Specifying real-time stochastic systems by timed automata

Jan Krčál

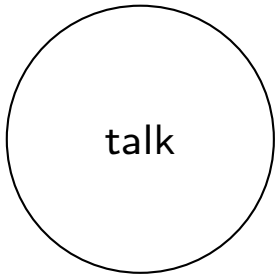
Faculty of Informatics,
Masaryk University, Brno, Czech Republic

DTEDI
November 11, 2010

Outline

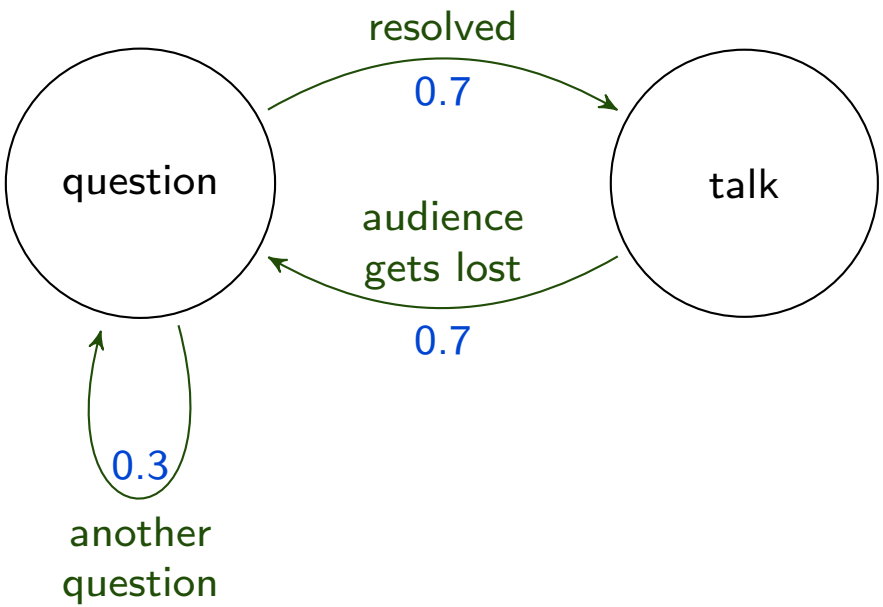
1. stochastic processes
   in particular continuous-time Markov chains (CTMCs)
2. observer timed automata
   as a specification formalism
3. results and problems
   state of the art

