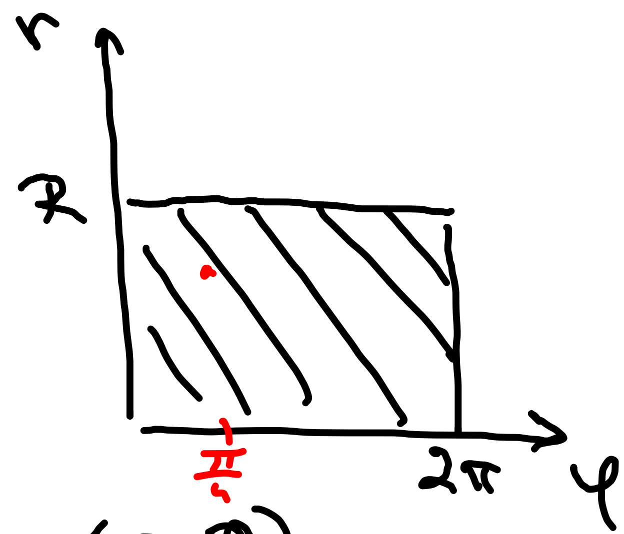
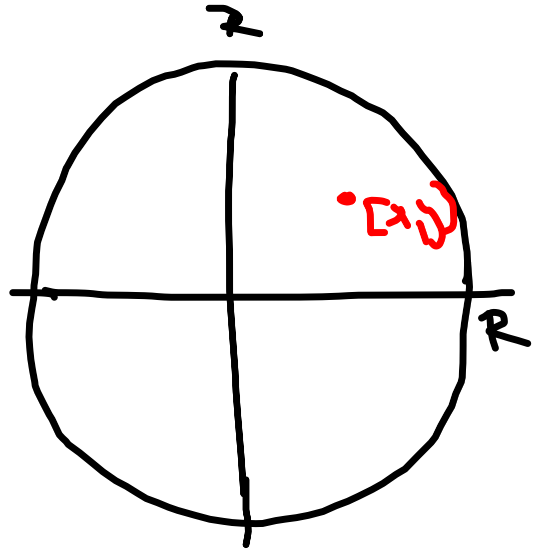




$r \in \langle 0, \infty)$

$\varphi \in \langle 0, 2\pi)$

$x = r \cos \varphi$

$y = r \cdot \sin \varphi$

$r = \sqrt{x^2 + y^2}$

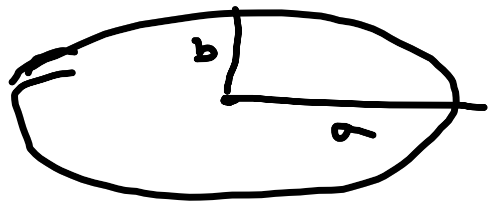
$\varphi = \operatorname{arctg} \frac{y}{x}$

