$$x + 2y + 5z = 22$$

$$x, y, z \in \mathbb{N}_0 \qquad x \in \{0, 1, 2, 3, 4\}$$
$$0 \le y \le 5$$
$$0 \le z \le 3$$

$(1, 6, 15)$ $(3, 4, 15)$

$$x^{22}: \quad x^1 \cdot x^6 \cdot x^{15} + x^3 \cdot x^4 \cdot x^{15} +$$
$$+ \quad x^2 \cdot x^{10} \cdot x^{10} + x^4 \cdot x^8 \cdot x^{10}$$

$(2, 10, 10)$ $(4, 8, 10)$

$$\frac{1}{1-x}\cdot\frac{1}{1-x^2}\cdot\frac{1}{1-x^5}\cdot\frac{1}{1-x^{10}}\cdot\frac{1}{1-x^{20}}\cdot\frac{1}{1-x^{50}}$$

koef. u x^{100} v rozvoji do Taylorovy řady

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$$\underbrace{(1+x)(1+x)\dots(1+x)}_{n} = \binom{n}{0} + \binom{n}{1} x + \dots + \binom{n}{n} x^n$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

n x^k:

$(1+x)^n$ vytváří posloupnost $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \dots, \binom{n}{n}, 0, \dots$

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n$$

derivace:

$$n(1+x)^{n-1} = \binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + \dots + n\binom{n}{n}x^{n-1}$$

dosadím $x=1$:

$$n\cdot 2^{n-1} = \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \dots + n\cdot\binom{n}{n}$$

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$$

$(1,1,1,1,1,\dots)$

$1 + 1\cdot x + 1\cdot x^2 + 1\cdot x^3 + \dots$

konverguje pro $x \in (-1,1)$ a součet je

$\frac{1}{1-x}$ $\qquad x \longmapsto \frac{1}{1-x}$

obráceně:

$\frac{1}{1-x}$ rozvineme do T. řady $\sum_{n=0}^{\infty} f^{(n)}(0)\cdot\frac{x^n}{n!}$

$\mathcal{D}$:

posloupnost

moc. řada

rozvoj do řady

vytv. fce

obyčejná vytvořující funkce

$$\sum_{n=0}^{\infty} a_n x^n$$

exponenciální vytv. fce

$$\sum_{n=0}^{\infty} a_n \frac{x^n}{n!}$$

$(a_0, a_1, a_2, \ldots)$

$$\sum_{n=0}^{\infty} a_n x^n + \sum b_n x^n = \sum (a_n + b_n) x^n$$

$$\alpha \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} (\alpha a_n) x^n$$

$$x^k \cdot \sum a_n x^n = \sum_{n=0}^{\infty} a_n x^{n+k}$$

$$(\underbrace{0, 0, 0, \ldots, 0}_{k}, a_0, a_1, \ldots)$$

$a(x)$ $a_0 + a_1 x + a_2 x^2 + \dots$

$\leftarrow$ 2 místa!

$$\frac{a(x) - a_0 - a_1(x)}{x^2}$$

$a_0 + a_1 x + a_2 x^2 + \dots$

$x \Leftarrow 2x$

$a_0 + a_1(2x) + a_2 \cdot (4x^2) + \dots$

$(a_0, 2a_1, 4a_2, 8a_3)$

$x \Leftarrow x^2$

$a_0 + a_1 x^2 + a_2 x^4 + \dots$

$(a_0, 0, a_1, 0, a_2, \dots)$

$$\frac{1}{1-x} \xleftrightarrow{\text{ogf}} (1,1,1,\ldots)$$

$$\frac{1}{1-2x} \xleftrightarrow{\text{ogf}} (1,2,4,8,\ldots)$$

$$a(x) \xleftrightarrow{\text{ogf}} (a_0,a_1,a_2,\ldots)$$

$$a(-x) \longleftrightarrow (a_0,-a_1,a_2,-a_3,\ldots)$$

$$\frac{a(x)+a(-x)}{2} \longleftrightarrow (a_0,0,a_2,0,a_4,\ldots)$$

$$\frac{1}{1-2x} \xleftrightarrow{\text{ogf}} (1, 2, 4, 8, \ldots)$$

$$\frac{1}{1-2x^2} \xleftrightarrow{\text{ogf}} (1, 0, 2, 0, 4, 0, 8, 0, \ldots)$$