Markov Chain: Talk example

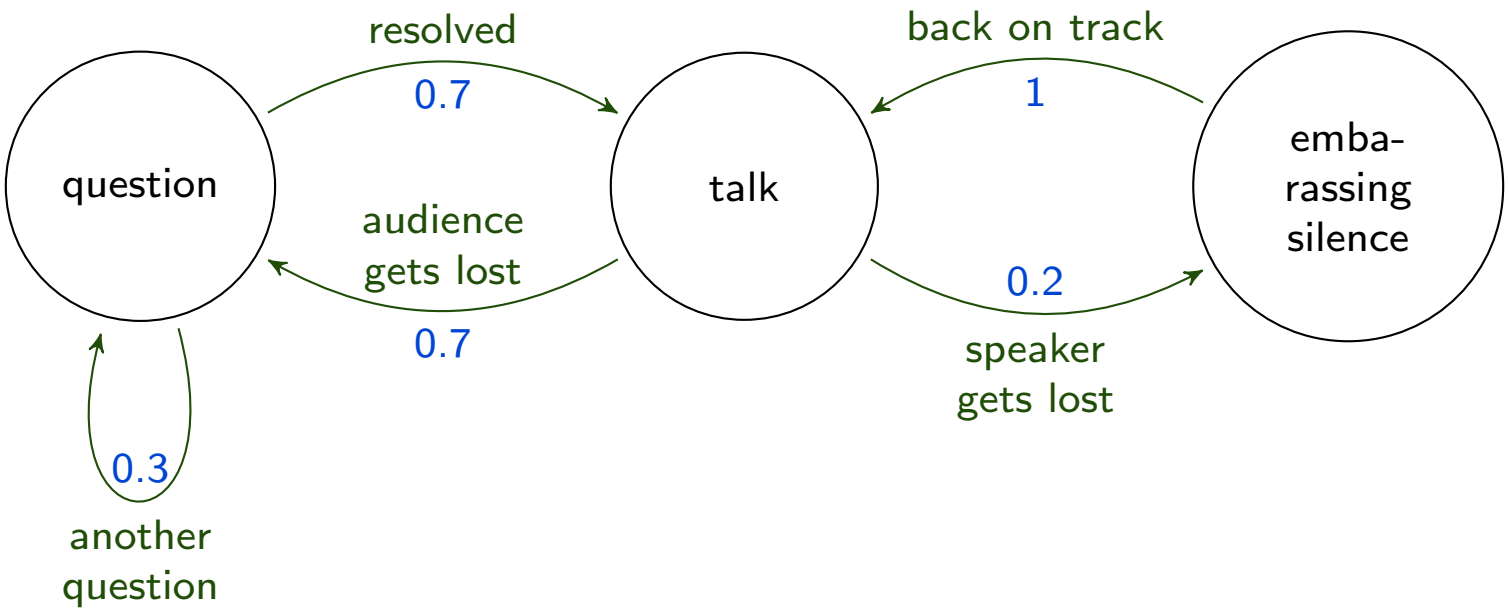


Markov Chain: Talk example

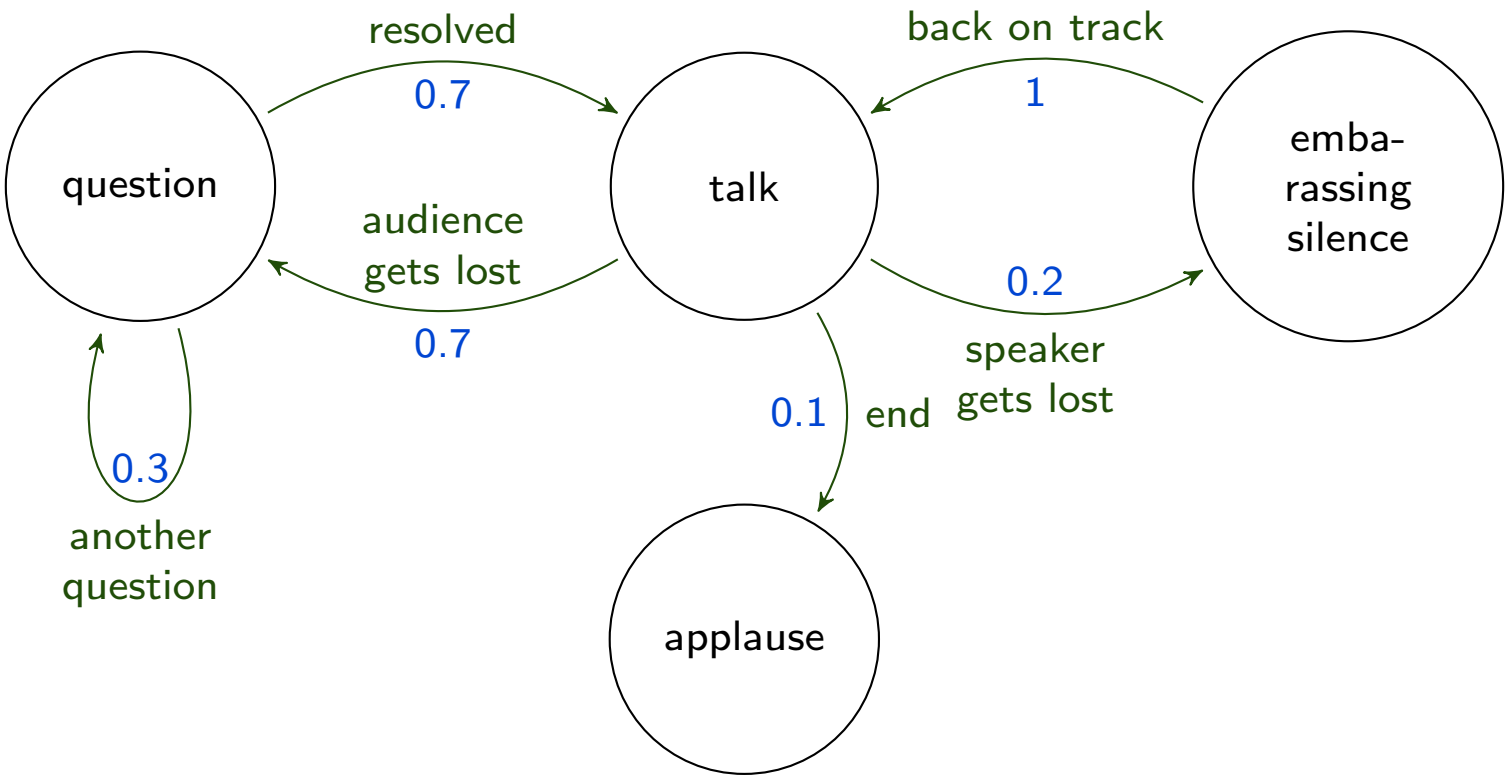
Markov Chain: Talk example



