

Důk. Eulerova věta indukci podle $|E|$:

I. $|E| = 0$

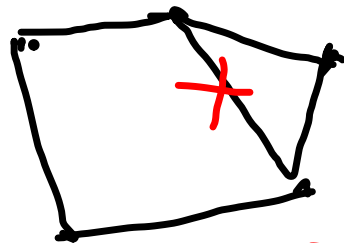
$|V| = 1$

$|S| = 1$

II. $|E| = n$, IP pro rovinný graf $G' = (V', E')$
 $|E'| < |E|$.

a) G je strom $\Rightarrow \left. \begin{matrix} |E| = |V| - 1 \\ |S| = 1 \end{matrix} \right\} |V| + |S| = |E| + 2$

b) G obsahuje kružnici C , zvolme hranu e ležící na C ,
 uvažme $G' = (V, E - \{e\})$



$\left. \begin{matrix} |V| \\ |S| - 1 \\ |E| - 1 \end{matrix} \right\} \text{počet stěn a směrů } 0 \uparrow$
 $\Rightarrow \text{IP: } |V'| + |S| = |E'| + 2 \Rightarrow \text{dokaz!}$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2} + \frac{1}{2} = 1$$

kdyby $d, k \geq 4 \Rightarrow \frac{1}{a} + \frac{1}{k} < \frac{1}{5} + \frac{1}{4} = \frac{1}{2}$

Přitom $d, k \geq 3$:

$d = k = 3$: $\frac{1}{3} + \frac{1}{3} = \frac{1}{2} + \frac{1}{6}$ $s = \frac{2a}{k} = 4 \Rightarrow$ čtyřlístek

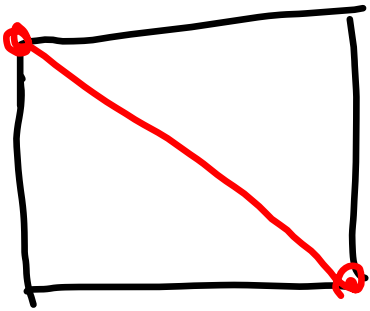
$d = 3, k = 4$: $\frac{1}{3} + \frac{1}{4} = \frac{1}{2} + \frac{1}{12}$ $s = \frac{2a}{k} = 6 \Rightarrow$ krychle

kdyby $d \geq 6$ nebo $k \geq 6$, pak $\frac{1}{a} + \frac{1}{k} \leq \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

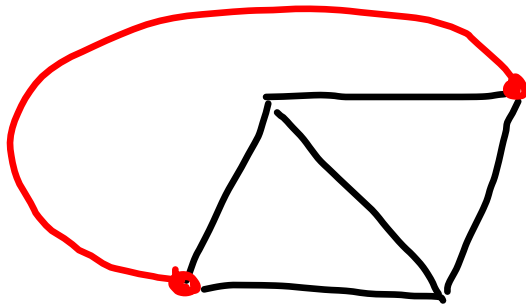
$d = 3, k = 5$: $\frac{1}{3} + \frac{1}{5} = \frac{1}{2} + \frac{1}{30}$ $s = \frac{2a}{k} = 12 \Rightarrow$ dvanáctistěn

$d = 4, k = 3$: $\frac{1}{4} + \frac{1}{3} = \frac{1}{2} + \frac{1}{12}$ $s = \frac{2a}{k} = 8 \Rightarrow$ osmistěn

$d = 5, k = 3$: $\frac{1}{5} + \frac{1}{3} = \frac{1}{2} + \frac{1}{30}$ $s = \frac{2a}{k} = 20 \Rightarrow$ dvacetišlák



přidáme hrany
 \Rightarrow není max.



není max.

