$$\frac{1}{1-x} \overset{v.f.p.}{\longleftrightarrow} 1,1,1,1,$$

$$S = a_0 + a_0 q + a_0 q^2 + \dots$$

$$qS = a_0 q + a_0 q^2 + \dots$$

$$S(1-q) = a_0$$

$$S = \frac{a_0}{1-q} \qquad \boxed{|q|<1}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} 1 \cdot x^n$$

$$\sum_{n=0}^{\infty} a_0 q^n = \frac{a_0}{1-q} \longleftrightarrow a_0, a_0 q, a_0 q^2, \dots$$

$$\sum_{n=0}^{\infty} \cancel{a_n} \cdot x^n \text{ konv.}$$

$$|x|<1$$

$$\sum_{n=0}^{\infty} a_n \cdot x^n$$

$$r = \frac{1}{\limsup_{n\to\infty} \sqrt[n]{|a_n|}}$$

$$f(x) = (1-x)^{-1} \qquad f'(x) = +1(1-x)^{-2}$$

$$f''(x) = +2(1-x)^{-3}$$

$$f^{(n)}(x) = \dots = n!(1-x)^{-(n+1)} = \frac{n!}{(1-x)^{n+1}}$$

$$T_{f_0}(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} \cdot x^n = \sum \frac{n!}{n!} x^n = \sum 1 \cdot x^n$$

$$\sum a_n x^n$$

$$\sum \frac{a_n}{n!} x^n$$

$$(1,1,1,\dots) \overset{\text{o.v.f.}}{\longleftrightarrow} \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$(1,1,1,\dots) \overset{\text{e.v.f.}}{\longleftrightarrow} e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

$$|a_n| \le K$$

$$\left|\sum a_n x^n\right| \le \sum |a_n x^n| = \sum |a_n|\cdot|x|^n \le \sum (K\cdot|x|)^n$$

geom. řada

s $q = K\cdot|x|$

konv. $\Leftrightarrow$ $|K\cdot x| < 1$

$$\boxed{|x| < \frac{1}{K}}$$

$$a_0, a_1, \ldots \longleftrightarrow \sum a_n x^n = a(x)$$

$$b_0, b_1, \ldots \longleftrightarrow \sum b_n x^n = b(x)$$

$$a_0+b_0, a_1+b_1, \ldots \longleftrightarrow \sum (a_n+b_n)x^n = \sum a_n x^n + \sum b_n x^n = a(x)+b(x)$$

$$\alpha a_0, \alpha a_1, \ldots \longleftrightarrow \sum \alpha a_n x^n = \alpha \sum a_n x^n = \alpha \cdot a(x)$$

$$\underbrace{0, 0, \ldots, 0}_{k}, a_0, a_1, \ldots \longleftrightarrow \underline{\sum_{n=0}^{\infty} a_n x^{n+k}} = \sum_n a_n x^n \cdot x^k = x^k \cdot \sum a_n x^n = x^k \cdot a(x)$$

$$x^0, x^1, x^2, \ldots$$

$$\cancel{1}, \cancel{2}, \cancel{3}, 4, 5, 6, \ldots \longrightarrow \underline{4, 5, 6}$$

$$\cancel{\ldots} \quad \sum_{n=0}^{\infty} (n+1) \cdot x^n \qquad \frac{\sum (n+1) x^n - (1 \cdot x^0 + 2 \cdot x^1 + 3x^2)}{x^3}$$

$$\sum a_n \cdot x^n \overset{x = \alpha \cdot y}{=\!=} \sum \boxed{a_n \cdot \alpha^n} \cdot y^n$$

$$\sum a_n \cdot x^n \overset{x = y^k}{=\!=} \sum_n a_n \cdot y^{k \cdot n}$$

$$\frac{1}{1-x} \longleftrightarrow 1, 1, 1, \ldots$$

$$\frac{1}{1-2x} \longleftrightarrow 1, 2, 4, 8, \ldots$$

$$\frac{1}{1-x^2} \longleftrightarrow 1, 0, 1, 0, 1, 0, \ldots$$

$$\sum_{n=0}^{\infty} a_n x^n = a(x) \quad \text{v oboru konv. platí!}$$

$$\sum_{n=1}^{\infty} n \cdot a_n \cdot x^{n-1} = a'(x)$$

$(-1)^k$

$$\frac{1}{(1-x)^n} = (1-x)^{-n} = \sum_{k=0}^{\infty} \binom{-n}{k} (-x)^k, \quad \text{stačí } \binom{-n}{k} = \binom{n+k-1}{n-1} = \binom{n+k-1}{k}$$

$$\frac{(-n)(-n-1)\cdots(-n-k+1)}{k!} = \frac{n(n+1)\cdots(n+k-1)\cdot(-1)^k}{k!} = (-1)^k \binom{n+k-1}{k}$$

$$\sum_{n=0}^{\infty} [n=1]\, x^n = x$$

$$F_n = F_{n-1} + F_{n-2} + \overline{[n=1]}^{\,n\geq 2}$$

$\forall n \in \mathbb{N}_0$

$F_0 = 0$

$F_1 = 1$

$F_{-n} = 0$

$$\sum_{n=0}^{\infty} F_n x^n = \sum_{n=0}^{\infty} F_{n-1} x^n + \sum_{n=0}^{\infty} F_{n-2} x^n + \sum_{n=0}^{\infty} [n=1] x^n$$

$$F_{-1}x^0 + F_0 x^1 + F_1 x^2 + \dots$$

$$x(\underbrace{F_{-1}x^{-1}}_{0} + \underbrace{F_0 x^0 + F_1 x + \dots}_{F(x)})$$