Markov Chain: Talk example

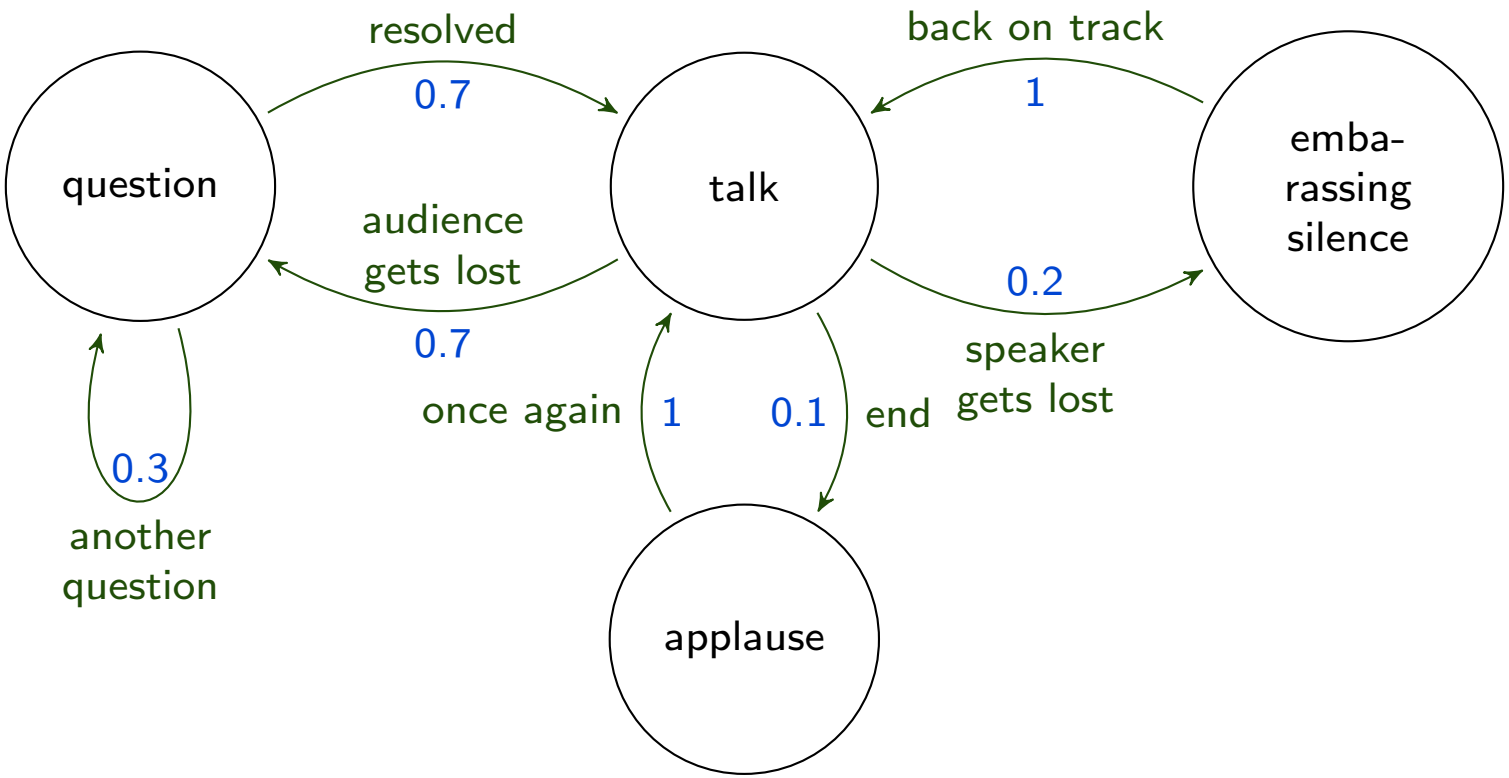


Markov Chain: Talk example



Run of the process: *talk*, *question*, *question*, *talk*, *silence*, . . .