$$3|S| = 2|E| \Rightarrow |S| = \frac{2}{3}|E|$$

$$|E| + 2 = |S| + |V| = \frac{2}{3}|E| + |V|$$

$$\frac{1}{3}|E| = |V| - 2$$

$$|E| = 3|V| - 6$$

$$2|E| \geq 4|S|$$

2. případ

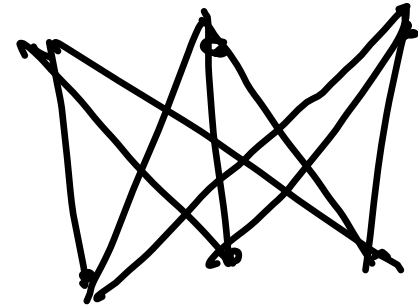
$$K_5: |V|=5 \quad |E| = \binom{5}{2} = 10$$

$$\text{rovinný} \Rightarrow |E| \leq 3 \cdot 5 - 6 = 9, \text{ spor.}$$

$K_{3,3}$ needs $K_3 \Rightarrow$

$$|E| \leq 2 \cdot |V| - 4$$

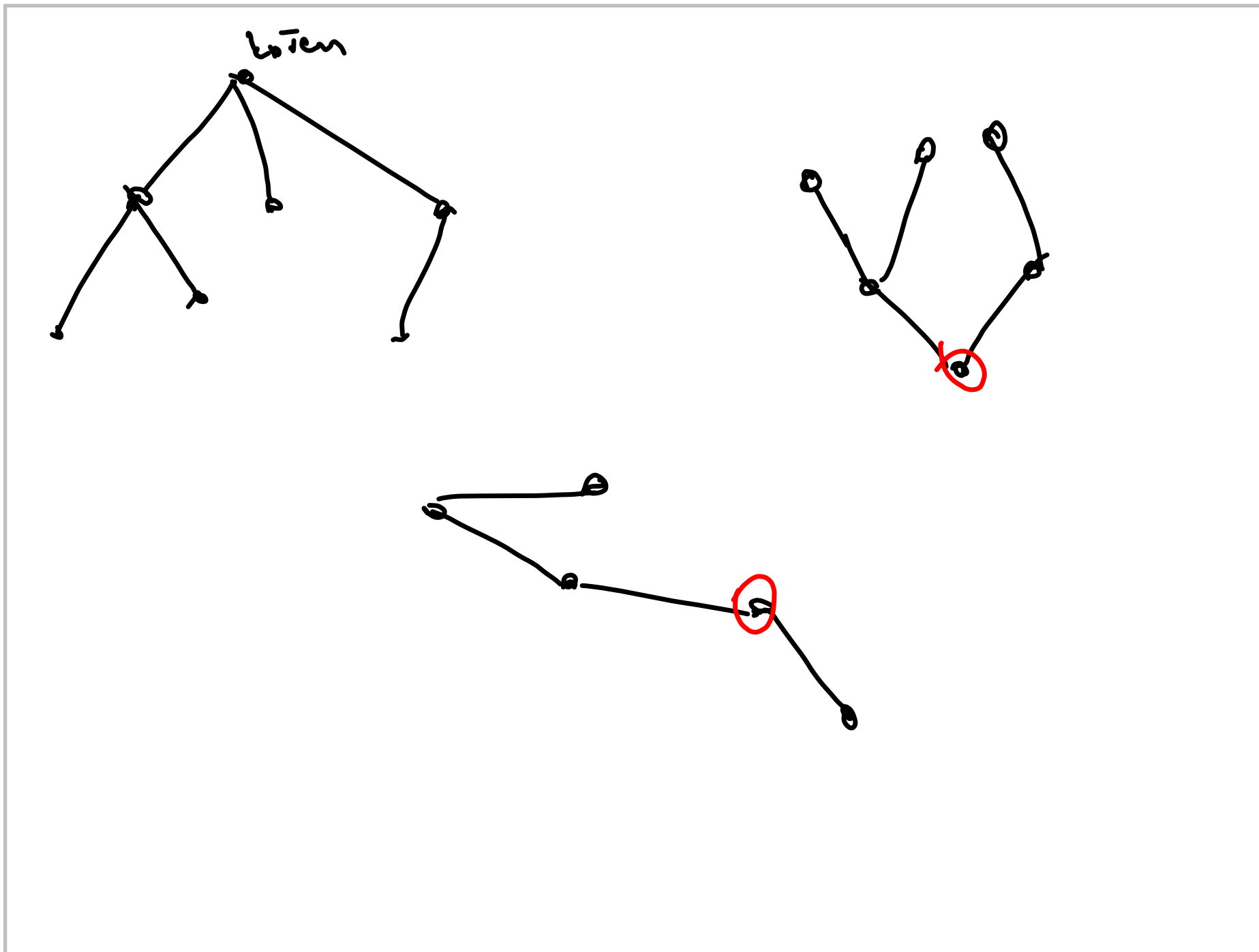
$$9 \quad 2 \cdot 6 - 4, \text{ spor.}$$

