- parciální derivace $\frac{\partial}{\partial x} f(x,y)$

$\Uparrow$

- derivace ve směru (vektoru v) $d_v f(x,y)$

$\Uparrow$

- diferenciál

$v \longmapsto d_v f(x,y)$

$d(v)$

$\exists$ v okolí bodu a jsou tam spojité

Pak

$\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_n}\right) \cdot v$

skal. součin

$$T_n(x; x_0) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(x_0)(x-x_0)^k$$

$$f^{(0)}(x_0) = f(x_0)$$

$$(\xi\ \eta)\begin{pmatrix} f''_{xx}(x_0,y_0) & f''_{xy}\cdots \\ f''_{yx}\cdots & f''_{yy}\cdots \end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix} =$$

$$= \left(f''_{xx}\xi + f''_{yx}\eta\ ,\ f''_{xy}\xi + f''_{yy}\eta\right)\begin{pmatrix}\xi\\ \eta\end{pmatrix}$$

$$= f''_{xx}\xi^2 + \underline{f''_{yx}\eta\xi} + \underline{f''_{xy}\xi\eta} + f''_{yy}\eta^2$$

$$= f''_{xx}\xi^2 + 2 f''_{xy}\xi\eta + f''_{yy}\eta^2$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k =$$

$$= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \dots + \binom{n}{n} y^n$$

$$\left(\frac{\partial}{\partial x}\xi + \frac{\partial}{\partial y}\eta\right)^3 = \left(\frac{\partial}{\partial x}\xi\right)^3 + 3\left(\frac{\partial}{\partial x}\xi\right)^2\left(\frac{\partial}{\partial y}\eta\right) +$$

$$+ 3\left(\frac{\partial}{\partial x}\xi\right)\cdot\left(\frac{\partial}{\partial y}\eta\right)^2 + \left(\frac{\partial}{\partial y}\eta\right)^3 =$$

$$= \frac{\partial^3}{\partial x^3}\cdot\xi^3 + 3\frac{\partial^3}{\partial x^2\partial y}\xi^2\eta + 3\frac{\partial^3}{\partial x\partial y^2}\xi\eta^2 +$$

$$+ \frac{\partial^3}{\partial y^3}\eta^3 = d^3 f(x,y)(\xi,\eta)$$

$df(x^*) = (-1, 0, 2)$

$d_v f(x^*) > 0$

$d_w f(x^*) < 0$

$v_1 = (0, 0, 1)$ nebo

$v_2 = (-1, 0, 0)$

$w = (1, 0, 0)$

kvadratická forma (vzhledem ke zvolené bázi)

$$h(u) = u^T \cdot A \cdot u$$

$$h((u_1, u_2)) = u_1^2 + u_2^2$$ je poz. def.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

hlavní minory

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \\ \vdots & & & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{pmatrix}$$

$\det a_{11}$

$\det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$\vdots$

$\det A$

$$(u_1,u_2)\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} = 2u_1u_2$$

pre $(1,1)$ je hodnotou 2

$(1,-1)$ — " — -2

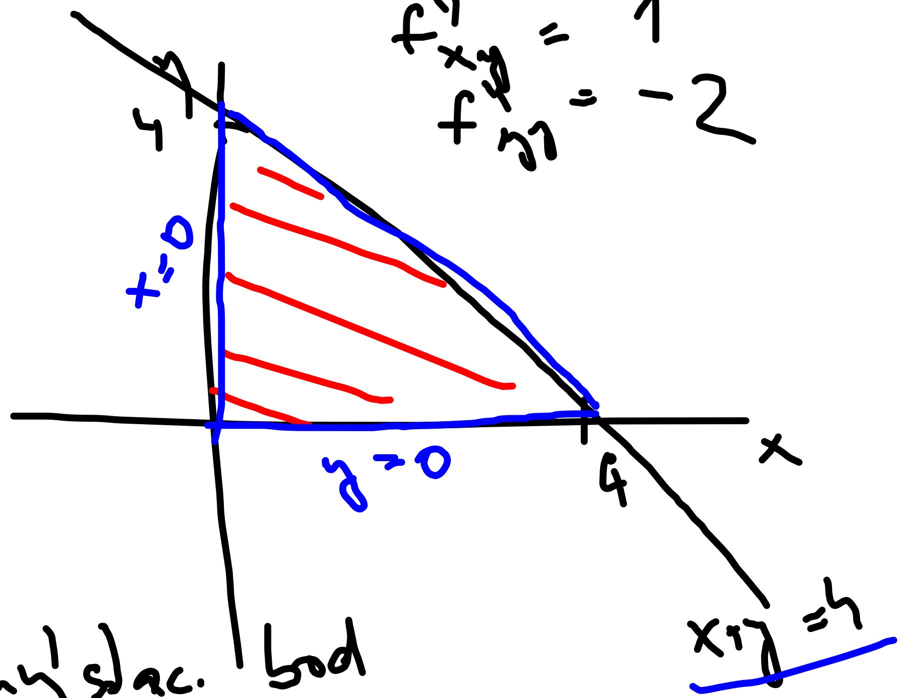
4

$\Rightarrow$ není extrém

$$f(x,y) = xy - x^2 - y^2 + x + y$$

$$f''_{xx} = -2$$
$$f''_{xy} = 1$$
$$f''_{yy} = -2$$

$$f'_x(x,y) = y - 2x + 1$$
$$f'_y(x,y) = x - 2y + 1$$



$$
\begin{aligned}
y - 2x + 1 &= 0 \\
x - 2y + 1 &= 0 \quad /\cdot 2 \\
\hline
-3y + 3 &= 0
\end{aligned}
$$

$$y = 1$$
$$x = 1$$

$[1,1]$ jediný stac. bod

$$Hf(x,y) = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$$ je negativně def. (hlavní minory)

$\Rightarrow [1,1]$ je lok. maximum $\quad f(1,1) = 1$

1. $y = 0$: $f(x,0) = -x^2 + x$
$x \in [0,4]$ $f'(x,0) = -2x \Rightarrow$ stac. v 0 $f(0,0) = 0$

hranica $f(4,0) = -12$
$f(0,0) = 0$

2. $x = 0$: $f(0,y) = -y^2 + y$
$y \in [0,4]$

analogicky jako v 1.

3. $x + y = 4$ $f(x, 4-x) = x(4-x) - x^2 - (4-x)^2 + x + 4 - x$
$y = 4 - x$
$x \in [0,4]$
$= 4x - x^2 - x^2 - 16 + 8x - x^2 + 4 =$
$= -3x^2 + 12x - 12$
$f'(x, 4-x) = -6x + 12 = 0 \Rightarrow \underline{x = 2}$
$f(2,2) = 2 \cdot 2 - 2^2 - 2^2 + 4 = 0$
$\underline{f(0,4) = f(4,0) = -12}$ abs. minimum