$$\begin{aligned} 1 &\mapsto 0\cdot 1 + 0\cdot x + 0\cdot x^2 \\ x &\mapsto 1\cdot 1 + 0\cdot x + 0\cdot x^2 \\ x^2 &\mapsto 0\cdot 1 + 2\cdot x + 0\cdot x^2 \end{aligned} \quad \sim \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} =: D$$

$$\underline{f} = \Big(\underset{1+x^2}{\underset{\wr}{(1,0,1)}}, \underset{x}{\underset{\wr}{(0,1,0)}}, \underset{x+x^2}{\underset{\wr}{(0,1,1)}} \Big).$$

Matice přechodu od $\underline{f}$ k $\underline{e}$ je

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$T^{-1} \circ D \circ T = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 1 & 3 \\ 0 & -1 & -1 \end{pmatrix}$$

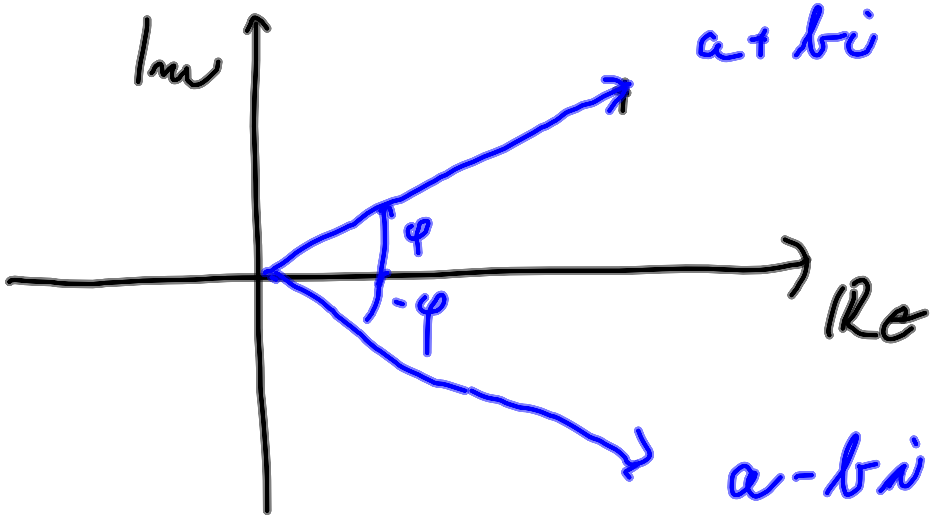
$$r_n = F_{n+6} = F_{n+5} + F_{n+4} = (F_{n+4} + F_{n+3}) + F_{n+4} =$$

$$= 2F_{n+4} + F_{n+3} = 2(F_{n+3} + F_{n+2}) + F_{n+3} =$$

$$= 3F_{n+3} + 2F_{n+2} = 5F_{n+2} + 3F_{n+1} =$$

$$= 8F_{n+1} + 5F_n$$

$$(5, 8)$$



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(a+bi)\cdot(2+i) = 2a-b+(a+2b)i$$

$$(a\overset{2}{,}b) \qquad (2a-b\overset{2}{,}a+2b)$$

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$\underline{f} : (1-i,\ 1+i) \sim \left((1,-1),\ (1,1) \right)$$

Matice přechodu od $\underline{f}$ k $\underline{e}$ je

$$T = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Matice konjugace v bázi $\underline{f}$ je

$$T^{-1} \circ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \circ T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Matice násobení číslem $(2+i)$ v bázi $\underline{f}$ je

$$T^{-1} \circ \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \circ T = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

Vlastní vektor $\vec{v} \in \mathbb{R}^n$ matice A je takový vektor $(\vec{v} \neq \vec{o})$

$$A \cdot \vec{v} = k \cdot \vec{v} \quad , \quad \lambda \in \mathbb{R}$$

$$A\vec{v} - \lambda \cdot E \cdot \vec{v} = \vec{o}$$

$$(A - \lambda E)\,\vec{v} = \vec{o}$$

jestliže $\vec{v}$ je nenulovým řešením, pak

$$|A - \lambda E| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 - 4 = \lambda^2 - 2\lambda - 3 = (\lambda-3)(\lambda+1) = 0$$

$$\Rightarrow \lambda_1 = 3, \quad \lambda_2 = -1$$

Spočtejme vlastní vektor odpovídající vlastní hodnotě $\lambda = 3$:

$$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow 2x - 2y = 0 \Leftrightarrow$$

$$\Leftrightarrow x = y$$

$$\langle (1; 1) \rangle$$

$\lambda = -1$: $\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow x + y = 0 \Leftrightarrow x = -y$