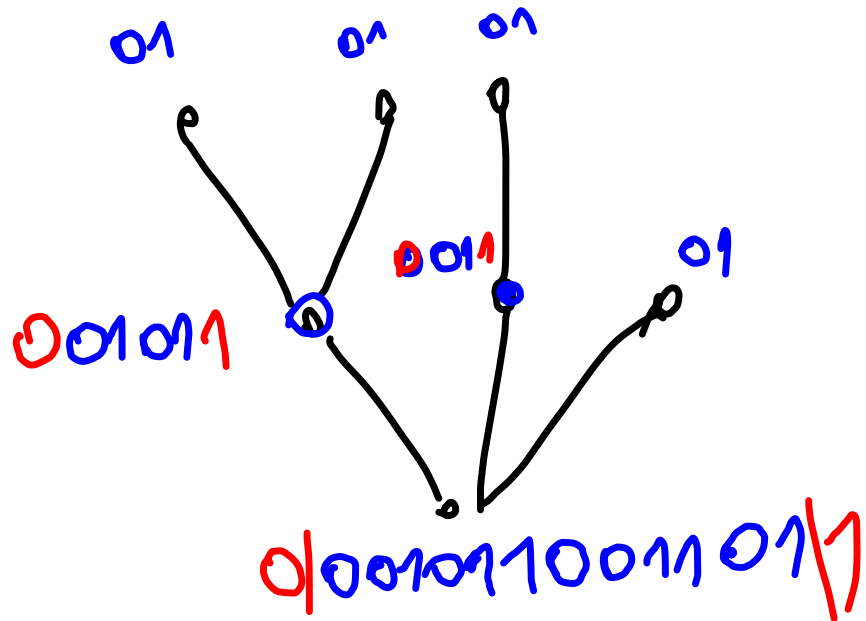


$$\text{kdýby } 6: \forall v: \deg(v) \geq 6 \Rightarrow |E| = \sum_{v \in V} \deg(v)$$

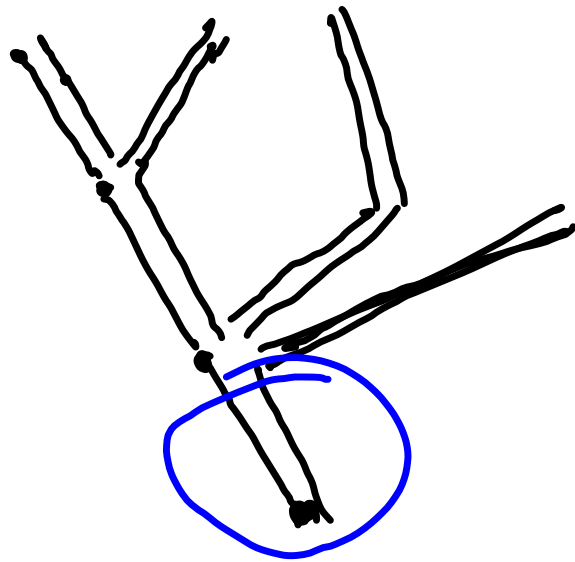
$$= \sum_{v \in V} \deg(v) \geq 6|V| \Rightarrow |E| \geq 3|V|$$

