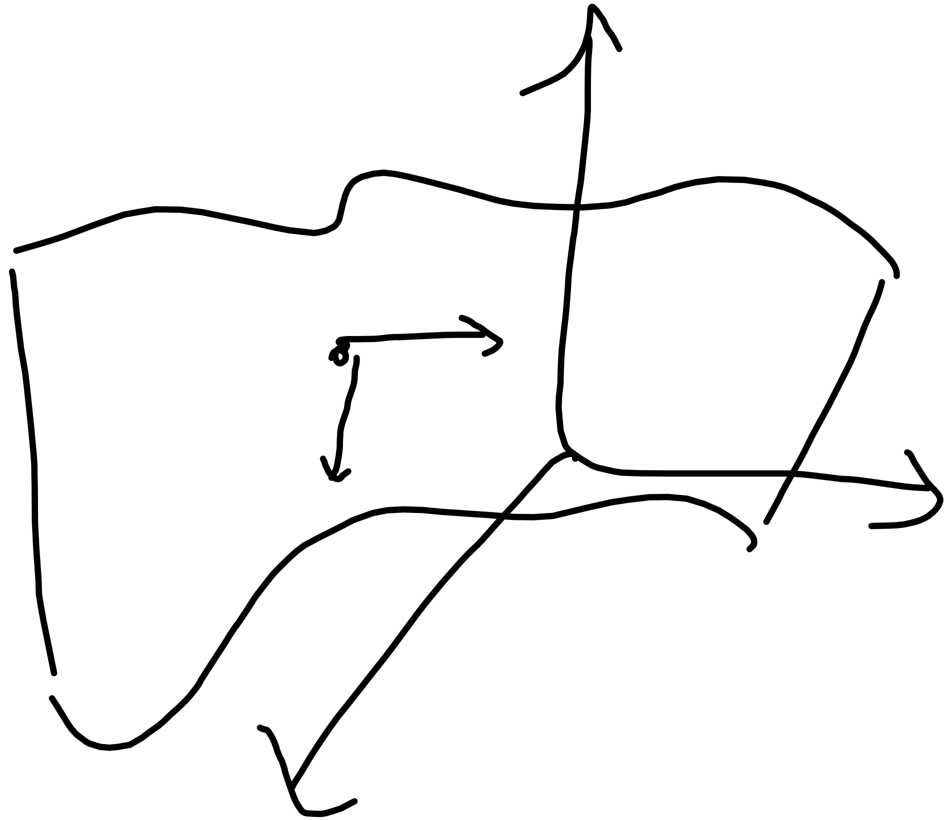
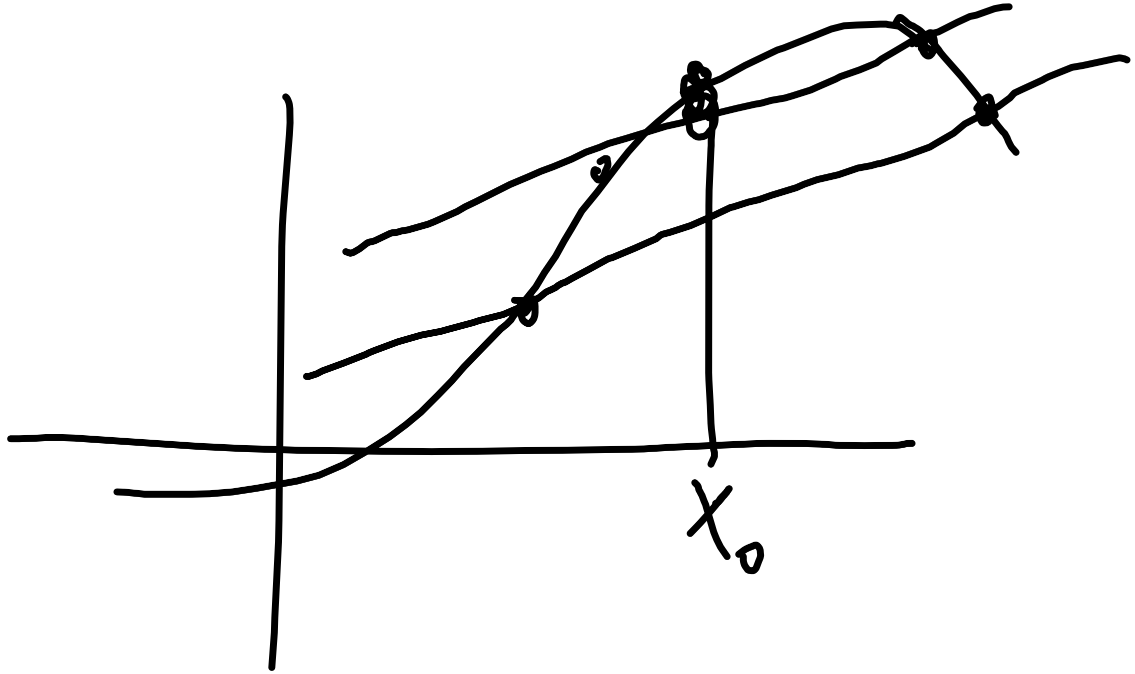