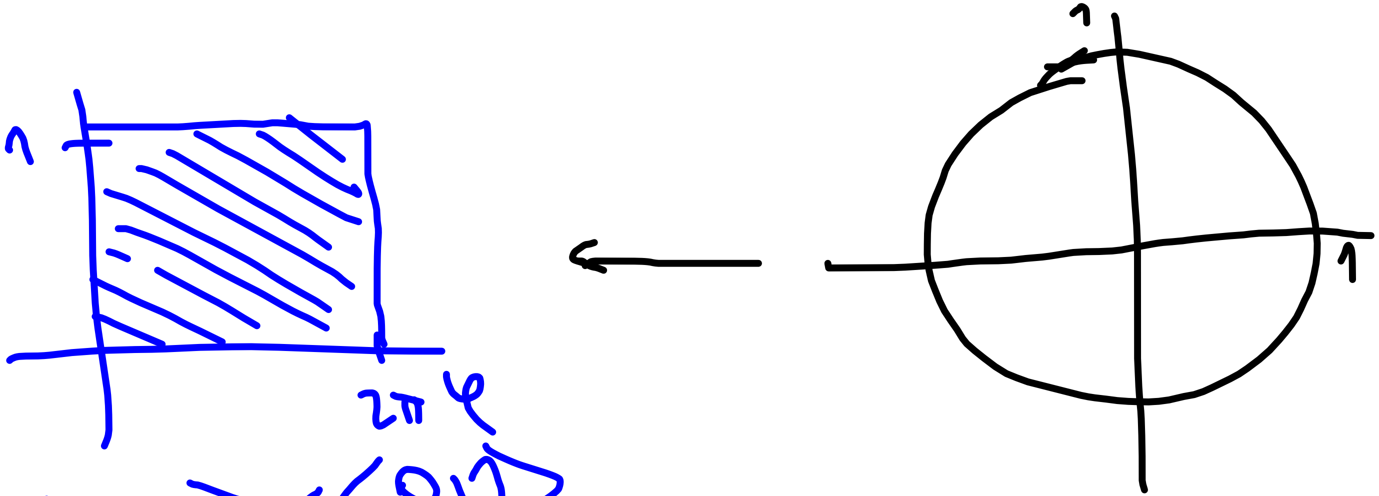
$[x \neq 0]$

zobecněné polární souřadnice

$x = a \cdot r \cdot \cos \varphi$

$y = b \cdot r \cdot \sin \varphi$



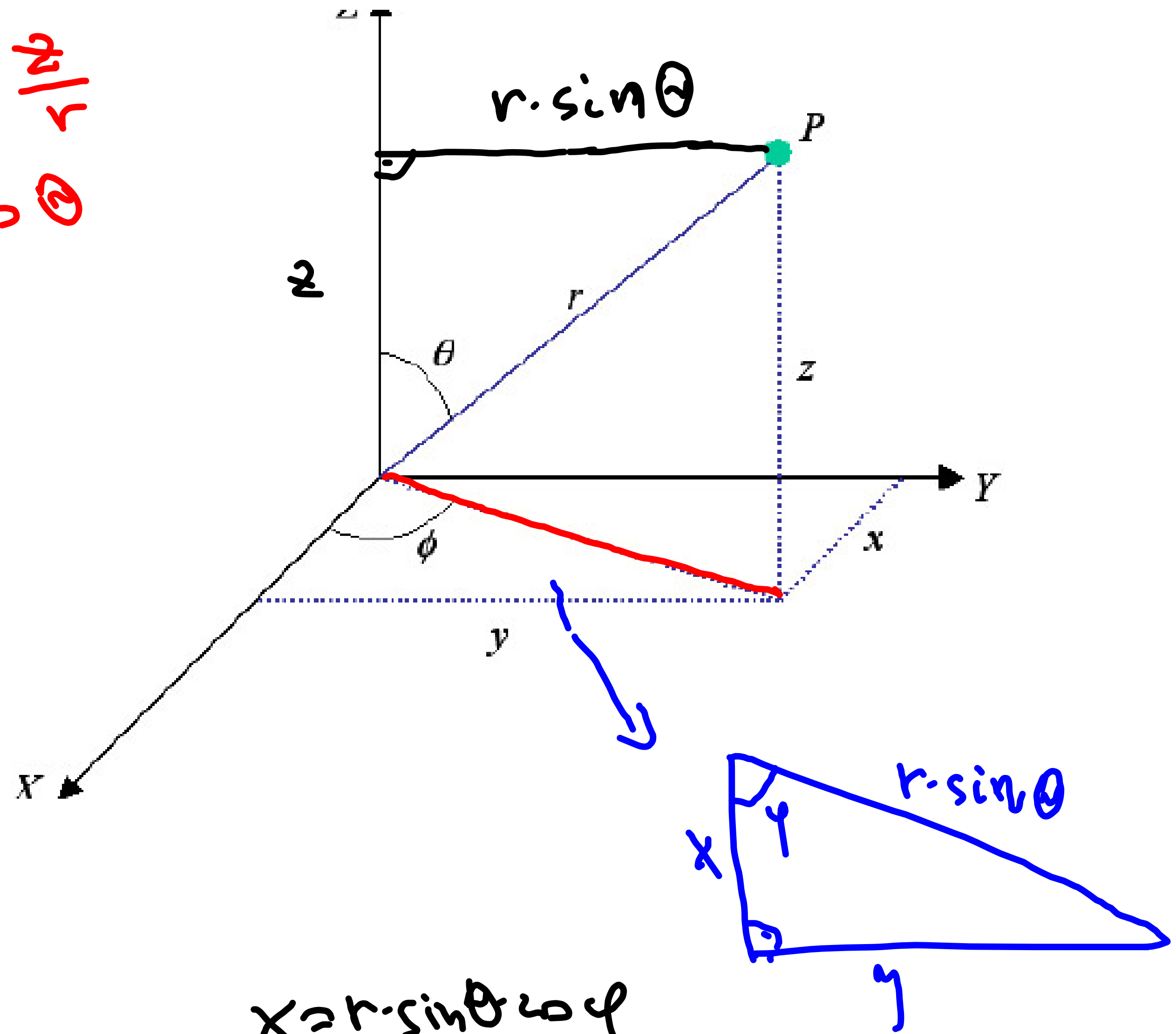


$$\langle 0, 2\pi\rangle \times \langle 0, 1\rangle$$

φ $\quad$ r

$$\cos\theta = \frac{z}{r}$$

$$z = r\cdot\cos\theta$$



$$x = r\cdot\sin\theta\cos\varphi$$

$$y = r\cdot\sin\theta\cdot\sin\varphi$$

$$\int_B 1\,dx\,dy\,dz = \int_U r^2 \sin\theta\,dr\,d\theta\,\mathrm{d}\varphi =$$
$$= \int_0^R r^2 dr \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\varphi = \frac{4}{3}R^3\pi.$$

$$*$$

$$* \left[\frac{r^3}{3}\right]_0^R \cdot \left[-\cos\Theta\right]_0^\pi \cdot \left[\varphi\right]_0^{2\pi} =$$
