

$F: V \rightarrow W$
 $F(u+v) = F(u) + F(v)$
 $F(au) = aF(u)$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$(u_1, u_2, u_3) = u$ $f(u) = a_{11}u_1 + a_{21}u_2$
 $(v_1, v_2) = v$ $f(v) = a_{12}v_1 + a_{22}v_2$
 $x_1u_1 + x_2u_2 + x_3u_3 = u$ $f(u) = a_{13}x_1 + a_{23}x_2$

$$F(u) = x_1(a_{11}u_1 + a_{21}u_2) + x_2(a_{12}u_1 + a_{22}u_2) + x_3(a_{13}u_1 + a_{23}u_2)$$

$$= (a_{11}x_1 + a_{12}x_2 + a_{13}x_3)u_1 + (a_{21}x_1 + a_{22}x_2 + a_{23}x_3)u_2$$

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$V \xrightarrow{f} W \xrightarrow{g} Z$
 $\downarrow \mu \quad \downarrow \nu \quad \downarrow \rho$
 $K^m \xrightarrow{A} K^m \xrightarrow{B} K^l$
 $\downarrow \tau \quad \downarrow \delta$
 $A \cdot x = y$
 $B \cdot y = (B \cdot A) \cdot x = z = C \cdot x$

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$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
 $A^2 = A \cdot A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = A$

$B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $R_1(x) = \langle 1, x \rangle$
 $B^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $ax + b \mapsto a$
 $R_2(x) = \langle x, x^2 \rangle$

diagonal: $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

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$C = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

$D = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
 $D \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix} \in \mathbb{C}^2$
 $v = \begin{pmatrix} 1 \\ i \end{pmatrix}$
 $D \cdot v = \begin{pmatrix} -i \\ 1 \end{pmatrix} = i \cdot \begin{pmatrix} 1 \\ i \end{pmatrix} = i \cdot v$

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$A - \lambda E = \begin{pmatrix} a_{11} - \lambda & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} - \lambda \end{pmatrix}$

$\det(A - \lambda E) = (-1)^n \lambda^n + (-1)^{n-1} (a_{11} + a_{22} + \dots + a_{nn}) \lambda^{n-1} + \dots + |A| \lambda^0$

polynomial in λ of degree n

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$f(x_i) = a_i x_i$ $a_i \neq a_j, i \neq j$

$c_1 u_1 + c_2 u_2 + \dots + c_n u_n = 0$ f $c_i \neq 0$

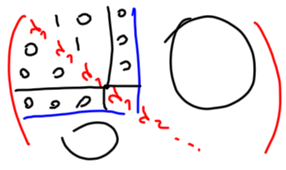
$c_1 f(u_1) + \dots + c_n f(u_n) = 0$

~~$c_1 a_1 u_1 + \dots + c_n a_n u_n = 0$~~

~~$c_1 a_1 u_1 + c_2 a_2 u_2 + \dots + c_n a_n u_n = 0$~~

~~$c_1 (a_2 - a_1) u_1 + \dots + c_n (a_n - a_1) u_n = 0$~~

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$$\begin{aligned} \begin{vmatrix} a-\lambda & b \\ 0 & b-\lambda \end{vmatrix} &= (a-\lambda)(b-\lambda) = 0 \\ \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} &= \lambda^2 + 1 = 0 \Rightarrow \lambda_{1,2} = \pm i \\ \begin{vmatrix} -\lambda & -1 \\ 0 & -\lambda \end{vmatrix} &= +\lambda^2 = 0 \Rightarrow \lambda_{1,2} = 0 \end{aligned}$$


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$$\begin{aligned} (x \ y) \begin{pmatrix} x' \\ y' \end{pmatrix} &= xx' + yy' \\ V &= \langle u_1, \dots, u_n \rangle \\ V^* &= \langle u_1^*, \dots, u_n^* \rangle \\ u_i^*(u_j) &= \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases} \\ &\langle u, \quad v \rangle \end{aligned}$$

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$$\begin{aligned} (x_1 \ \dots \ x_n) \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} &= x_1 y_1 + \dots + x_n y_n \\ \|x\|^2 &= x_1^2 + \dots + x_n^2 \end{aligned}$$

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