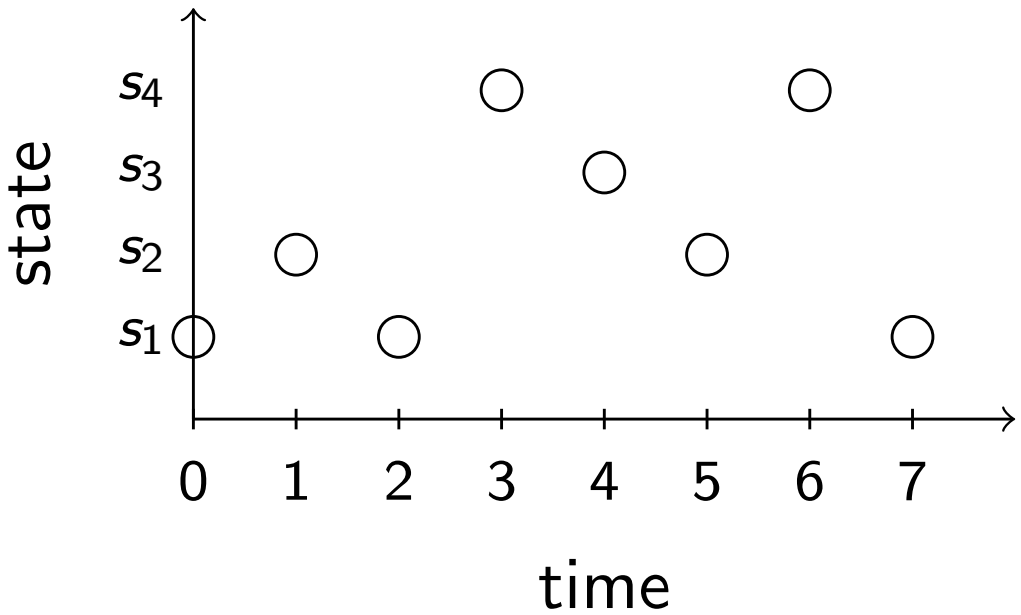
Markov Chain: Talk example



Run of the process: *talk*, *question*, *question*, *talk*, *silence*, . . .

Discrete Time Stochastic Process

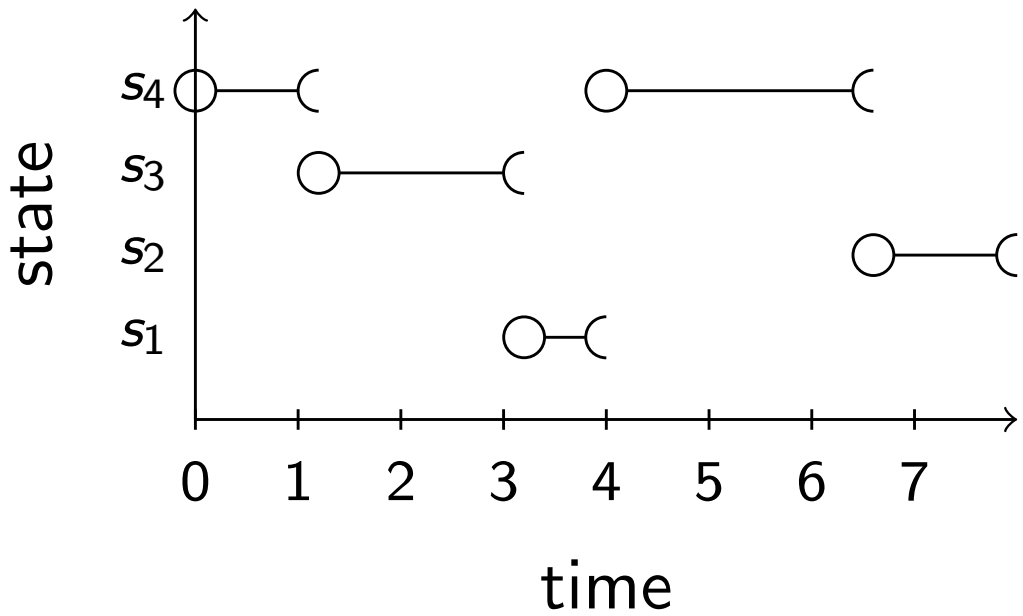
- set of states (usually finite)
- stochastic "rules" for transitions