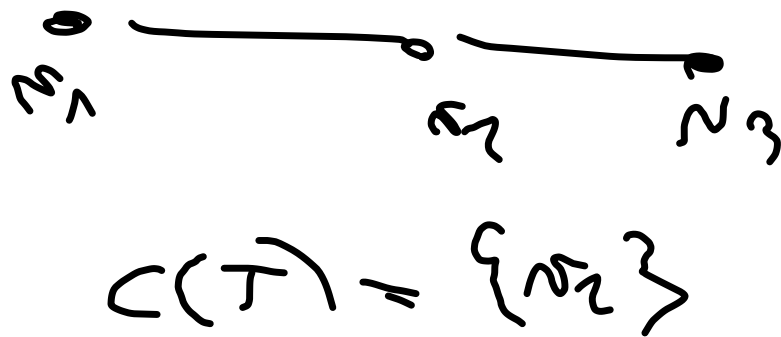
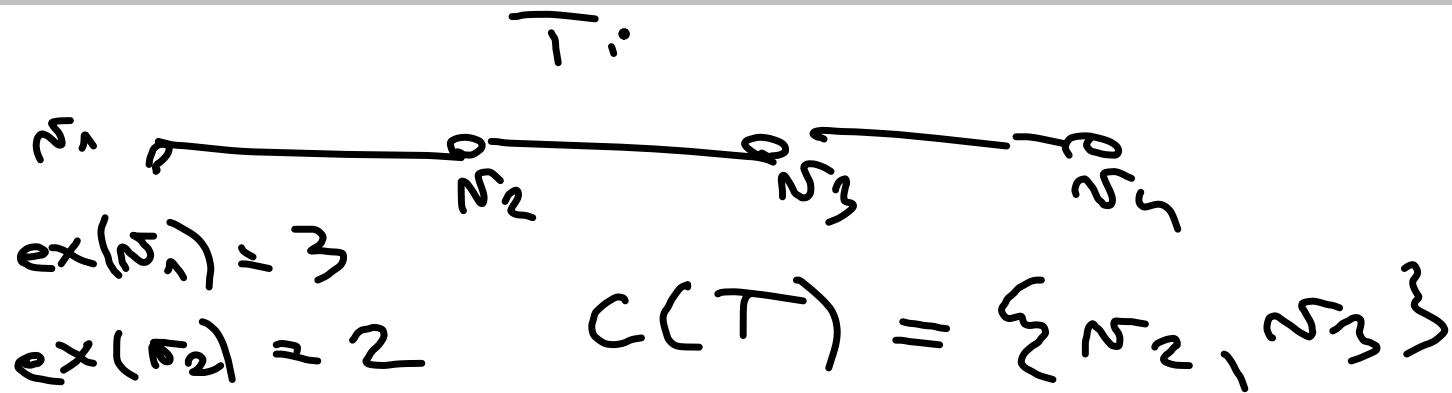
\Rightarrow 6 není rovinný

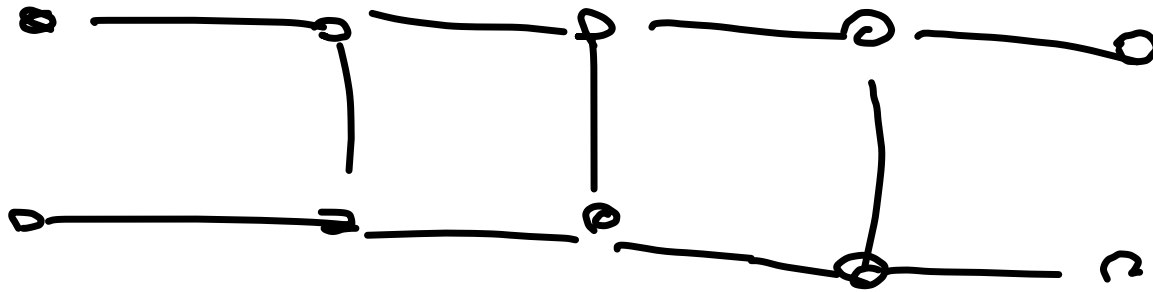




kód přístřešného stromu







3 p̄eřl kōlepn na K_n ?