$f : \mathbb{R} \to \mathbb{R}$



$$y = k \cdot x + q$$

směrnice

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

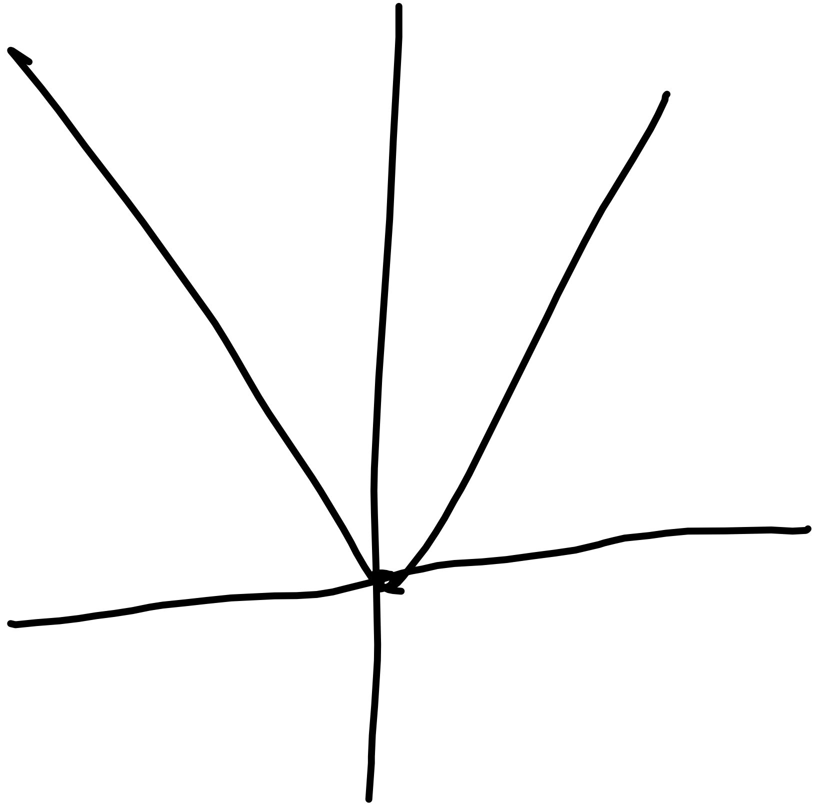
$f(x,y) = \cos x \cdot \sin y$

fix

$$\frac{\partial}{\partial x} f(x,y) = -\sin x \cdot \sin y$$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{t \to 0} \frac{f(x_0 + t) - f(x_0)}{t}$$

$$= \lim_{t \to 0} \frac{1}{t} \big(f(x_0 + t) - f(x_0) \big)$$



$y = |x|$

$\not\Leftarrow$

pro $n = 1$: difer. $\Rightarrow$ spoj.