$$= \frac{R^3}{3} \cdot (1+1) \cdot 2\pi = \frac{4}{3}\pi R^3$$

$x^2+y^2+z^2=z$

$z=0$: $x^2+y^2=0 \Leftrightarrow x=y=0$

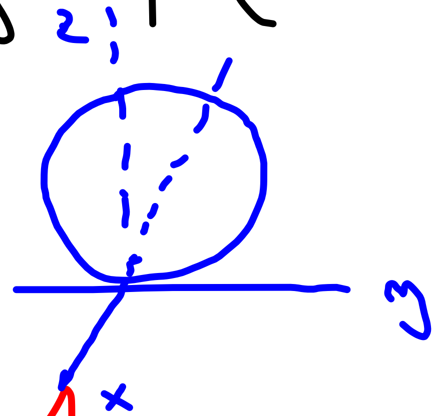
$z=1$: $x^2+y^2+1=1 \Leftrightarrow x=y=0$

$z=2$: $x^2+y^2+4=2$ $\emptyset$

$z=\frac{1}{2}$: $x^2+y^2+\frac{1}{4}=\frac{1}{2} \Leftrightarrow x^2+y^2=\frac{1}{4}$ (kružnice $k([0,0],\frac{1}{2})$)

$z\in(0,1)$: $x^2+y^2=\underbrace{z-z^2}_{\in(0,1)}$

tj. $z^2<z$:



Koule: $x^2+y^2+\left(z-\frac{1}{2}\right)^2=\frac{1}{4}$

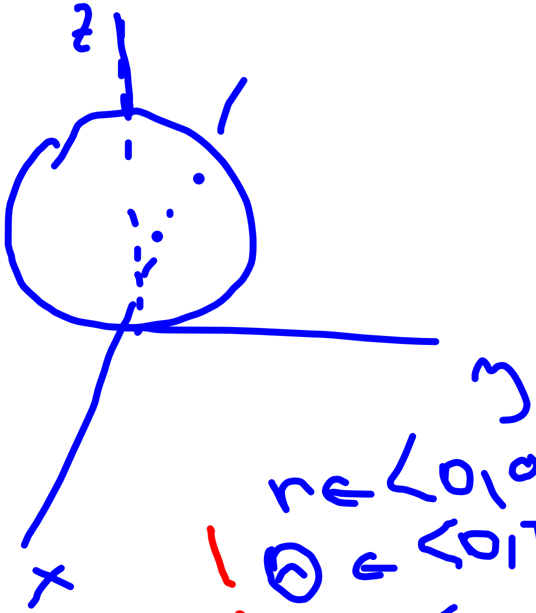
střed $[0,0,\frac{1}{2}]$, poloměr $\frac{1}{2}$

$$x^2+y^2+z^2 = z \Leftrightarrow r^2 = r\cos\Theta$$

$$z \geq 0$$

Jrace:

$$z = r\cdot\cos\Theta$$
$$y = r\cdot\sin\Theta\sin\varphi$$
$$z = r\cdot\sin\Theta\cos\varphi$$



