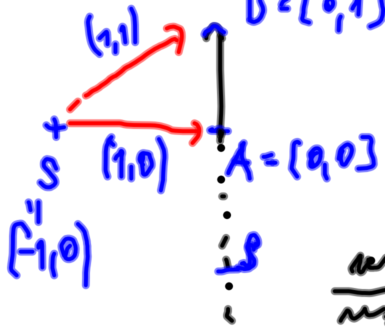


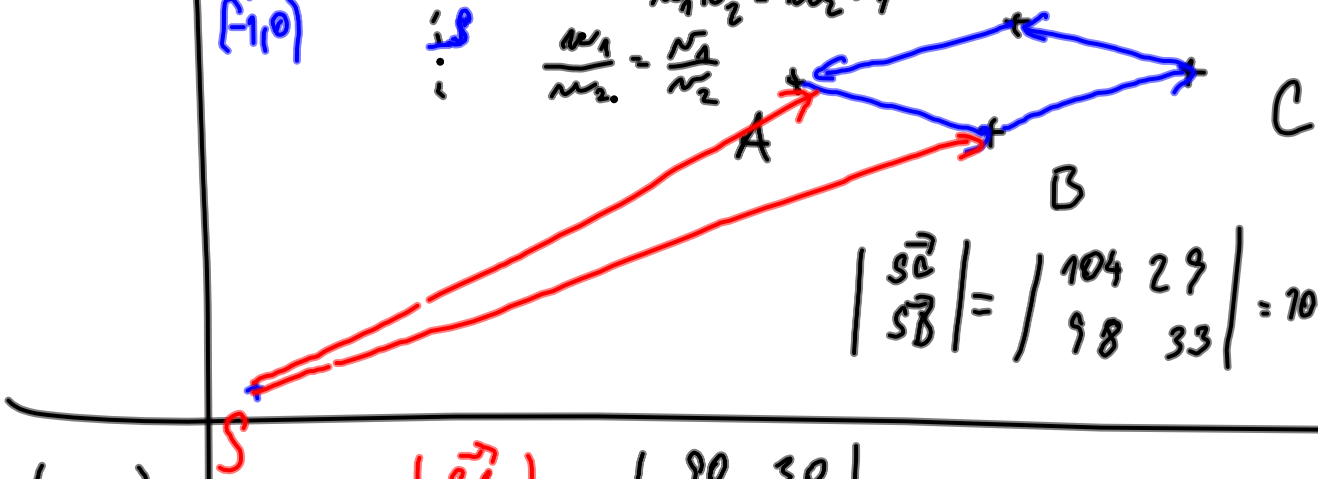

$$\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} = u_1 v_2 - u_2 v_1 = 0$$

$$u_1 v_2 = u_2 v_1$$

$$\frac{u_1}{u_2} = \frac{v_1}{v_2}$$

$\vec{u} = (u_1, u_2)$

$\vec{v} = (v_1, v_2)$




$$\begin{vmatrix} \vec{SC} \\ \vec{SB} \end{vmatrix} = \begin{vmatrix} 104 & 29 \\ 98 & 33 \end{vmatrix} = 104 \cdot 33 - 98 \cdot 29 > 0$$

$\vec{SA} = (90, 30)$

$\vec{SB} = (97, 27)$

$\vec{SC} = (104, 29)$

$\vec{SD} = (98, 33)$

$$\begin{vmatrix} \vec{SA} \\ \vec{SB} \end{vmatrix} = \begin{vmatrix} 90 & 30 \\ 97 & 27 \end{vmatrix} = 90 \cdot 27 - 30 \cdot 97 < 0$$