$$f(x,y) = \begin{cases} 1 & x=0 \vee y=0 \\ 0 & \text{jinak} \end{cases}$$

$$f'_x(0,0) = \lim_{t \to 0} \frac{f(0+t,0) - f(0,0)}{t} =$$

$$= \lim_{t \to 0} \frac{0}{t} = 0$$

$$\lim_{(x,y) \to (0,0)} f(x,y) \cancel{=} \text{neexistuje}$$

$$\varphi(t) = f(\underset{\substack{\cap \\ E_m}}{x} + t \cdot \underset{\substack{\cap \\ \mathbb{R}^n}}{v})$$

$$\varphi : \mathbb{R} \to E_m \to \mathbb{R}$$

$$\varphi_0'(t) = \lim_{t \to 0} \frac{1}{t}(\varphi(t) - \varphi(0))$$

$$\lim \frac{1}{t}(f(x + t \cdot v) - f(x))$$

$$v = (1, 0, \cdots, 0)$$

$$x + tv = (x_1 + t, x_2, \dots, x_n)$$

$$\lim_{t \to 0} \frac{f(x_0 + tv) - f(x_0)}{t} = f'_{x_1}(x_0)$$

$$d_{k \cdot v} = k \cdot d_v$$

$$d_v(f \cdot g) = d_v f \cdot g + f \cdot d_v g$$

$$\lim_{t \to 0} \frac{f(x + t \cdot \vec{v}) - f(x)}{t} =$$

$$\left[v = (v_1, v_2) \right] =$$

$$= \lim_{t \to 0} \frac{1}{t} \cdot \left(f(t \cdot v_1, t \cdot v_2) - 0 \right) =$$

$$= \lim_{t \to 0} \frac{1}{t} \cdot \frac{t^4 v_1^4 \cdot t^2 v_2^2}{t^8 v_1^8 + t^4 v_2^4} = \lim_{t \to 0}$$