$$\frac{x}{1-2x^2} \longleftrightarrow (0, 1, 0, 2, 0, 4, 0, 8, \ldots)$$

$$\frac{x+1}{1-2x^2} \longleftrightarrow (1, 1, 2, 2, 4, 4, 8, 8, \ldots)$$

$$a(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$a'(x) = \sum_{n=1}^{\infty} n \cdot a_n \cdot x^{n-1}$$

$$(a_0, a_1, a_2, a_3, \dots)$$

$$\downarrow$$

$$(a_1, 2a_2, 3a_3, \dots)$$

$$a(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$\int_0^y a(x)\,dx = \int_0^y \sum_{n=0}^{\infty} a_n x^n =$$

$$= \sum_{n=0}^{\infty} \int_0^y a_n x^n =$$

$$\sum_{n=0}^{\infty} \frac{a_n}{n+1} \cdot x^{n+1}$$

$$a(x) = \sum_{n=0}^{\infty} a_n x^n \qquad b(x) = \sum_{n=0}^{\infty} b_n x^n$$

$$(a(x)) \cdot (b(x)) = \left(\sum a_n x^n\right)\left(\sum b_n x^n\right) =$$

$$= \sum_{n=0}^{\infty} c_n x^n$$

$$c_n = \sum_{k=0}^{n} a_k b_{n-k}$$

$$\frac{1}{1-x} \longleftrightarrow (1,1,1,\dots)$$

$$\left(\frac{1}{1-x}\right)' \longleftrightarrow (1,2,3,\dots)$$

$$\left(\frac{1}{1-x}\right)'' = \left(+\frac{1}{(1-x)^2}\right)' = +2\cdot(1-x)^{-3} =$$

$$= \frac{2}{(1-x)^3} \longleftrightarrow (1\cdot 2, 2\cdot 3, 3\cdot 4, \dots)$$

$$\frac{1}{(1-x)^2} \longleftrightarrow \text{není}\quad 1\cdot 1\ \ 2\cdot 1,\ 3\cdot 1, \dots$$

$$(1^2, 2^2, 3^2, \dots)$$

$$\ln \frac{1}{1-x} \xleftrightarrow{\text{ogf}} \left(0, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\right)$$

$$e^x \xleftrightarrow{\text{egf}} \left(\frac{1}{0!}, \frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}, \dots\right)$$

$$\binom{n}{k} = \frac{\cancel{n!}}{k!\,\cancel{(n-k)!}} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

$n \in \mathbb{N}$

$$(1-x)^{-n} = \sum_{k\geq 0} \binom{-n}{k}(-x)^k =$$

$$= \sum_{k\geq 0} \underbrace{\binom{-n}{k}\cdot(-1)^k}\, x^k =$$

$$\binom{-n}{k}\cdot(-1)^k =$$

$$= \frac{(-n)\cdot(-n-1)\cdots(-n-k+1)}{k!}\cdot(-1)^k =$$

$$= \frac{n(n+1)\cdots(n+k-1)}{k!} =$$

$$\frac{(n+k-1)(n+k-2)\cdots n}{k!} =$$

$$= \binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

Př:

$$\frac{1}{(1-x)^2} = \binom{1}{1} + \binom{2}{1}x + \binom{3}{1}x^2 + \binom{4}{1}x^3 + \dots$$

