

$$x + 2y + 5z = 22$$

$$x, y, z \in \mathbb{F}_0$$

$$x \in \{0, 1, 2, 3, 4\}$$

$$0 \leq y \leq 5$$

$$0 \leq z \leq 3$$

$$x^{22} : \quad \underbrace{x^1 \cdot x^6 \cdot x^{15}}_{(1, 6, 15)} + x^3 \cdot x^4 \cdot x^{15} +$$
$$+ x^2 \cdot x^{10} \cdot x^{10} + x^4 \cdot x^8 \cdot x^{10}$$

$(2, 10, 10)$ $(4, 8, 10)$

$$\frac{1}{1-x} \cdot \frac{1}{1-x^2} \cdot \frac{1}{1-x^3} \cdot \frac{1}{1-x^4} \cdot \frac{1}{1-x^5} \cdot \frac{1}{1-x^6}$$

koef. u x^{100} v rozvoji
do Taylorovy řady

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$$

$$\underbrace{(1+x)(1+x)\dots(1+x)}_n = \binom{n}{0} + \binom{n}{1}x + \dots + \binom{n}{n}x^n$$

$$n x^k \rightarrow \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$(1+x)^n$ vytváří posloupnost $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \dots$
 $, \binom{n}{n}, 0, \dots$

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n$$

derivative:

$$n(1+x)^{n-1} = \binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + \dots + n\binom{n}{n}x^{n-1}$$

dosadím $x=1$:

$$n \cdot 2^{n-1} = \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \dots + n\binom{n}{n}$$

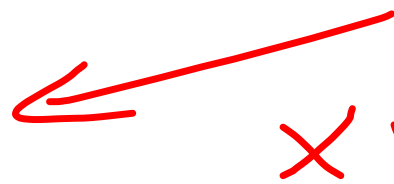
$$\sum_{k=0}^n k \binom{n}{k} = n 2^{n-1}$$

$(1, 1, 1, 1, \dots)$

$$1 + 1 \cdot x + 1 \cdot x^2 + 1 \cdot x^3 + \dots$$

konverguje pro $x \in (-1, 1)$ a součet je

$$\frac{1}{1-x}$$



$x \mapsto$

$$\frac{1}{1-x}$$

obrácení :

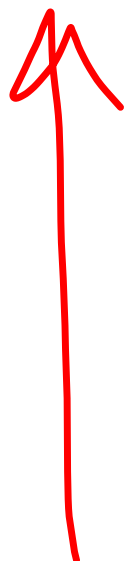
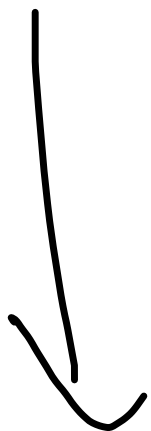
$\frac{1}{1-x}$
Přijímá

rozvíjíme do Třady

$$\sum_{n=0}^{\infty} f^{(n)}(0) \cdot \frac{x^n}{n!}$$

po sloupnost

mol.
řada



rozvoj
do řady

vytv. fce

obvyčejná vytvořující funkce

$$\sum_{n=0}^{\infty} a_n x^n$$

exponenciální vytv. fce

$$\sum_{n=0}^{\infty} a_n \frac{x^n}{n!}$$

$$\begin{aligned}
 & (a_0, a_1, a_2, \dots) \\
 \sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} b_n x^n &= \sum_{n=0}^{\infty} (a_n + b_n) x^n \\
 \sum_{n=0}^{\infty} a_n x^n &= \sum_{n=0}^{\infty} (\alpha a_n) x^n \\
 x^k \cdot \sum_{n=0}^{\infty} a_n x^n &= \sum_{n=0}^{\infty} a_n x^{n+k} \\
 & (0, 0, \dots, 0, a_0, a_1, \dots)
 \end{aligned}$$

$$a(x) = a_0 + a_1x + a_2x^2 + \dots$$

← 2 místa:

$$\frac{a(x) - a_0 - a_1(x)}{x^2}$$

$$a_0 + a_1x + a_2x^2 + \dots$$

$$x \leftarrow 2x$$

$$a_0 + a_1(2x) + a_2(4x^2) + \dots$$

$$(a_0, 2a_1, 4a_2, 8a_3)$$

$$x \leftarrow x^2$$

$$a_0 + a_1x^2 + a_2x^4 + \dots$$

$$(a_0, 0, a_1, 0, a_2)$$

$$\begin{array}{l}
 \frac{1}{1-x} \xrightarrow{\text{ogf}} (1, 1, 1, \dots) \\
 \frac{1}{1-2x} \xrightarrow{\text{ogf}} (1, 2, 4, 8, \dots) \\
 a(x) \xrightarrow{\text{ogf}} (a_0, a_1, a_2, \dots) \\
 a(-x) \xrightarrow{\text{ogf}} (a_0, -a_1, a_2, -a_3, \dots) \\
 \hline
 \frac{a(x) + a(-x)}{2} \xrightarrow{\text{ogf}} (a_0, 0, a_2, 0, a_4, \dots)
 \end{array}$$

$$\frac{1}{1-2x} \xrightarrow{\text{ogf}} (1, 2, 4, 8, \dots)$$

$$\frac{1}{1-2x^2} \xrightarrow{\text{ogf}} (1, 0, 2, 0, 4, 0, 8, 0, \dots)$$

$$\frac{x}{1-2x^2} \xrightarrow{\text{ogf}} (0, 1, 0, 2, 0, 4, 0, 8, \dots)$$

$$\frac{x+1}{1-2x^2} \xrightarrow{\text{ogf}} (1, 1, 2, 2, 4, 4, 8, 8, \dots)$$

$$a(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$a'(x) = \sum_{n=1}^{\infty} n \cdot a_n \cdot x^{n-1}$$

$$(a_0, a_1, a_2, a_3, \dots)$$



$$(a_1, 2a_2, 3a_3, \dots)$$

$$\begin{aligned}
 a(x) &= \sum_{n=0}^{\infty} a_n x^n \\
 \int_0^z a(x) dx &= \int_0^z \sum_{n=0}^{\infty} a_n x^n dx \\
 &= \sum_{n=0}^{\infty} \int_0^z a_n x^n dx \\
 &= \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}
 \end{aligned}$$

$$a(x) = \sum_{n=0}^r a_n x^n \quad b(x) = \sum_{n=0}^s b_n x^n$$

$$(a(x)) \cdot (b(x)) = \left(\sum a_n x^n \right) \left(\sum b_n x^n \right) = \\ = \sum_{n=0}^{\infty} c_n x^n$$

$$c_n = \sum_{k=0}^n a_k b_{n-k}$$

$$\frac{1}{1-x} \longleftrightarrow (1, 1, 1, \dots)$$

$$\left(\frac{1}{1-x}\right)' \longleftrightarrow (1, 2, 3, \dots)$$

$$\left(\frac{1}{1-x}\right)'' = \left(\frac{1}{(1-x)^2}\right)' = +2 \cdot (1-x)^{-3} =$$

$$= \frac{2}{(1-x)^3}$$

\longleftrightarrow

$$\begin{array}{l} (1 \cdot 2, 2 \cdot 3, 3 \cdot 4, \dots) \\ \text{nové} \quad (1 \cdot 1, 2 \cdot 1, 3 \cdot 1, \dots) \\ \hline (1^2, 2^2, 3^2, \dots) \end{array}$$

$$\lim_{x \rightarrow 0} \frac{1}{1-x} \xrightarrow{\text{Taylor}} \left(0, \frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \frac{1}{5!}, \dots \right)$$

$$e^x \xrightarrow{\text{Taylor}} \left(\frac{1}{0!}, \frac{x}{1!}, \frac{x^2}{2!}, \frac{x^3}{3!}, \dots \right)$$

$$\binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{k!}$$

$$\begin{aligned} (1-x)^{-n} &= \sum_{k=0}^{\infty} \binom{-n}{k} (-x)^k \\ &= \sum_{k=0}^{\infty} \underbrace{\binom{-n}{k}}_{\frac{(-n)(-n-1)\dots(-n-k+1)}{k!}} (-1)^k x^k \\ &= \sum_{k=0}^{\infty} \frac{(-n)(-n-1)\dots(-n-k+1)}{k!} \cdot (-1)^k x^k \end{aligned}$$

$$\frac{(n+k-1)(n+k-2)\dots n}{k!} =$$

$$= \binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

Pr:

$$\frac{1}{(1-x)^2} = \binom{1}{1} + \binom{2}{1}x + \binom{3}{1}x^2 + \binom{4}{1}x^3 + \dots$$