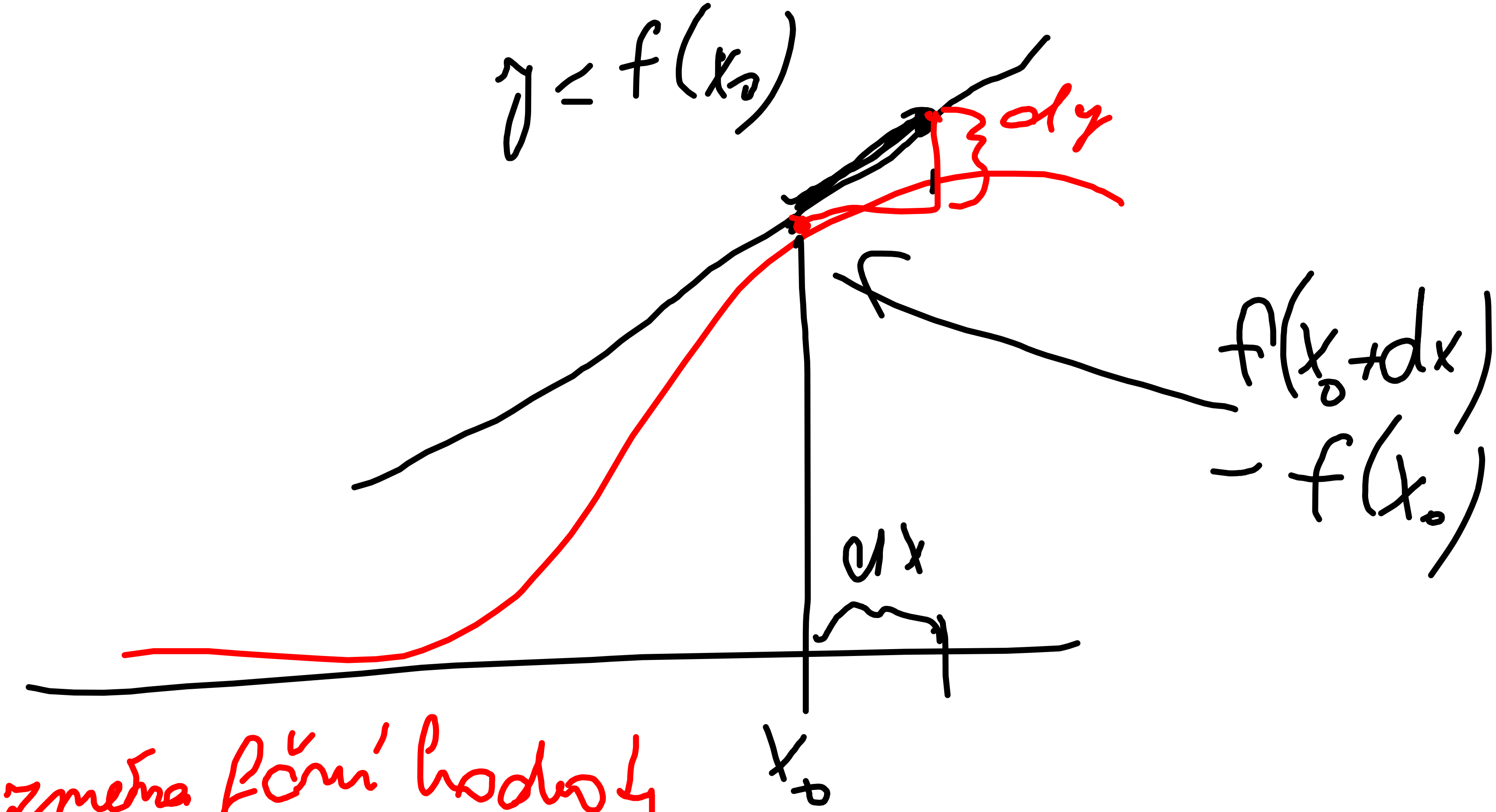
$$\lim_{t \to 0} \frac{1}{1} \cdot \frac{t\, v_1^4 \cdot v_2^2}{t^4 v_1^8 + v_2^4} = \frac{0}{v_2^4} = 0$$

není spojitá! pro $[x,y]=[x,k\cdot x^2]$

$$\lim_{x\to 0}\frac{x^4\cdot k^2x^4}{x^8+k^4x^8}=\lim_{x\to 0}\frac{k^2}{1+k^4}=\frac{k^2}{1+k^4}$$

$\Rightarrow$ neex. limita

v $[0,0]$



změna funkční hodnoty na tečně

$$dy = f'(x_0) \cdot dx$$

$$f(x_0+h) - f(x_0) = A \cdot h + \tau(h)$$

z def. $\lim\limits_{h \to 0} \frac{\tau(h)}{h} = 0$ chyba aprox.

$$\frac{\tau(h)}{h} = \frac{f(x_0+h) - f(x_0) - A \cdot h}{h}$$

$$\lim \frac{\tau(h)}{h} = \lim \text{—}\,''\,\text{—} = 0$$

$f(x) \to 0$ $g(x)$ ohran.

$\Rightarrow f(x) \cdot g(x) \to 0$

$\frac{\tau(h)}{h} \to 0$ h ohran.

