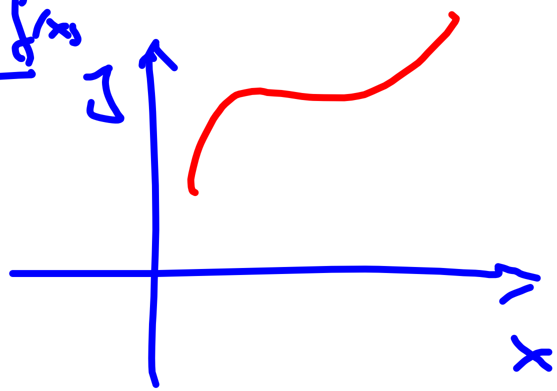
$$D : C^1(\mathbb{R}) \longrightarrow C(\mathbb{R}) \quad \text{lineární nad } \mathbb{R}$$

$$f \longmapsto f'$$

$$y = f(x)$$



$$f' \simeq \frac{\Delta y}{\Delta x} \simeq \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$y = f(x) \cdot g(x)$$

$$\Delta y = f(x+\Delta x)\, g(x+\Delta x) - f(x)\, g(x)$$

$$= f(x+\Delta x)\big(g(x+\Delta x) - g(x)\big)$$

$$+ \big(f(x+\Delta x) - f(x)\big)\, g(x)$$

$$\mathbb{R} \xrightarrow{f} \mathbb{R} \xrightarrow{h} \mathbb{R} \qquad y = f(x)$$

$$z = h(y) = h(f(x))$$

$$\frac{\Delta z}{\Delta x} = \frac{\Delta z}{\Delta y} \cdot \frac{\Delta y}{\Delta x}$$

$$h'(y) \cdot f'(x)$$

Jak vypadá důkaz doopravdy?

1) spojitost $f'(x_0)$ ex., tj. ex. $\lim\limits_{x\to x_0} \dfrac{f(x)-f(x_0)}{x-x_0}$

$$f(x) = \frac{f(x)-f(x_0)}{x-x_0} \cdot (x-x_0) + f(x_0)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\underbrace{f'(x_0) \cdot 0}_{=0} \qquad f(x_0)$$

$$\Rightarrow \lim_{x\to x_0} f(x) = f(x_0)$$

pro součin:

$$\frac{(f\cdot g)(x) - (f\cdot g)(x_0)}{x-x_0} = \overbrace{f(x)\cdot\frac{g(x)-\cancel{g(x_0)}}{x-x_0}} + \overbrace{+\frac{\cancel{f(x)}-f(x_0)}{x-x_0}\cdot g(x)}$$

$$\longrightarrow f(x_0)\cdot g'(x_0) + f'(x_0)\cdot g(x_0)$$

$$h(y) = h(y_0) + \varphi(y)(y-y_0), \qquad f(x_0)$$

$$f(x) = f(x_0) + \psi(x)(x-x_0)$$

$$\frac{h(f(x)) - h(f(x_0))}{x-x_0} = \varphi(f(x) - f(x_0)) = \frac{\varphi(f(x))\psi(x)\cancel{(x-x_0)}}{\cancel{(x-x_0)}}$$

$$\frac{f}{g} = f \cdot g^{-1}, \qquad x \overset{h}{\longmapsto} x^{-1} \qquad g^{-1} = h \circ g$$