Run of the process: $s_1, s_2, s_1, s_4, s_3, s_2, s_4, s_1$

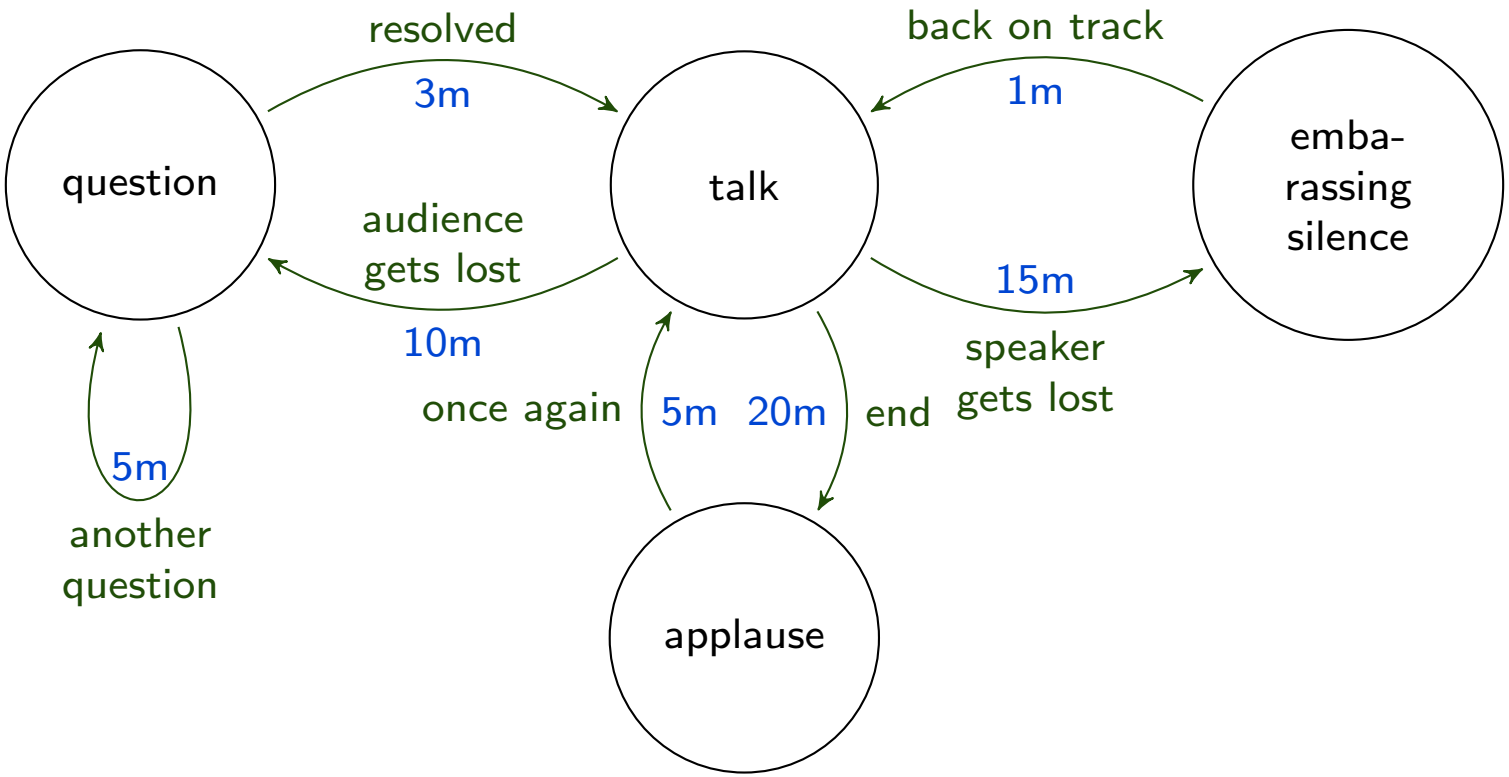
Continuous Time Stochastic Process

- set of states (usually finite)
- stochastic "rules" for transitions and delays

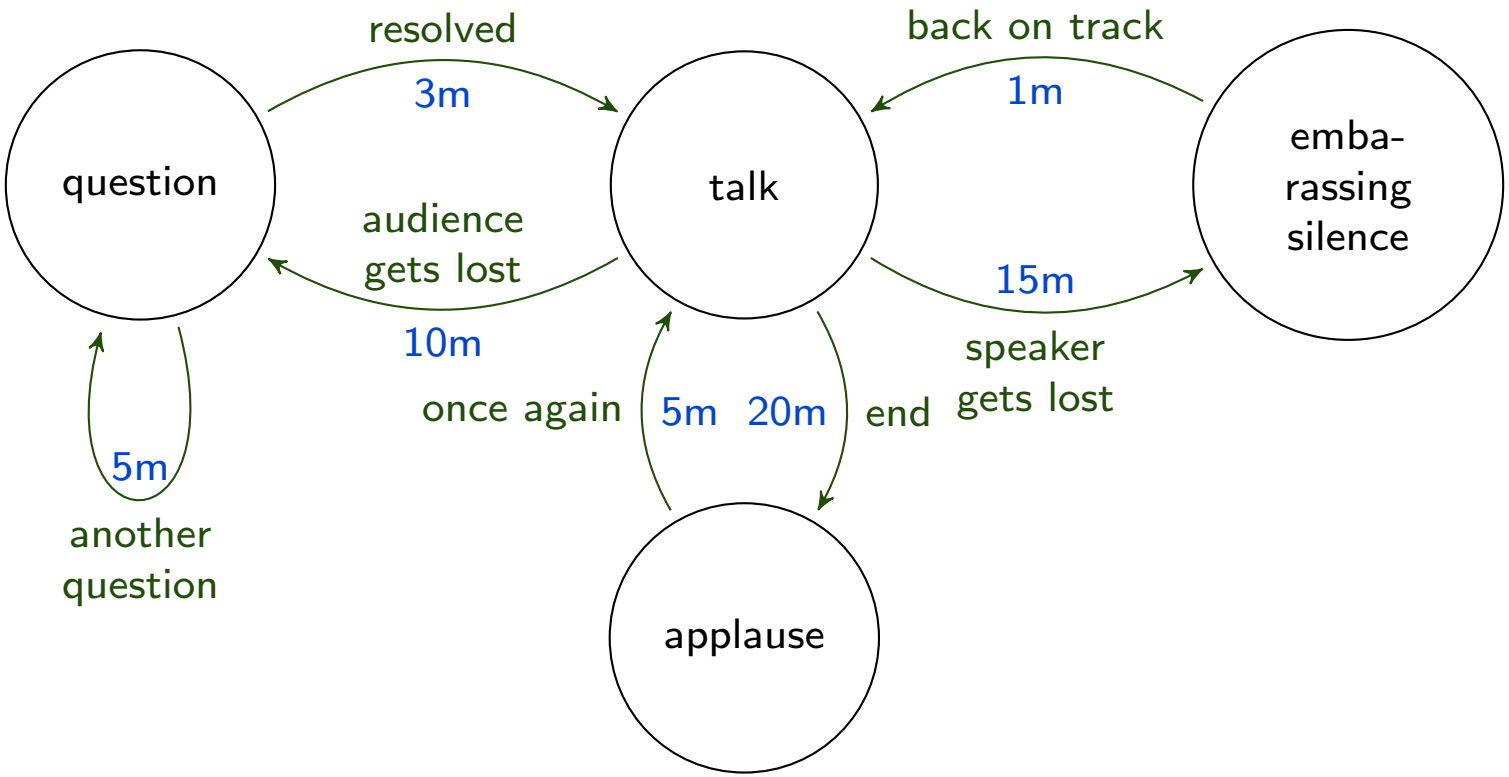


Run of the process: $s_4, 1.2, s_3, 2, s_1, 0.8, s_4, 2.6, s_2, 1.4, \ldots$

Continuous Time Markov Chain: Talk example



Continuous Time Markov Chain: Talk example



Run of the process:
$talk, 8m, question, 2m, question, 4m, talk, 17m, silence, \ldots$

