$$(1+x)^r = \sum_{k\geq 0} \binom{r}{k} x^k$$

$$r = -n,\ n \in \mathbb{N}: \quad (1+x)^{-n} = \sum_{k\geq 0} \binom{-n}{k} x^k$$

$$\binom{-n}{k} = \frac{(-n)(-n-1)\cdot(-n-2)\cdots(-n-k+1)}{k!} =$$

$$= \frac{n\cdot(n+1)\cdots(n+k-1)\cdot(-1)^k}{k!} =$$

$$= (-1)^k \frac{(n+k-1)\cdots n\cdot(n-1)!}{(n-1)!\,k!} =$$

$$= (-1)^k \frac{(n+k-1)!}{(n-1)!\,k!} = (-1)^k \cdot \binom{n+k-1}{n-1}$$

$$\Rightarrow (1-x)^{-n} = \sum_{k\geq 0} \binom{-n}{k} (-1)^k x^k = \sum_{k\geq 0} \binom{n+k-1}{n-1} x^k$$

$F_0 = 0,\ F_1 = 1,\ F_{n+2} = F_{n+1} + F_n$

$F_n = F_{n-1} + F_{n-2} \quad n \geq 2$

upravíme: $F_n = F_{n-1} + F_{n-2} + [n=1] \quad \Big| \cdot x^n \quad \Big| \sum$

$$\sum F_n x^n = \sum F_{n-1} x^n + \sum F_{n-2} x^n + x$$

$$F(x) = x \cdot F(x) + x^2 F(x) + x$$

odtud $F(x) = \dfrac{x}{1 - x - x^2}$

$$x^2 + x - 1 = 0 \iff x_{1,2} = \frac{-1 \pm \sqrt{5}}{2} \quad \begin{cases} \dfrac{-1+\sqrt{5}}{2} \\ \dfrac{-1-\sqrt{5}}{2} \end{cases}$$

$$\frac{x}{1-x-x^2} = \frac{A}{x-x_1} + \frac{B}{x-x_2} = \frac{x(A+B) - Ax_2 - Bx_1}{(x-x_1)(x-x_2)} =$$

$$= \frac{-x(A+B) - Ax_2 - Bx_1}{1-x-x^2} \iff \begin{matrix} A + B = -1 \\ Ax_2 + Bx_1 = 0 \end{matrix}$$

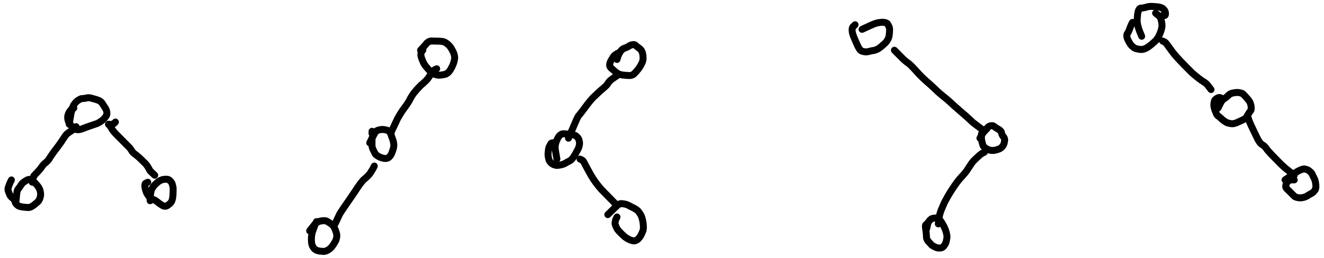
$B = -1 - A$

$Ax_2 - (A+1)x_1 = 0 \qquad A(x_2 - x_1) - x_1 = 0$

$-\sqrt{5}A = -\dfrac{1+\sqrt{5}}{2}$

$b_1 = 1$ 

$b_2 = 2$ 

$b_3 = 5$ 

$b_4 = b_3 + b_3 + b_2 \cdot b_1 + b_1 \cdot b_2 = 14$

$$B(x) = x \cdot B(x)^2 + 1$$

$$x \cdot B(x)^2 - B(x) + 1 = 0$$

$$B(x) = \frac{1 \pm \sqrt{1-4x}}{2x}$$

Přitom musí být $B(x) = b_0 + b_1 x + b_2 x^2 + \dots$

kde $b_0 = 1,\ b_1 = 1,\ b_2 = 2, \dots$

$$\lim_{x \to 0^+} \frac{1 + \sqrt{1-4x}}{2x} = +\infty$$

$$\lim_{x \to 0^+} \frac{1 - \sqrt{1-4x}}{2x} = \lim_{x \to +0} \frac{1-(1-4x)}{2x\left(1+\sqrt{1-4x}\right)} = \lim_{x \to 0} \frac{2}{1+\sqrt{1-4x}} =$$