$$F(x) = xF(x) + x^2 F(x) + x$$

$$F(x)(1 - x - x^2) = x$$

$$F(x) = \frac{x}{1 - x - x^2}$$

$$x^2 + x - 1 = 0 \iff$$

$$x_{1,2} = \frac{-1 \pm \sqrt{5}}{2}$$

Ideme $\dfrac{a}{1 - \lambda \cdot x} = a \sum \lambda^n \cdot x^n$

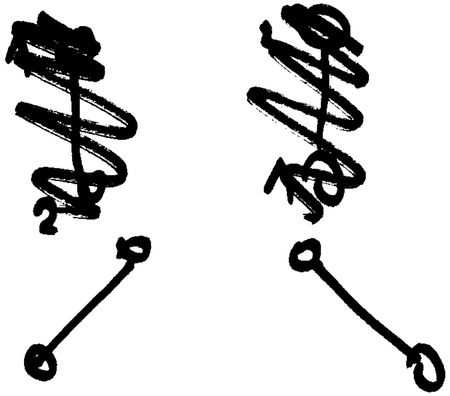
$$\frac{A}{(x-\alpha)^k} = \frac{A/(-\alpha)^k}{\left(1 - \frac{x}{\alpha}\right)^k}$$

$= a_0$

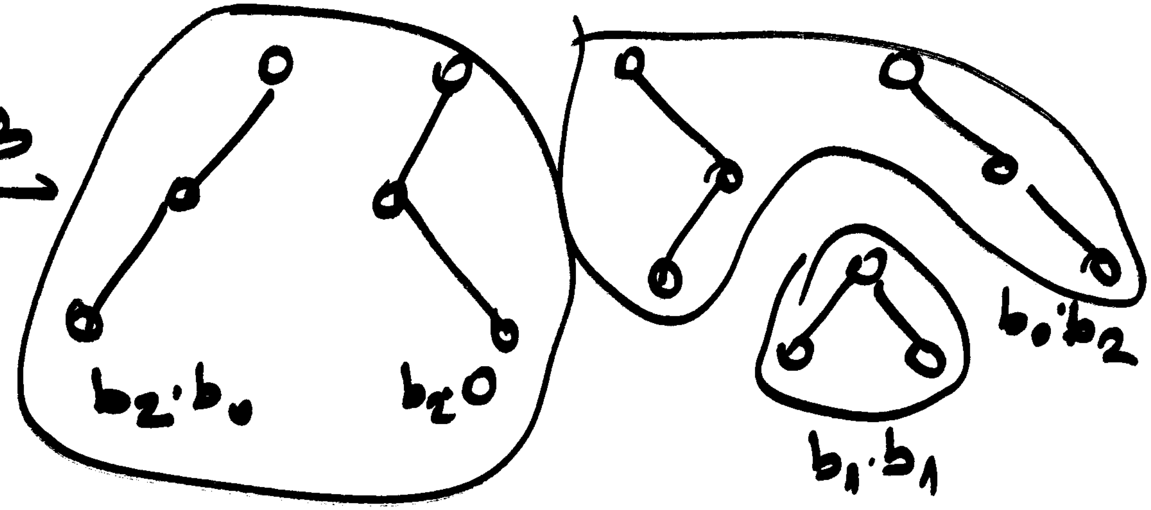
γ

$$F_n = a \cdot \lambda_1^n + b \cdot \lambda_2^n$$

$n=2$



$n=3$



$$\underline{b_n = \sum_{k=0}^{n-1} b_k \cdot b_{n-k-1}} + [n=0]$$

$$B(x) = x \cdot B(x)^2 + 1$$

$$x \cdot B(x)^2 - B(x) + 1 = 0 \Rightarrow B(x) = \frac{1 \pm \sqrt{1-4x}}{2x}$$

$$\lim_{x \to 0^+} \frac{1+\sqrt{1-4x}}{2x} = +\infty$$

$$\lim_{x \to 0^+} \frac{1-\sqrt{1-4x}}{2x} = \lim_{x \to 0^+} \frac{1-(1-4x)}{2x(1+\sqrt{1-4x})} = \lim_{x \to 0^+} \frac{4\cancel{x}}{2\cancel{x}(1+\sqrt{1-4x})} = \frac{4}{2 \cdot 2} = \underline{1}$$

$$B(x) = \frac{1-\sqrt{1-4x}}{2x}$$

$$\binom{1/2}{k} = \frac{\frac{1}{2}\cdot\left(\frac{1}{2}-1\right)\cdots\left(\frac{1}{2}-k+1\right)}{k!} = \frac{1}{2k}\binom{-\frac{1}{2}}{k-1}$$

$\underline{k \geq 1}$

$$\binom{-\frac{1}{2}}{n} = \frac{-\frac{1}{2}\cdot\left(-\frac{3}{2}\right)\cdots\left(-\frac{1}{2}-n+1\right)}{n!} = (-1)^n \cdot \frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdots\frac{2n-1}{2}}{n!} =$$

$$= \frac{(-1)^n}{2^n}\cdot\frac{1 \overset{2}{\cdot} 3 \overset{4}{\cdot} 5 \overset{6}{\cdot} \cdots (2n-1)\overset{2n}{\cdot}}{n!}\cdot\frac{2^n\cdot n!}{2^n\cdot n!} = \frac{(-1)^n\cdot 2n!}{\cancel{2^{2n}}\,4^n\cdot(n!)^2} =$$

$$\left(-\frac{1}{4}\right)^n\cdot\binom{2n}{n}$$