$\Downarrow$

$\tau(h) \to 0$

$$f(x+h)-f(x)=a\cdot h+\tau(h)$$

$$a\cdot h = d_h f(x)$$

$$\lim_{h\to 0}\tau(h)=0$$

$$\lim_{h\to 0}a\cdot h=0$$

$$df(x) : \mathbb{R}^n \to \mathbb{R}$$

$$v \longmapsto d_v f(x)$$

$$v \longmapsto a \cdot v$$

$$df(x)(t\cdot v) =$$

$$= t\cdot df(x)(v)$$

$$\lim_{t\to 0} \frac{\tau(tv)}{t} = \lim_{t\to 0} \frac{\tau(t\cdot v)}{\|tv\|}\cdot\|v\|$$

$$= \|v\|\cdot\lim \dots = 0$$

$$f'_{x_1}(x) = d_v f(x)$$

$$v = (1, 0, \dots, 0)$$

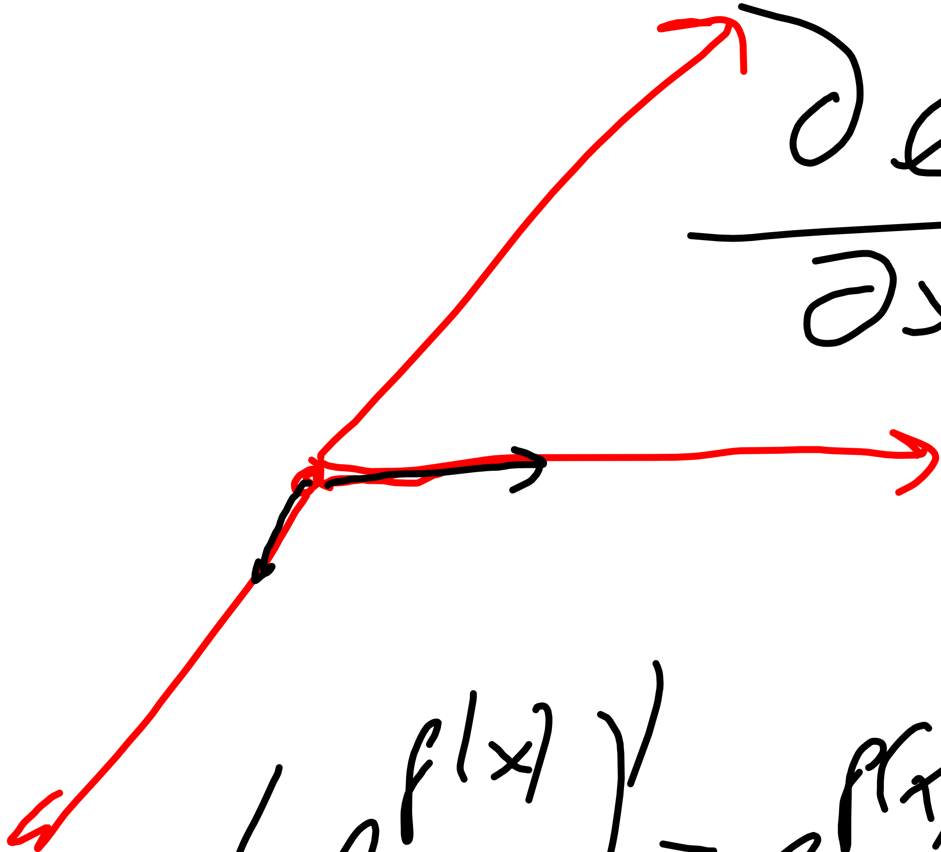
$$d_v f(x) = a \cdot v = a_1$$

$$\sum a_i \cdot v_i$$

$$f'(x) = (f'_{x_1}(x), f'_{x_2}(x), \dots) = (a_1, a_2, \dots) = a$$

$$df = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) \cdot (dx, dy)$$

$$v \longmapsto a \cdot v$$



$$\frac{\partial e^{x^3+y}}{\partial x} = e^{x^3+y} \cdot 3x^2$$

$$\frac{\partial e^{x^3+y}}{\partial y} = e^{x^3+y}$$

$$\left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x)$$

$$f(x_0, y_0) + df(\underbrace{dx}_{x - x_0}, \underbrace{dy}_{y - y_0})$$

$$df = \left(\frac{\partial f}{\partial x}\ \frac{\partial f}{\partial y}\right)$$

$$\boxed{\frac{\partial^2 f}{\partial x_i \partial x_j} \stackrel{?}{=} \frac{\partial^2 f}{\partial x_j \partial x_i}}$$