$$(g^{-1})' = -g^{-2} \cdot g'$$

$$\left(\frac{f}{g}\right)' = f' \cdot g^{-1} + f \cdot (-1) g^{-2} g'$$

$$= \frac{f' \cdot g - g' \cdot f}{g^2}$$

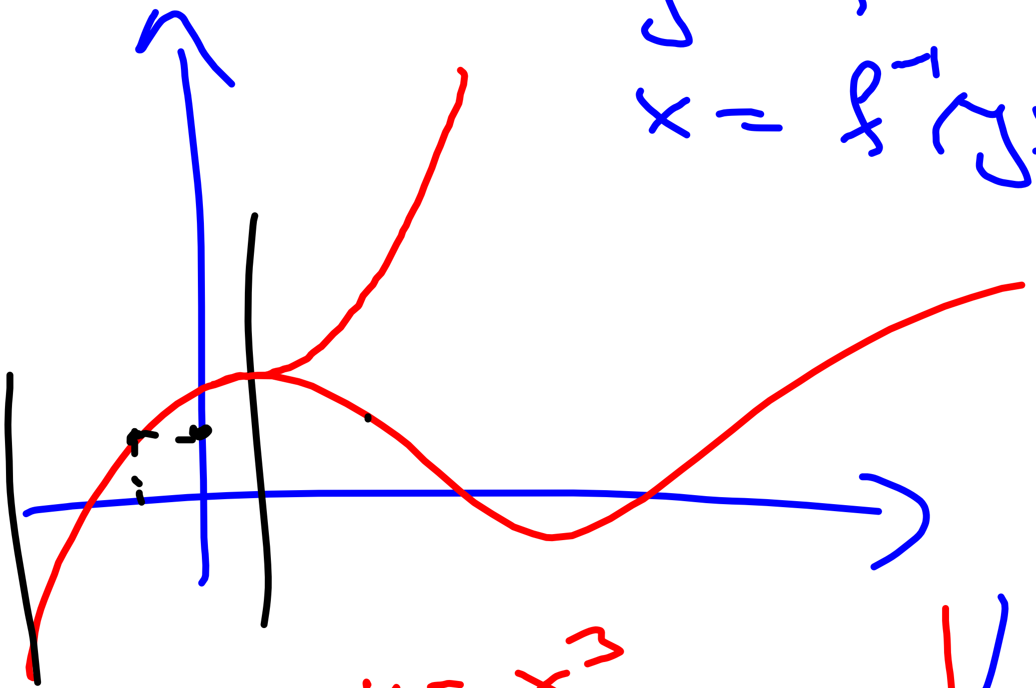
✓

$$x = f^{-1} \circ f(x) \quad /\,'$$

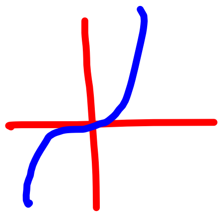
$$1 = (f^{-1})'(f(x)) \cdot f'(x) \implies (f^{-1})'(f(x_0)) = \frac{1}{f'(x_0)}$$

$y = f(x)$

$x = f^{-1}(y)$



$y = x^3$



$$f(x) = f(x_0) + \varphi(x)(x - x_0)$$

$$\varphi(x_0) = f'(x_0)$$

$$D : C^\infty(\mathbb{R}) \circlearrowleft$$

$$f \longmapsto f'$$

$$f', \quad f'', \quad \ldots \quad f^{\cancel{||||}\;|||}$$

$$f^{(8)}$$

a^b pro $a, b \in \mathbb{Q}$, $a > 0$

a^b pro $a, b \in \mathbb{R}$, $a > 0$

$$f(x) = x^2 + 1 = 0 \Rightarrow x_{1,2} = \pm i$$

$$f = (x-a_1)^{c_1} \cdots (x-a_q)^{c_q}$$

$$f' = c_1 (x-a_1)^{c_1 - 1} \cdots + \cdots$$

$c_1 = 1$:

$$= \boxed{1 \cdot \cancel{(x-a_1)}^{0}} (x-a_2) \cdots + (x-a_1)^1 \cdots$$

$x = a_1$:

$$= (a_1 - a_2) \cdots (a_1 - a_q) + 0 \neq 0$$

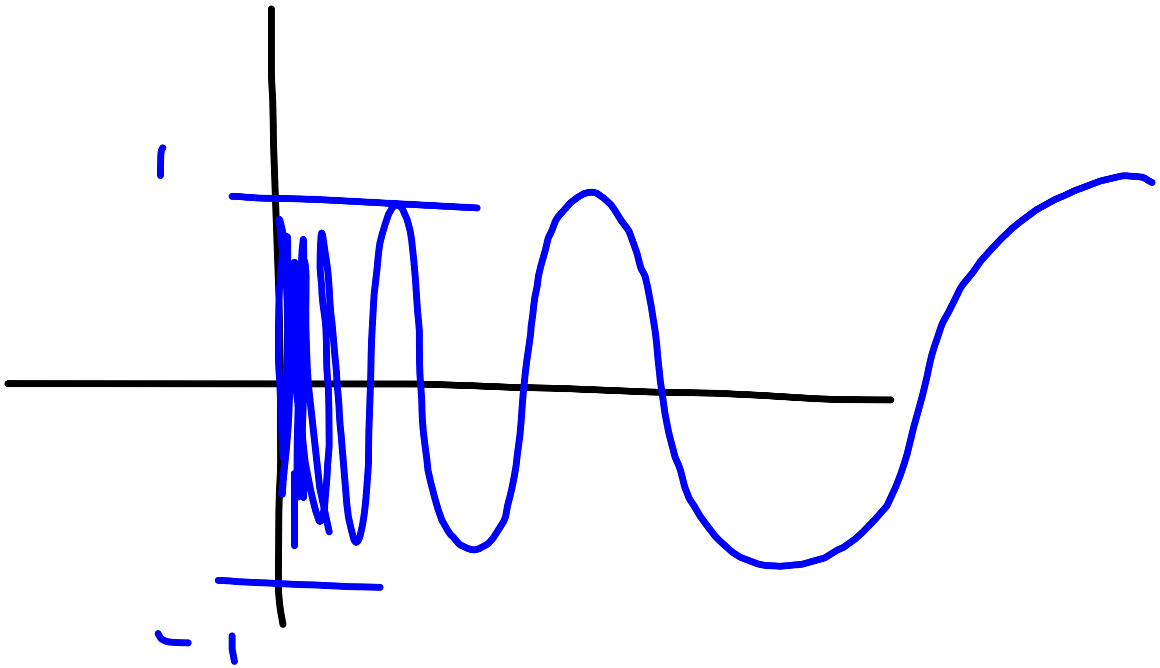
$(x^n)' = n x^{n-1}$ $\quad x \in \mathbb{R},\ n \in \mathbb{N}$

$x^{n+m} = x^n \cdot x^m$ $\quad$ pro $n, m \in \mathbb{Q}$

$y = g(x) = x^q$ $\quad g = h^{-1},\ h = g^{-1}$

$h'(g(x)) = \dfrac{1}{g'(x)}$ $\quad g'(x) = q\, x^{q-1}$

$f(x) = \sin\left(\frac{1}{x}\right)$



$$(1+b)^n = 1^n + nb + \underbrace{\binom{n}{2} b^2 + \dots}_{>0}$$

$$\boxed{a_n = \left(1 + \frac{1}{n}\right)^n}$$

$$\frac{a_n}{a_{n-1}} = \frac{\left(1+\frac{1}{n}\right)^n}{\left(1+\frac{1}{n-1}\right)^{n-1}} > 1$$

$$a_n > a_{n-1} > a_{n-2} \dots$$

$$b_n = \underbrace{\left(1+\frac{1}{n}\right)}_{\substack{n\to\infty \\ \to 1}} a_n \Rightarrow \lim_{n\to\infty} b_n = \lim_{n\to\infty} a_n$$

$$\frac{e^x - 1}{x} \to 1$$

$$e^{\ln x} = x$$

$$\underline{e^{\ln x + \ln y}} = e^{\ln x} \cdot e^{\ln y} = x \cdot y$$

$$= \underline{e^{\ln (x \cdot y)}}$$

$$\underline{e^{y \cdot \ln x}} = \left(e^{\ln x}\right)^{y} = x^{y} = \underline{e^{\ln x^{y}}}$$

$$e^{a \ln x} (a \ln x)' = x^{a} \cdot a \cdot \frac{1}{x} = a \cdot x^{a-1}$$