FIBONACCI

$$F_0 = 0$$
$$F_1 = 1$$

$$F_{n+2} = F_{n+1} + F_n$$

$$F(x) \longleftrightarrow (F_0, F_1, F_2, F_3, \dots)$$

$$F(x) \longleftrightarrow (0, 1, 1, 2, 3, 5, 8, \dots)$$

$$x \cdot F(x) \longleftrightarrow (0, 0, 1, 1, 2, 3, 5, \dots)$$

$$(x^2 + x) \cdot F(x) \longleftrightarrow (0, 1, 2, 3, 5, 8, 13, \dots)$$

$$(x^2 + x)F(x) = F(x) - x \cdot \frac{F(x) - 0}{x} - 1$$

$$\frac{x}{1-x-x^2} = \frac{A}{x-x_1} + \frac{B}{x-x_2}$$

$$x^2+x-1$$

$$x_{1,2} = \frac{-1 \pm \sqrt{5}}{2} \quad \left| \begin{array}{l} \frac{-1+\sqrt{5}}{2} \\ \frac{-1-\sqrt{5}}{2} \end{array} \right.$$

$$-x = A(x-x_2) + B(x-x_1)$$

$$-x+0 = x(A+B) + (-Ax_2 - Bx_1)$$

$$-1 = A+B$$

$$0 = +Ax_2 + Bx_1$$

} \Rightarrow spočítáme x, B

$$x_1 = \frac{1}{x_1} \quad x_2 = \frac{1}{x_2}$$

$$\frac{x}{1-x-x^2} = \frac{A}{x-x_1} + \frac{B}{x-x_2} =$$

$$= \frac{\left(\frac{A}{-x_1}\right)}{\frac{x-x_1}{-x_1}} + \dots = \frac{\left(\frac{A}{x_1}\right)}{1 - \frac{x}{x_1}} + \dots$$

$x_1 x$

$$\frac{a}{1-\lambda_1 x} \rightarrow (a \cdot x_1^0, a \cdot x_1^1, a \cdot x_1^2, \dots)$$

$$(1-x)^{-1} \leftrightarrow (1, 1, 1, \dots)$$

Podobně

$$\frac{b}{1 - \lambda_2 x} \iff (b \cdot \lambda_2^0, b \cdot \lambda_2^1, \dots)$$

$$\frac{x}{1 - x - x^2} \iff (a \lambda_1^0 + b \lambda_2^0, a \lambda_1^1 + b \lambda_2^1, \dots)$$

$$\stackrel{=}{=} F(x)$$

$$\implies F_n = a \cdot \lambda_1^n + b \cdot \lambda_2^n$$

$$\frac{x}{1-x-x^2} = \frac{a}{1-\lambda_1 x} + \frac{b}{1-\lambda_2 x}$$

$$(1-\lambda_1 x)(1-\lambda_2 x) = 1 - (\lambda_1 + \lambda_2)x + \lambda_1 \lambda_2 x^2$$

$1-x-x^2$ má kořeny x_1, x_2 $x_1 + x_2 = -1$
 $x_1 x_2 = -1$

$$\frac{1-x-x^2}{x^2+x-1}$$

$$\lambda_1 + \lambda_2 = \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 x_2} = 1$$

$$\lambda_1 \cdot \lambda_2 = \frac{1}{x_1} \cdot \frac{1}{x_2} = \frac{1}{x_1 x_2} = -1$$

Vynásobíme $1-x-x^2$:

$$x = a(1-\lambda_2 x) + b(1-\lambda_1 x)$$

$$1 = -a \cdot \lambda_2 - b \cdot \lambda_1$$

$$0 = a + b$$

$$x_1 = \frac{-1 + \sqrt{5}}{2}$$
$$x_2 = \frac{-1 - \sqrt{5}}{2}$$

$$\lambda_1 = \frac{2}{-1 + \sqrt{5}}$$

$$\lambda_2 = \frac{2}{-1 - \sqrt{5}}$$

$$1 = -a \left(\frac{2}{-1 - \sqrt{5}} \right) - b \left(\frac{2}{-1 + \sqrt{5}} \right)$$

$$a = -b$$

$$1 = b \left(\frac{2}{-1 - \sqrt{5}} \right) - b \cdot \frac{2}{-1 + \sqrt{5}} \Rightarrow \frac{1}{2} = b \left(\frac{2\sqrt{5}}{(\sqrt{5}+2)(\sqrt{5}-2)} \right) =$$
$$\Rightarrow b = \frac{\sqrt{5}}{5}$$