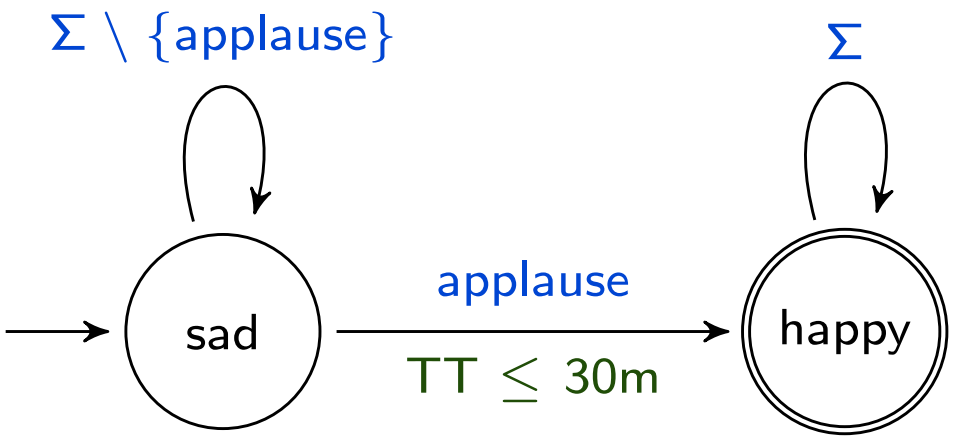
Outline

1. stochastic processes
   in particular continuous-time Markov chains (CTMCs)
2. observer timed automata
   as a specification formalism
3. results and problems
   state of the art

Specifying property: The speaker gets applause

Run of the process:
$talk, 23m, question, 2m, question, 4m, talk, 26m, silence, \ldots$

total time
$TT = 0$

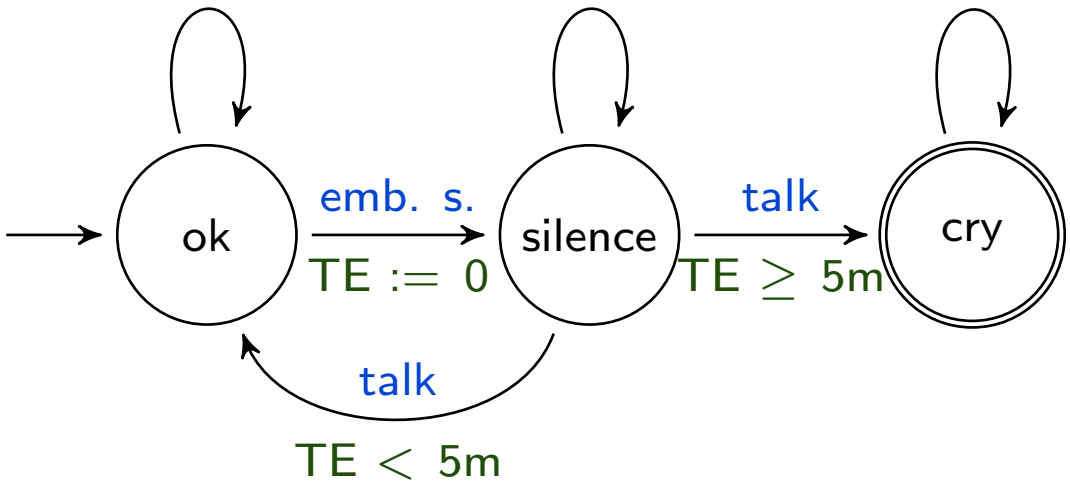


The question

Probability of reaching happiness.