$r \in \langle 0,\infty)$

! $\Theta \in \langle 0,\pi\rangle$

$\varphi \in \langle 0,2\pi)$

Podmínky:

$$r^2 \leq r\cos\Theta$$

$r \neq 0$

$$\boxed{0 \leq r \leq \cos\Theta} \Rightarrow \Theta \leq \frac{\pi}{2}$$

$$\sqrt{x^2+y^2+z^2} = r$$

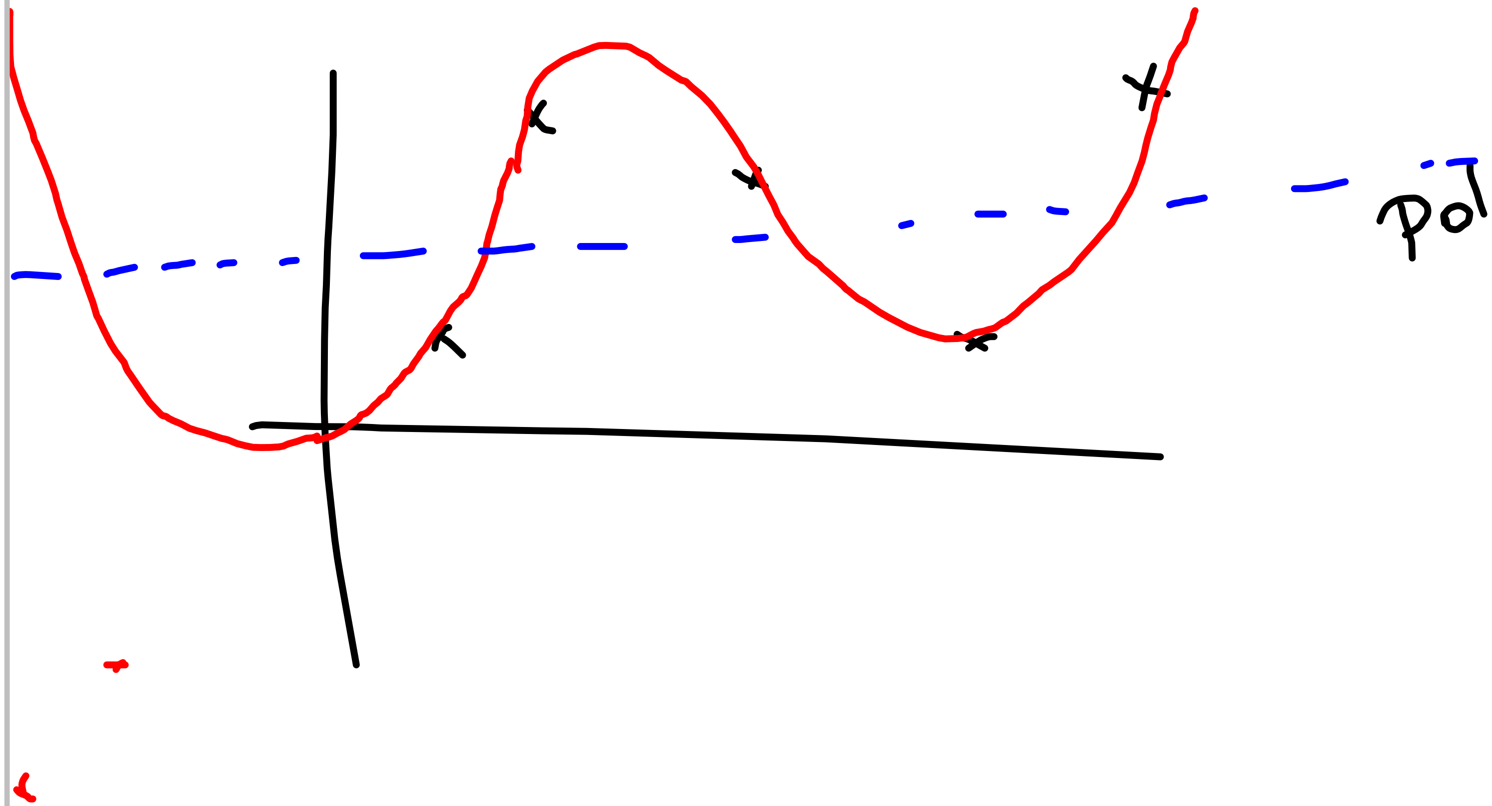
$$0 \leq \varphi \leq 2\pi$$
$$0 \leq \Theta \leq \frac{\pi}{2}$$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{\cos\Theta} r\, r^2 \sin\Theta \, dr\, d\Theta\, d\varphi = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \sin\Theta \left[\frac{r^4}{4}\right]_0^{\cos\Theta} d\Theta =$$