FIBONACCI $\quad F_0 = 0 \quad F_1 = 1 \quad F_{n+2} = F_{n+1} + F_n$

$$F(x) \longleftrightarrow (F_0, F_1, F_2, F_3, \ldots)$$

$$F(x) \longleftrightarrow (0, 1, 1, 2, 3, 5, 8, \ldots)$$

$$x \cdot F(x) \longleftrightarrow (0, 0, 1, 1, 2, 3, 5, \ldots)$$

$$(x+1) \cdot F(x) \longleftrightarrow (0, 1, 2, 3, 5, 8, 13, \ldots)$$

$$\updownarrow$$

$$(x^2+x)F(x) = F(x) - x \qquad \frac{F(x) - 0}{x} - 1$$

$$\frac{x}{1-x-x^2} = \frac{A}{x-x_1} + \frac{B}{x-x_2}$$

$$\boxed{x^2+x-1}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{5}}{2} \quad \frac{-1+\sqrt{5}}{2}, \quad \frac{-1-\sqrt{5}}{2}$$

$$-x = A(x-x_2) + B(x-x_1)$$

$$-x+0 = x(A+B) + (-Ax_2 - Bx_1)$$

$$\left.\begin{array}{l} -1 = A+B \\ 0 = +Ax_2 + Bx_1 \end{array}\right\} \Rightarrow \text{spočítáme } A, B$$

$$\lambda_1 = \frac{1}{x_1} \qquad \lambda_2 = \frac{1}{x_2}$$

$$\frac{x}{1-x-x^2} = \frac{A}{x-x_1} + \frac{B}{x-x_2} =$$

$$= \frac{\left(-\frac{A}{x_1}\right)}{\frac{x}{-x_1} - \frac{x_1}{-x_1}} + \ldots = \frac{\left(-\frac{A}{x_1}\right)}{1 - \frac{x}{x_1}} + \ldots$$

$\lambda_1 x$

$$\frac{a}{1-\lambda_1 x} \longrightarrow \left(a\cdot\lambda_1^0,\ a\cdot\lambda_1^1,\ a\cdot\lambda_1^2,\ \ldots\right)$$

$$(1-x)^{-1} \longleftrightarrow (1,1,1,\ldots)$$

Podobně

$$\frac{b}{1-\lambda_2 x} \longleftrightarrow \left(b\cdot\lambda_2^0,\ b\cdot\lambda_2^1,\ \dots\right)$$

$$\frac{x}{1-x-x^2} \longleftrightarrow \left(a\lambda_1^0+b\cdot\lambda_2^0,\ a\cdot\lambda_1^1+b\cdot\lambda_2^1,\ \dots\right)$$

$$\parallel$$

$$F(x) \Rightarrow F_n = a\cdot\lambda_1^n + b\cdot\lambda_2^n$$

$$\frac{x}{1-x-x^2} = \frac{a}{1-\lambda_1 x} + \frac{b}{1-\lambda_2 x}$$

$$(1-\lambda_1 x)(1-\lambda_2 x) = 1-(\lambda_1+\lambda_2)x + \lambda_1\lambda_2 x^2$$

$$1 - x - x^2$$

$$\frac{1-x-x^2}{x^2+x-1}$$ má kořeny x_1, x_2

$$x_1 + x_2 = -1$$

$$x_1 x_2 = -1$$

$$\lambda_1 + \lambda_2 = \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1+x_2}{x_1 x_2} = 1$$

$$\lambda_1 \cdot \lambda_2 = \frac{1}{x_1} \cdot \frac{1}{x_2} = \frac{1}{x_1 x_2} = -1$$

Vynásobíme $1-x-x^2$:

$$x = a(1-\lambda_2 x) + b(1-\lambda_1 x)$$

$$1 = -a\cdot\lambda_2 - b\cdot\lambda_1$$

$$0 = a+b$$

$$x_1 = \frac{-1+\sqrt{5}}{2}$$

$$x_2 = \frac{-1-\sqrt{5}}{2}$$

$$\lambda_1 = \frac{2}{-1+\sqrt{5}}$$

$$\lambda_2 = \frac{2}{-1-\sqrt{5}}$$

$$1 = -a\left(\frac{2}{-1-\sqrt{5}}\right) - b\left(\frac{2}{-1+\sqrt{5}}\right)$$

$$a = -b$$

$$1 = b\left(\frac{2}{-1-\sqrt{5}}\right) - b\cdot\frac{2}{-1+\sqrt{5}} \Rightarrow \frac{1}{2} = b\left(\frac{2\sqrt{5}}{(\sqrt{5}+1)(\sqrt{5}-1)}\right) =$$

$$\Rightarrow b = \frac{1}{\sqrt{5}}$$