$\Rightarrow$ S je napravo od $\vec{AB}$ $\Rightarrow$

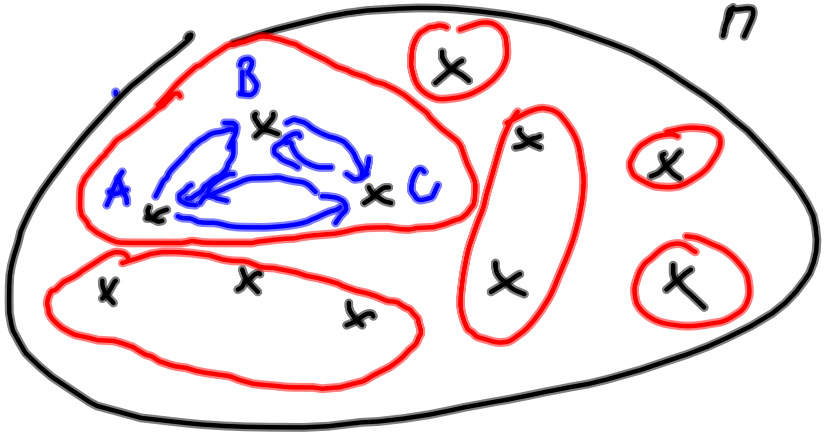
$$\begin{vmatrix} \vec{SB} \\ \vec{SC} \end{vmatrix} = \begin{vmatrix} 97 & 27 \\ 104 & 29 \end{vmatrix} = 97 \cdot 29 - 27 \cdot 104 =$$ $\Rightarrow$ AB je vidět z S

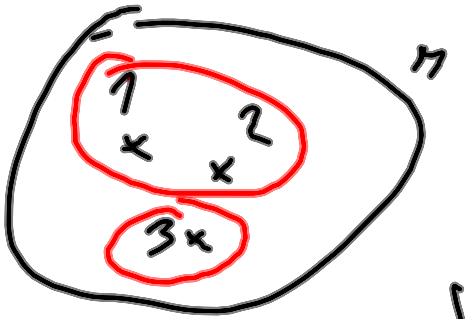
$$= \cancel{97 \cdot 27} + 97 \cdot 2 - \cancel{27 \cdot 97} - 7 \cdot 27 > 0 \Rightarrow$$

$\Rightarrow$ BC není vidět z S

$$\begin{vmatrix} \vec{SD} \\ \vec{SA} \end{vmatrix} = \begin{vmatrix} 98 & 33 \\ 90 & 30 \end{vmatrix} = 98 \cdot 30 - 90 \cdot 33 = 8 \cdot 30 - 3 \cdot 90 < 0$$

$\Rightarrow$ AD je vidět z S

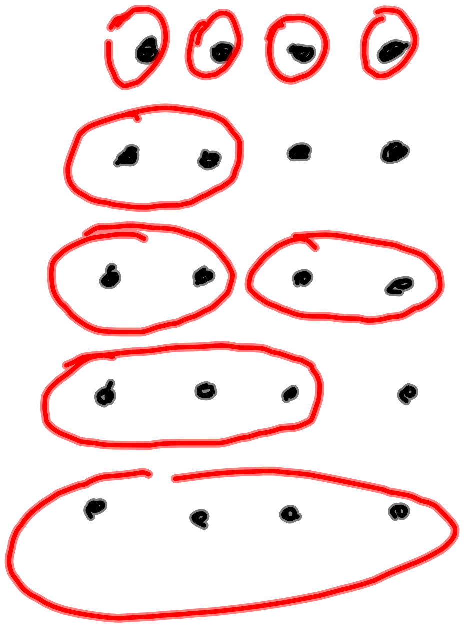



$$\{(1,2),(2,1),(1,1),(2,2),(3,3)\}$$

$$x = 3k + r_1$$
$$y = 3l + r_2$$
$$3 \mid (x-y) = 3(k-l) + (r_1 - r_2) \iff 3 \mid r_1 - r_2 \Rightarrow$$
$$x \equiv y \pmod 3 \qquad \iff r_1 = r_2$$

Spočítáme počet rozkladů čtyřprvkové množiny:



1

6

3

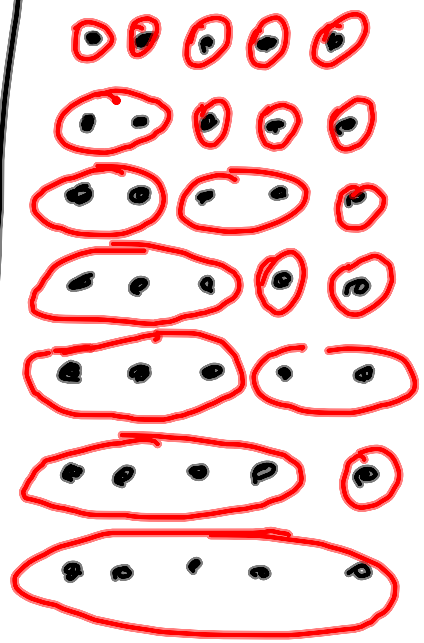
4

1

---

15

Nejdříve spočítejme počet relací ekvivalence na 5-ti prvkové množině:



1

10

15

10

10

5

1

---

52

---

Relací splňujících podmínky ze zadání je
$52 - 15 = 37.$

$$\begin{pmatrix} a & b & c & d & e & f \\ 1 & 2 & 8 & \cdot & \cdot & \cdot \end{pmatrix} + \begin{pmatrix} x & x & x & x & x & x \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} a+x & b+x \\ & \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

---

$$A \cdot A^{-1} = A^{-1} A = E$$

$$ax = b \Rightarrow x = \frac{b}{a} = b \cdot a^{-1}$$

$$\frac{A \cdot \vec{x} = \vec{b}}{\vec{x} = A^{-1} \cdot \vec{b}}$$

$$\begin{aligned} x + y + 2z &= 1 \\ 2x - y - 3z &= 1 \\ x + y + z &= 2 \end{aligned} \quad \sim \quad \overbrace{\begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & -3 \\ 1 & 1 & 1 \end{pmatrix}}^{A} \overbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}^{\vec{x}} = \overbrace{\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}}^{\vec{b}}$$

$$\underbrace{\begin{pmatrix}1&2&4\\1&0&1\\-1&1&0\end{pmatrix}}_{A}\;\left|\;\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right.$$

$$\underbrace{P_k \cdots P_3 P_2 P_1}_{A^{-1}} A = E$$

$$\begin{pmatrix}1&2&4\\0&-2&-3\\0&3&4\end{pmatrix}\;\left|\;\begin{pmatrix}1&0&0\\-1&1&0\\1&0&1\end{pmatrix}\right.$$

$$\begin{pmatrix}1&2&4\\0&1&\frac{3}{2}\\0&3&4\end{pmatrix}\;\left|\;\begin{pmatrix}1&0&0\\\frac{1}{2}&-\frac{1}{2}&0\\1&0&1\end{pmatrix}\right.$$

$$\begin{pmatrix}1&2&4\\0&1&\frac{3}{2}\\0&0&-\frac{1}{2}\end{pmatrix}\;\left|\;\begin{pmatrix}1&0&0\\\frac{1}{2}&-\frac{1}{2}&0\\-\frac{1}{2}&\frac{3}{2}&1\end{pmatrix}\right.$$

$$\begin{pmatrix}1&2&4\\0&1&\frac{3}{2}\\0&0&1\end{pmatrix}\;\left|\;\begin{pmatrix}1&0&0\\\frac{1}{2}&-\frac{1}{2}&0\\1&-3&-2\end{pmatrix}\right.$$

$$\begin{pmatrix}1&2&0\\0&1&0\\0&0&1\end{pmatrix}\;\left|\;\begin{pmatrix}-3&12&8\\-1&4&3\\1&-3&-2\end{pmatrix}\right.$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\;\left|\;\begin{pmatrix}-1&4&2\\-1&4&3\\1&-3&-2\end{pmatrix}\right.$$

Pr.:

$$\begin{pmatrix}1&2&4\\1&0&1\\-1&1&0\end{pmatrix}\begin{pmatrix}-1&4&2\\-1&4&3\\1&-3&-2\end{pmatrix} = \Big($$

$$A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 2 \\ -1 & 4 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ -5 \end{pmatrix}$$

$$\begin{aligned} x + y - z &= 1 \\ -x + 2y &= -1 \\ 2x - y - z &= 2 \end{aligned}$$

$$3y - z = 0 \Rightarrow z = 3y$$

$$x + y - z = 1$$

$$x = 1 - y + z = 1 + 2y$$

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$A = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ -1 & 2 & 0 & -1 \\ 2 & -1 & -1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & -3 & 1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Řešení soustavy jsou trojice typu $(1+2y, y, 3y)$, $y \in \mathbb{R}$.