$$= \frac{2\pi}{4} \int_0^{\frac{\pi}{2}} \sin\Theta \cdot \cos^4\Theta \, d\Theta \; .=$$

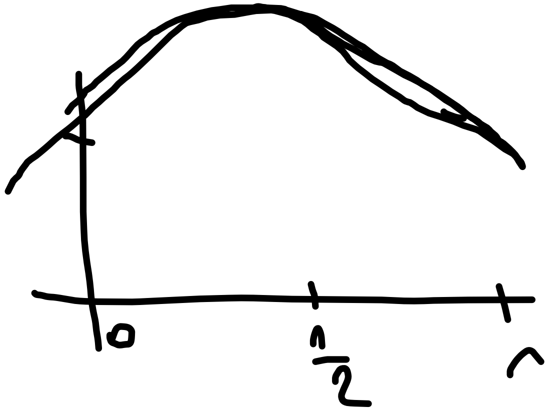
$$\left| \begin{array}{l} t = \cos\Theta \\ dt = -\sin\Theta \, d\Theta \end{array} \right| = \frac{2\pi}{4} \int_1^0 -t^4 \, dt =$$

$$= \frac{2\pi}{4} \int_0^1 t^4 \, dt = \frac{2\pi}{4} \cdot \left[\frac{t^5}{5} \right]_0^1 = \frac{\pi}{10} .$$



polynom st. 4

Odvození Simpsonova pravidla:



$y_0 = f(0)$, $f\left(\frac{1}{2}\right)$, $f(1)$

$\qquad\qquad y_{1/2} \qquad y_1$

$$y = ax^2 + bx + c$$

$$\int_0^1 ax^2 + bx + c \, dx = \underline{\alpha} \cdot f(0) + \underline{\beta} \cdot f\left(\frac{1}{2}\right) + \underline{\gamma} \cdot f(1)$$

$\approx \int_0^1 f(x)\, dx$ Zajímají nás α, β, γ, chceme funkčnost pro každé f!

$f(x) \equiv 1$: $\int_0^1 1 \, dx \approx \alpha \cdot 1 + \beta \cdot 1 + \gamma \cdot 1 \iff 1 = \alpha + \beta + \gamma$

$f(x) \equiv x$: $\int_0^1 x \, dx \approx \alpha \cdot 0 + \beta \cdot \frac{1}{2} + \gamma \cdot 1 \iff \frac{1}{2} = \frac{1}{2}\beta + \gamma$

$f(x) \equiv x^2$: $\int_0^1 x^2 \, dx \approx \alpha \cdot 0^2 + \beta \cdot \left(\frac{1}{2}\right)^2 + \gamma \cdot 1^2 \iff \frac{1}{3} = \frac{1}{4}\beta + \gamma$

$$\frac{1}{4}\beta = \frac{1}{6} \Rightarrow \beta = \frac{2}{3}, \quad \gamma = \frac{1}{6}, \quad \alpha = \frac{1}{6}$$

$$\frac{1}{6}\left(f(0) + 4f\left(\frac{1}{2}\right) + f(1) \right)$$