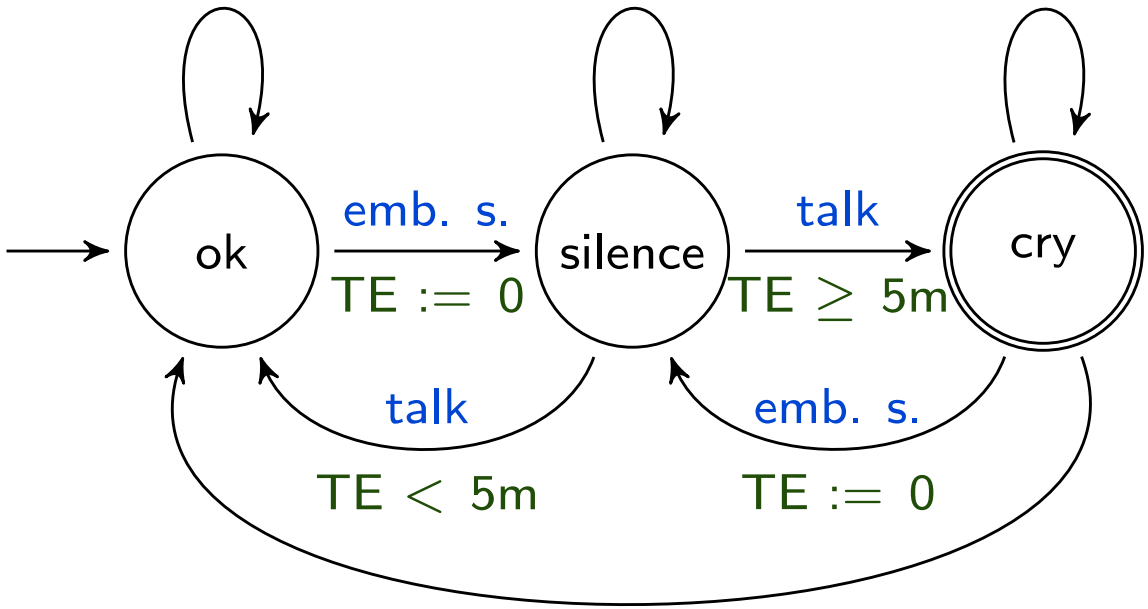
Specifying property: The speaker never cries!

time of embarassement
$TE = 0$

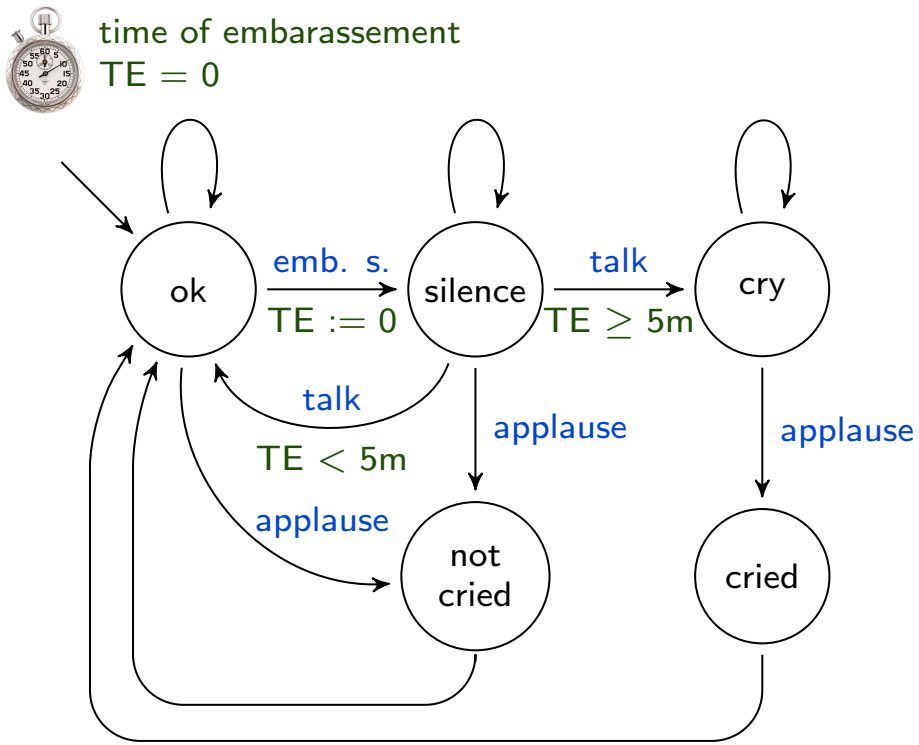


Specifying property: Does the speaker cry infinitely often?

time of embarassement
$TE = 0$



Measuring performance: How often does the speaker cry?

