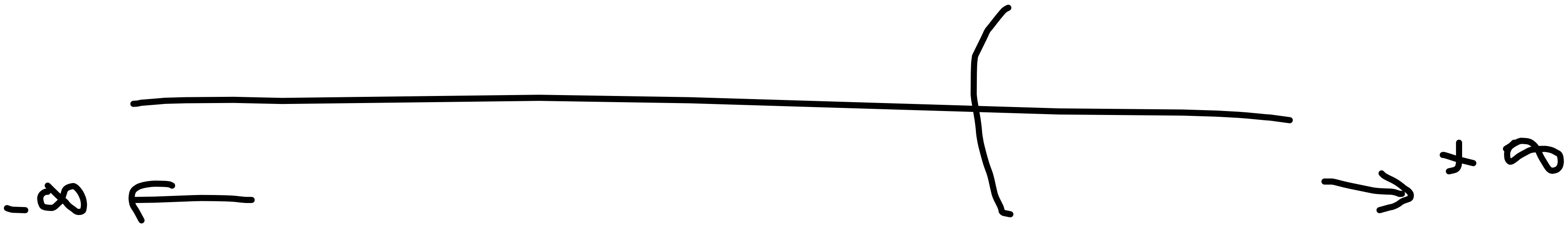


(N,∞) $\quad N\in\mathbb{R}$ ... okolí $+\infty$

$(x,y)\in(N,+\infty)\times(-\infty,P)$

$x\to+\infty$

$y\to-\infty$ $\quad$ ... okolí $(\infty,-\infty)$

$$f(\vec{x})\le g(\vec{x})\le h(\vec{x})$$

$$\lim_{\vec{x}\to a} f(\vec{x}) = \lim h(\vec{x}) = L$$

$$\Rightarrow \lim_{x\to a} h(\vec{x}) = L$$

$$f(x,y) = (x+y)\sin\frac{1}{x}\sin\frac{1}{y}$$

$$\lim_{x\to(0,0)} f(x,y) = 0$$

$$\varphi(x,y) = x+y$$

$$\psi(x,y) = \sin\frac{1}{x}\sin\frac{1}{y}$$

$$\lim_{(x,y)\to(0,0)} \varphi(x,y) = 0$$

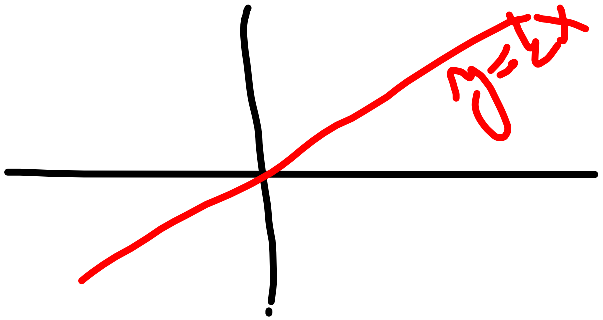
$$|\psi(x,y)| \leq 1$$ ... ohraničená

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{x\to 0} \frac{k\cdot x^2}{x^2+k^2x^2} = \lim_{x\to 0} \frac{k}{1+k^2} = \frac{k}{1+k^2}$$

$y = kx$

$k \in \mathbb{R}$

konst.



závisí na k

$\Rightarrow$ limita $\frac{xy}{x^2+y^2}$ neexistuje

$$\lim_{t\to 0}\frac{1}{t}\left(f(x_1,x_2,\dots,x_i+t,\dots,x_n)-f(x_1,\dots,x_n)\right)$$

$$\lim_{t\to 0}\frac{f(x_0+t)-f(x_0)}{t}=:f'(x_0)$$

$$\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}=:f'(x_0)$$

směrnice sečny $(x,f(x))$ a $(x_0,f(x_0))$

$k \in \mathbb{R} \setminus \{0\}$ konst

$$* = \lim_{t \to 0} \frac{1}{t} \cdot \left[f(x + t \cdot v) - f(x) \right] =$$

$$\lim_{t \to 0} \frac{1}{t} \left[f(x + kt \cdot v) - f(x) \right] =$$

$$= \lim_{k \cdot t \to 0} \frac{1}{k \cdot t} \left[f(x + \underline{t \cdot (k \cdot v)}) - f(x) \right] =$$

$$= \frac{1}{k} \lim_{t \to 0} \frac{1}{t} \left[\ldots \right]$$

$$= \frac{1}{k} d_{k \cdot v} f(x) = * = d_v f(x)$$

$$f(x,y) = \frac{x^4 y^2}{x^8 + y^4} \qquad f(0,0) = 0$$

? má $d_v f(0,0) = 0 \quad \forall v \in \mathbb{R}^n$

? není spojitá v $(0,0)$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim \frac{k^2 x^8}{x^8(1+k^2)} = \frac{k^2}{1+k^2}$$ – závisí na k

$\Rightarrow \lim$ neex.!!

„po parabolách"
$y = kx^2$

$$\varphi(t) = \lim_{t\to 0} \frac{f(x+t\cdot v_1, y+t\cdot v_2) - f(x,y)}{t} =$$

$$= \lim \frac{1}{t}\left(\frac{(x+t\cdot v_1)^4 \cdot (y+t\cdot v_2)^2}{(x+t\cdot v_1)^8 + (y+t\cdot v_2)^4} - \left(\frac{x^4 y^2}{x^8 + y^4} \right) \right) =$$

$f(x,y)$

$$\underset{(0,0)}{=} \lim_{t\to 0} \frac{1}{t}\left(\frac{t^4 v_1^4 \cdot t^2 v_2^2}{t^8 v_1^8 + t^4 v_2^4} - 0\right) =$$

$$= \lim_{t\to 0} \frac{1}{t} \, \frac{t^6 \, v_1^4 \cdot v_2^2}{t^8 v_1^8 + t^4 v_1^4} = \lim \frac{1}{t} t^2 \, \frac{v_1^4 v_2^2}{t^4 \cdot v_1^8 + v_2^4} =$$

$$= \lim_{t\to 0} \frac{t \, v_1^4 v_2^2}{t^4 v_1^8 + v_2^4} \underset{v_2 \neq 0}{=} 0$$

$v_2 = 0$ analogicky $= f'_x = 0$

$$df(x)(v) = a \cdot v,$$

kde $a = \left(f'_{x_1}(x), f'_{x_2}(x), \dots, f'_{x_n}(x) \right)$

$n = 2$:

$$a = \left(f'_x(x_0, y_0), f'_y(x_0, y_0) \right)$$

$$a \cdot v = f'_x(x_0, y_0) \cdot v_1 + f'_y(x_0, y_0) \cdot v_2$$

$$= \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy$$