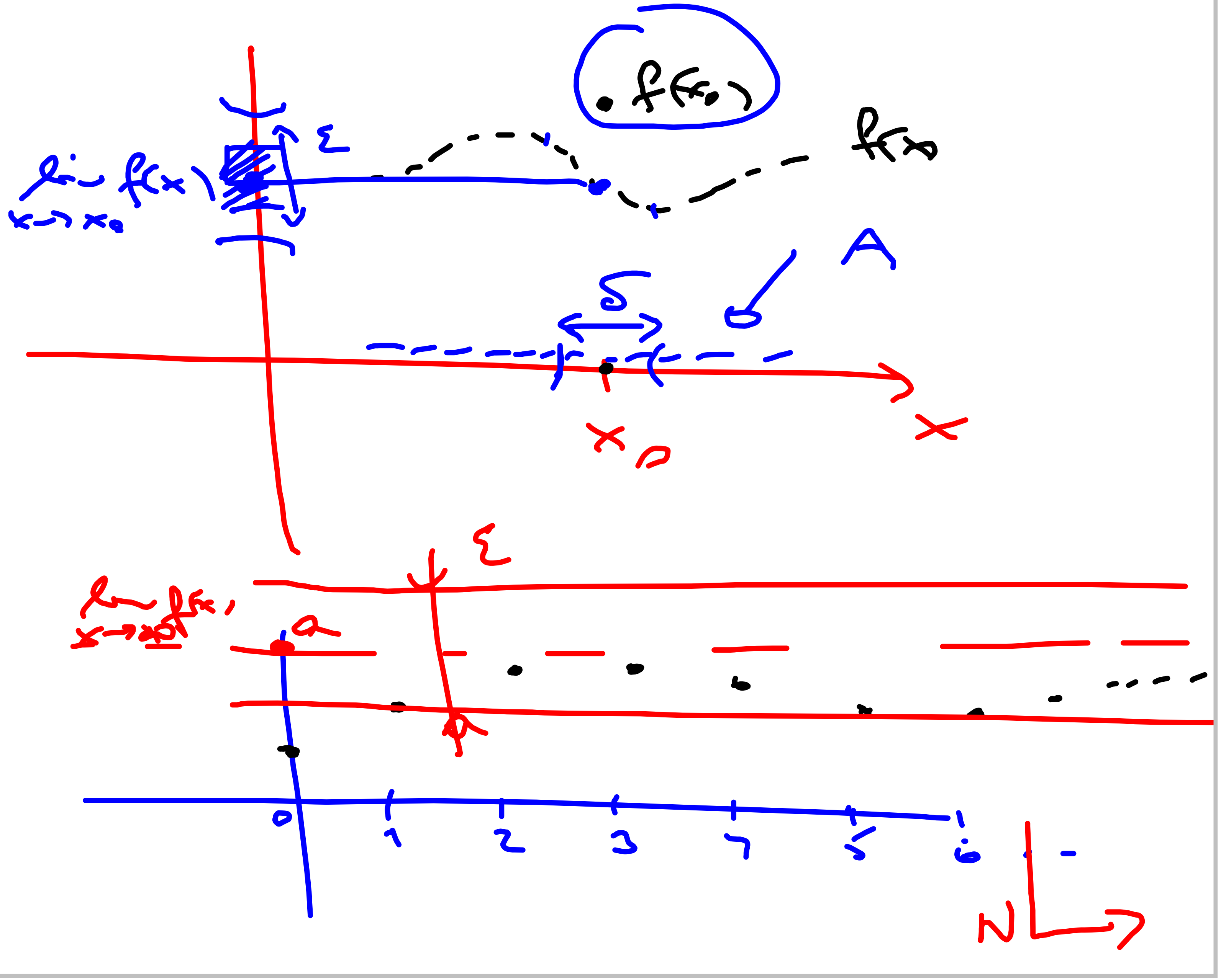
speciálně $A = \mathbb{N}$ dává opět posloupnosti

$f(0), f(1), \dots$

$a_0, a_1, \dots, a_i, \dots$ $i \to \infty$



$\lim_{x \to \infty} f(x)$ ← limita posloupnosti


lim f(x)
x→x₀
ε
f(x₀)
f(x)
A
δ
x₀
x
ε
lim f(x)
x→x₀
a
0
1
2
3
4
5
6
N

Příklad 1

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

ε-okolí 0 je $(-\varepsilon, \varepsilon)$ ~ k $N > \frac{1}{\varepsilon}$ platí

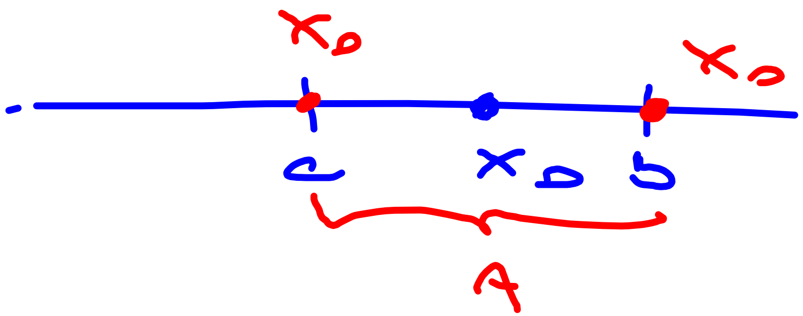
$\frac{1}{n} < \varepsilon$ pro $n > N$

$\lim\limits_{n\to\infty} a_n$ neexistuje !