$$= 1$$

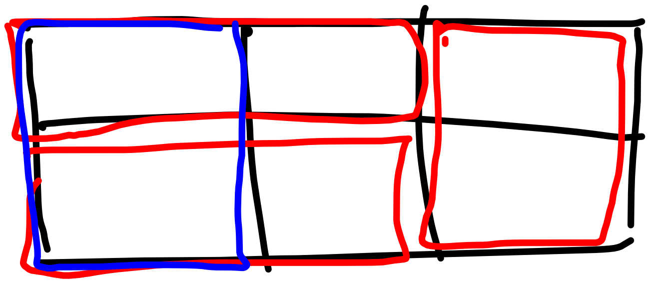
$$\binom{1/2}{k} = \frac{1/2 \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{3}{2}\right) \cdots \left(\frac{1}{2} - k + 1\right)}{k!} =$$

$$= \frac{1}{2k} \cdot \binom{-1/2}{k-1}$$

$$\binom{-1/2}{n} = \frac{\left(-\frac{1}{2}\right) \cdot \left(-\frac{3}{2}\right) \cdots \left(-\frac{1}{2} - n + 1\right)}{n!} =$$

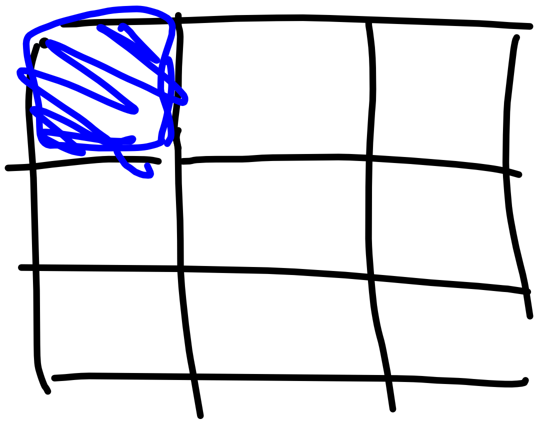
$$= (-1)^n \frac{\frac{1}{2} \cdot \frac{3}{2} \cdots \left(\frac{1+2n-2}{2}\right)}{n!} =$$

$$= \frac{(-1)^n}{2^n} \cdot \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) \cdot 2 \cdot 4 \cdot 6 \cdots 2n}{n! \quad 2^n \cdot n!} = \frac{(-1)^n}{4^n} \binom{2n}{n}$$