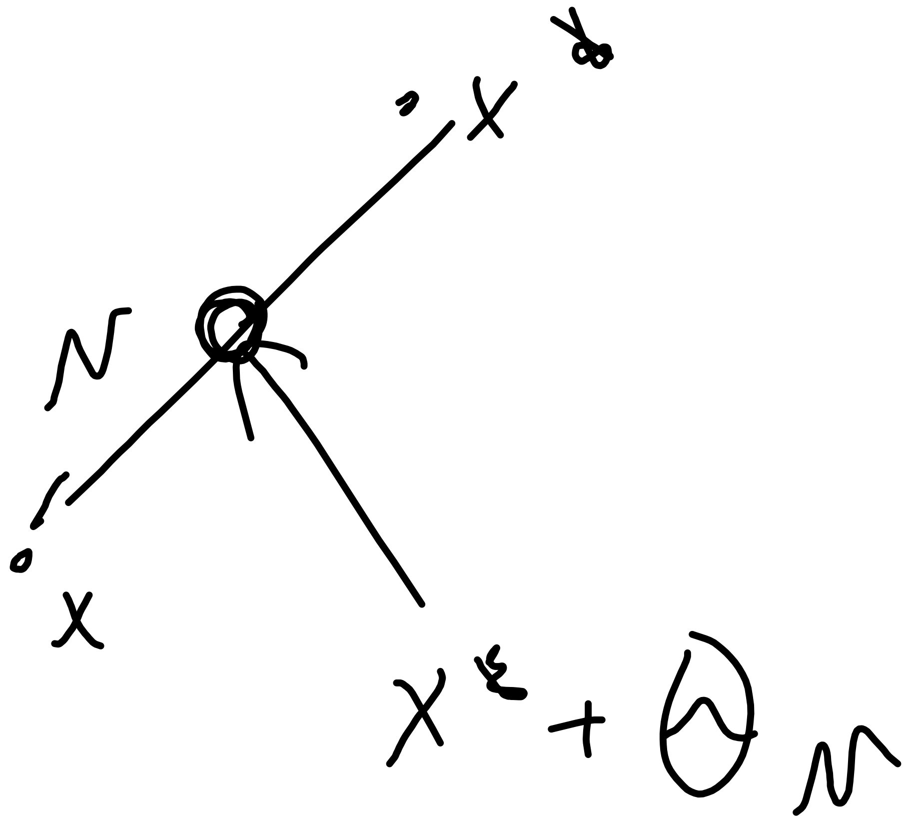
$$\frac{\partial^2 f}{\partial x^2}$$

$$f(x_0+h) < f(x_0) + f'(x_0)\cdot h + \frac{1}{2}\cdot f''(x_0)\cdot h^2 + \frac{1}{6} f'''(x_0)\cdot h^3$$

$\uparrow$

diference

$$f(x_0+h) \approx \sum_{k=0}^{m} \frac{1}{k!} f^{(k)}(x_0)\cdot h^k$$


X
N
X

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)(x+y) \cdot \ldots \cdot (x+y)$$

$$\left(\frac{\partial}{\partial x}\cdot\xi+\frac{\partial}{\partial y}\eta\right)^2=$$

$$=\frac{\partial^2}{\partial x^2}\cdot\xi^2+2\frac{\partial}{\partial x\partial y}\xi\eta\;+$$

$$+\frac{\partial^2}{\partial y^2}\cdot\eta^2$$

$$Hf(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(\xi, \eta) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \xi \\ \eta \end{pmatrix} =$$

$$= \eta^2$$

$$\approx 1 - 0{,}02 - \frac{1}{2} 0{,}02^2 =$$

$$= 0{,}9803$$