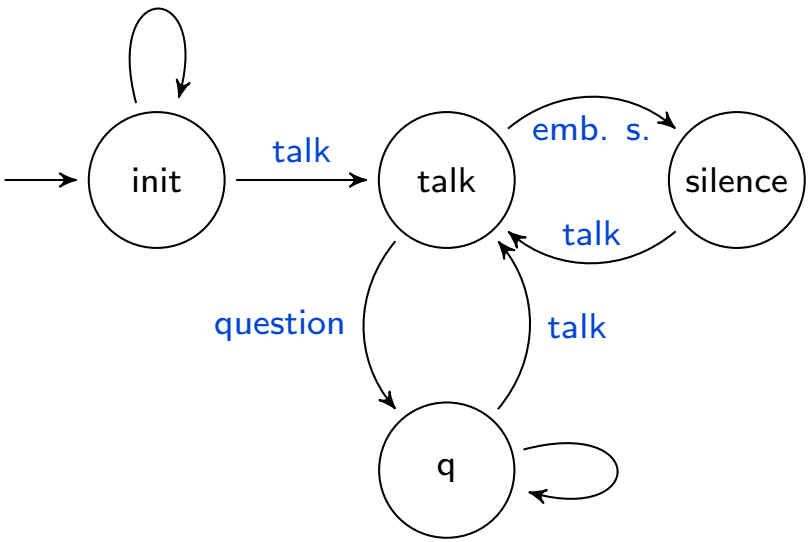


Interested in:

$$\frac{d_{cried}(\omega)}{d_{cried}(\omega) + d_{notcried}(\omega)}$$

Ratio of visits to location *cried* denoted by $d_{cried}(\omega)$ (for run ω).

Measuring performance: What percentage of time takes answering questions?



Interested in:

$$\frac{c_q(\omega)}{c_q(\omega) + c_{silence}(\omega) + c_{talk}(\omega)}$$

Ratio of time spent in location *question* denoted by $c_{question}(\omega)$ (for run ω).

Outline

1. stochastic processes
   in particular continuous-time Markov chains (CTMCs)
2. observer timed automata
   as a specification formalism
3. results and problems
   state of the art

Variants of the stochastic process

(not complete)

Events

- Exponentially distributed
  continuous-time Markov chain
- Arbitrary sequential
  semi-Markov process
- Arbitrary parallel
  generalized semi-Markov process

Game extension

- No player
  continuous-time Markov chain
- One player
  continuous-time Markov decision process
- Two players
  continuous-time stochastic games

Previous results

- arbitrary parallel events extension
  TA reachability & Muller

  R. Alur, C. Courcoubetis, D. Dill:
  Verifying Automata Specifications of Probabilistic Real-time Systems. REX Workshop 1991
  - qualitative: is the probability of accepting 1?

- CTMC
  TA reachability & Büchi

  T. Chen, T. Han, J.-P. Katoen, A. Mereacre:
  Quantitative Model Checking of Continuous-Time Markov Chains Against Timed Automata Specifications. LICS 2009
  - quantitative: approximate probability of accepting

- other results combining stochastic processes and TA directly

Our results

- 2 players extension
  trivial TA (time bounded reachability in the game)

  T. Brázdil, V. Forejt, J. Krčál, J. Křetínský, A. Kučera:
  Continuous-Time Stochastic Games with Time-Bounded Reachability. FSTTCS 2009
  - quantitative: compute optimal strategies, efficiently approximate them

Our results

- 2 players extension
  trivial TA (time bounded reachability in the game)

  T. Brázdil, V. Forejt, J. Krčál, J. Křetínský, A. Kučera:
  Continuous-Time Stochastic Games with Time-Bounded Reachability. FSTTCS 2009

  - quantitative: compute optimal strategies, efficiently approximate them

- 2 players extension + arbitrary parallel events extension
  TA reachability

  T. Brázdil, J. Krčál, J. Křetínský, A. Kučera, V. Řehák:
  Stochastic Real-Time Games with Qualitative Timed Automata Objectives. CONCUR 2010

  - qualitative: compute winning strategies, if exist

Our results

- arbitrary sequential events extension
  TA reachability + frequency measures

  T. Brázdil, J. Krčál, J. Křetínský, A. Kučera, V. Řehák:
  Measuring Performance of Continuous-Time Stochastic Processes using Timed Automata. submitted

  - quantitative: approximate probabilities + frequencies

Conclusions

Summary

- Model: *continuous-time stochastic process*
- Specification: observer *timed automata*
- Question: accepting probabilities and performance measures

Future work

- Many open problems
- Efficient algorithms (for subclasses)

Conclusions

Summary

- Model: *continuous-time stochastic process*
- Specification: observer *timed automata*
- Question: accepting probabilities and performance measures

Future work

- Many open problems
- Efficient algorithms (for subclasses)

Thank you for your attention!