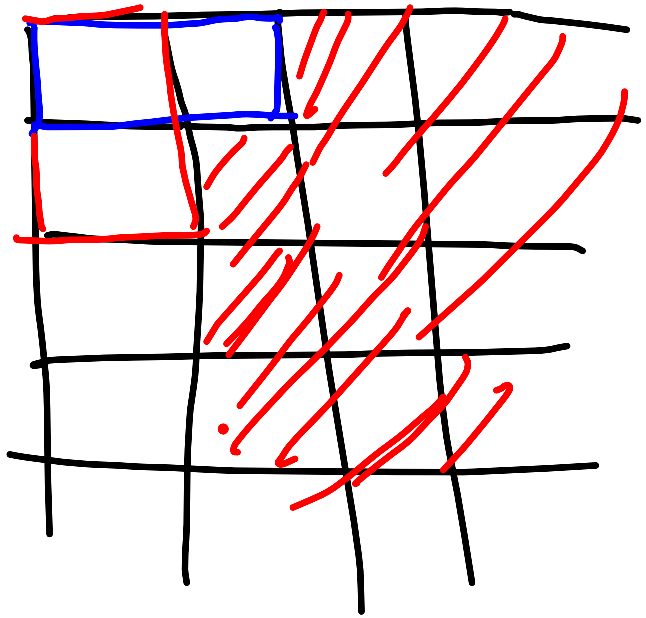


3 x 2

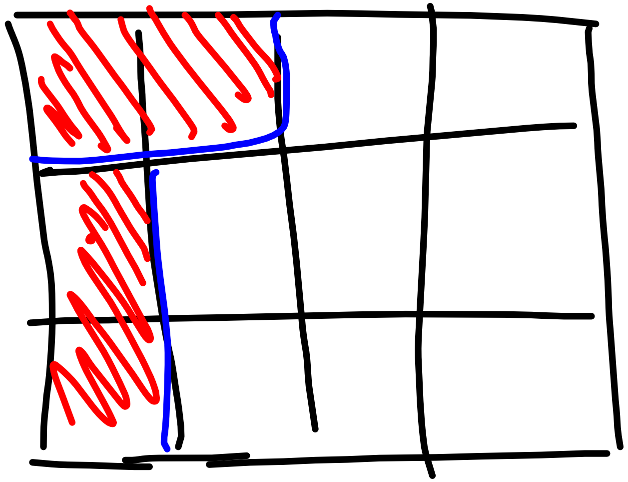
3 možnosti

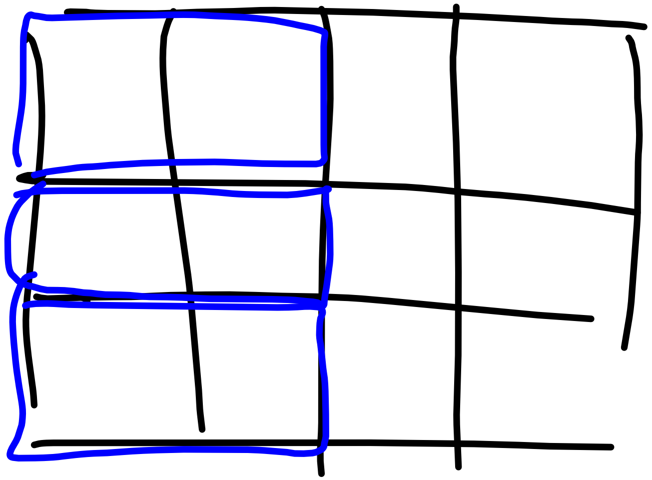


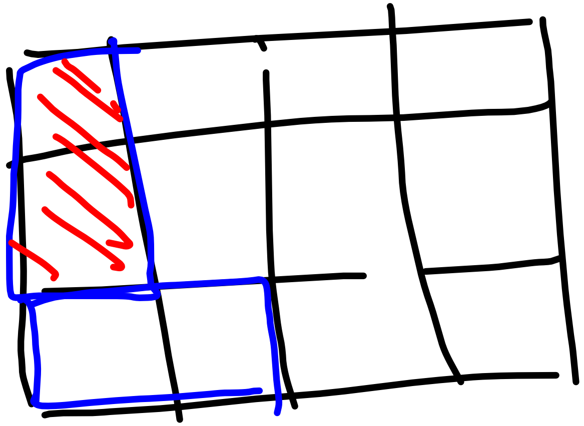
r_3



$$c_4 = r_3 + c_2 + r_3$$







$$C(x) = 2x R(x) + x^2 C(x) + 1$$
$$R(x) = x C(x) + x^2 R(x)$$

$$(x^2-1) C(x) + 2x \cdot R(x) = -1$$
$$x \cdot C(x) + (x^2-1) \cdot R(x) = 0$$

Cramerovo pravidlo:

$$a_{11} x_1 + \dots + a_{1n} x_n = b_1$$
$$\vdots$$
$$a_{n1} x_1 + \dots + a_{nn} x_n = b_n$$

$$D = \det \begin{pmatrix} a_{11} & \dots & a_{1n} \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \qquad x_i = \frac{A_i}{D}$$

$$A_i = \det \begin{pmatrix} a_{11} & b_1 & a_{1n} \\ a_{n1} & b_n & a_{nn} \end{pmatrix}$$

i-tý

$$D = (x^2-1)^2 - 2x^2 = x^4 - 4x^2 + 1$$

$$A_1 = 1 - x^2 \qquad A_2 = \begin{vmatrix} x^2-1 & -1 \\ x & 0 \end{vmatrix} = x$$