$\langle (1, -1) \rangle$

$$\begin{vmatrix} \frac{1}{3} - k & -\frac{1}{3} & -\frac{4}{3} \\ \frac{2}{3} & \frac{4}{3} - k & \frac{4}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{7}{3} - k \end{vmatrix} = 0 \qquad s = 3k$$

$$\begin{vmatrix} 1 - s & -1 & -4 \\ 2 & 4 - s & 4 \\ 2 & 1 & 7 - s \end{vmatrix} = 0 \Leftrightarrow$$

$$(1-s)(4-s)(7-s)-16+8(4-s)-4(1-s)+2(7-s)=$$

$$=(4-5s+s^2)(7-s)+26-6s=$$

$$=(28-39s+12s^2-s^3)+26-6s=$$

$$=-s^3+12s^2-45s+54=(s-3)(-s^2+9s-18)=$$

$$=-(s-3)(s-3)(s-6)$$

$$\begin{array}{c|cccc} & -1 & 12 & -45 & 54 \\ \hline 3 & -1 & 9 & -18 & 0 \end{array}$$

$$-s^3+12s^2-45s+54=$$

$$=s\overbrace{(-s^2+12s-45)}^{-18}+54=$$

$$=s(s\overbrace{(-s+12)}^{9}-45)+54=$$

$$=s(s(-1\cdot s+12)-45)+54$$

Vlastní čísla jsou $\lambda_{1,2} = 1,\ \lambda_3 = 2$

$\lambda = 1:$
$$\begin{pmatrix} -2 & -1 & -4 \\ 2 & 1 & 4 \\ 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff$$

$$2x + y + 4z = 0 \iff y = -2x - 4z$$

$$\left\langle (\underline{1}, -2, \underline{0}),\ (\underline{0}, -4, \underline{1}) \right\rangle$$

$$\begin{pmatrix} -5 & -1 & -4 \\ 2 & -2 & 4 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -5 & -1 & -4 \\ 2 & -2 & 4 \\ 2 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & -6 & 6 \\ 0 & 3 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad x - y + 2z = 0 \Rightarrow x = y - 2z = -z$$

$$y - z = 0 \Rightarrow y = z$$

$$\langle (-1, 1, 1) \rangle$$

Pr.: $\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & -\frac{5}{3} \\ \frac{2}{3} & \frac{4}{3} & \frac{4}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$

$$s = 2k$$

$$\begin{vmatrix} \frac{1}{2}-k & -\frac{1}{2} & \frac{7}{2} \\ 0 & 1-k & 1 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{7}{2}-k \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-s & -1 & 7 \\ 0 & 2-s & 2 \\ -1 & -1 & 7-s \end{vmatrix} = 0 \iff (1-s)(2-s)(7-s)+2$$

$$+7(2-s)+2(1-s) =$$

$$= (2-3s+s^2)(7-s) + 18 - 9s =$$

$$= (14-23s+10s^2-s^3) =$$

$$= -(s^3-10s^2+32s-32) = 0$$

$$\begin{array}{c|cccc} & 1 & -10 & 32 & -32 \\ \hline 2 & 1 & -8 & 16 & 0 \end{array}$$

$$s^3 - 10s^2 + 32s - 32 = (s-2)(1 \cdot s^2 - 8s + 16) =$$
$$= (s-2)(s-4)^2$$

$$\Rightarrow \lambda_1 = 1, \; \lambda_{2,3} = 2$$

$$\lambda_1 = 1: \quad \begin{pmatrix} -1 & -1 & 7 \\ 0 & 0 & 2 \\ -1 & -1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \quad \begin{aligned} z &= 0 \\ x &= -y \end{aligned}$$

$$\langle (1, -1, 0) \rangle$$

$\lambda_{2,3} = 2$

$$\begin{pmatrix} -3 & -1 & 7 \\ 0 & -2 & 2 \\ -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 & -1 & 7 \\ 0 & -2 & 2 \\ -1 & -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -3 \\ 0 & 2 & -2 \\ 0 & -2 & 2 \end{pmatrix} \Rightarrow$$

$$\Rightarrow y = z \ \& \ x + y - 3z = 0 \Rightarrow x = 3z - y = $$
$$= 2z$$

$$\langle (2,1,1) \rangle$$

Uvažme bázi $f = ((2,1,1),(1,0,1),(-1,1,0))$

V této bázi má uvažovaná projekce matici

$$\left.\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.\right\} P$$

Matice přechodu od f k e je

$$T = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$T \cdot P \cdot T^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \end{pmatrix}.$$