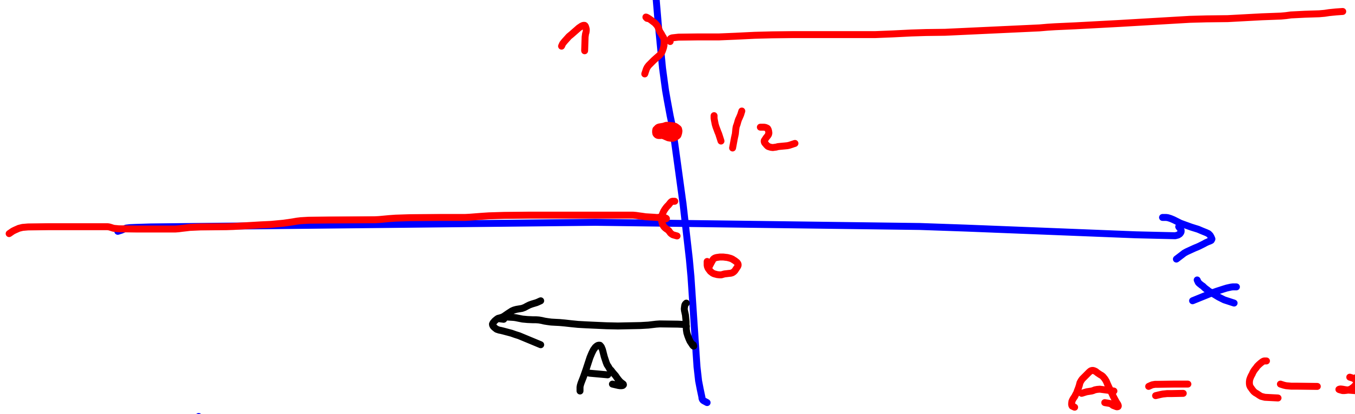
$a_n = \mathrm{sg}(n)$

$2n \quad 2n+1 \quad 2n+2 \quad \cdot \quad \cdot$

Příklad 2



$h(x)$:



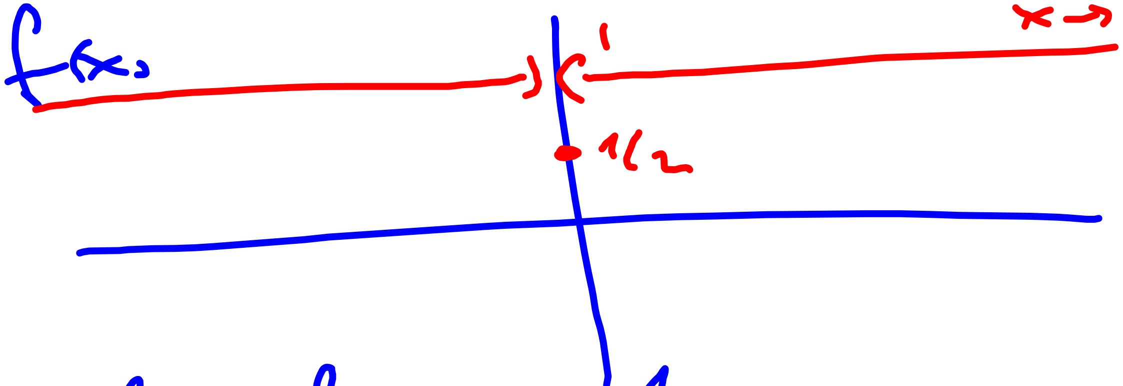
$\lim\limits_{x\to 0} h(x)$ neex.

$A = (-\infty, 0]$

$\lim\limits_{x\to 0^-} h(x) = 0$

$\lim\limits_{x\to 0^+} h(x) = 1$

$f(x)$



$\lim\limits_{x\to 0} f(x) = 1$

Příklad 5

$$\lim_{x\to x_0} f(x) = f(x_0)$$

$$f(x) = a_n x^n + \dots + a_0$$

$$f(x+\Delta x) = a_n (x+\Delta x)^n + a_{n-1}(x+\Delta x)^{n-1} + \dots$$

$$= a_n x^n + a_{n-1} x^{n-1} + \dots a_0$$

konečné [?]

$$\left\{\begin{aligned} &+ \Delta x \left(n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \dots a_1\right) \\ &+ \Delta x^2 \left(\dots \right) \\ &\ \ \vdots \\ &+ a_n \Delta x^n \end{aligned}\right\}$$

násobky [?] Δx změní [?] hodnotu [?] přesně [?] (!) limita [?]

PEVNÉ