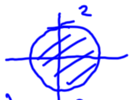
(98) $f(x,y) = (2x^2+3y^2)\cdot e^{-(x^2+y^2)}$

$M: x^2+y^2 \leq 4$




ST. BODY:

$f'_x = -2x e^{-x^2-y^2}(2x^2+3y^2-2) = 0$

$f'_y = -2y e^{-x^2-y^2}(2x^2+3y^2-3) = 0$

i) $x=0$

$-2y e^{-y^2}(0+3y^2-3)=0$

$2y e^{-y^2}(3y^2-3)=0$

$\Rightarrow \begin{cases} y=0 \\ 3y^2-3=0 \Rightarrow y=\pm 1 \end{cases}$

11 10-18:44

$A_1=[0,0]$ $\quad \boxed{f(A_1)=0}$ MIN.

$A_2=[0,1]$ $A_3=[0,-1]$ $\quad \boxed{f(A_2)=\frac{3}{e}=f(A_3)}$ MAX

(ii) $y=0$ $\quad -2x e^{-x^2}(2x^2-2)=0$

$\Rightarrow \begin{cases} x=0 \\ x=\pm 1 \end{cases}$

$A_4=[1,0]$ $A_5=[-1,0]$ $f(A_4)=\frac{2}{e}=f(A_5)$

(iii) $\left.\begin{array}{l} 2x^2+3y^2-2=0 \\ 2x^2+3y^2-3=0 \end{array}\right\} \ominus$

$1=0$ nemá řeš.

11 10-19:08

HRANICE

(i) horní půlkružnice

$x^2+y^2=4 \Rightarrow \boxed{y=\sqrt{4-x^2}}$ $\quad x\in\langle -2,2\rangle$

$f(x,\sqrt{4-x^2}) =$

$= e^{-(x^2+4-x^2)}(2x^2+3\sqrt{4-x^2}^2) = e^{-4}(12-x^2)$

$f'(x,\sqrt{4-x^2}) = -2x e^{-4}$

$-2x e^{-4}=0 \iff x=0$

$y=\sqrt{4-0}=2$

$A_6=[0,2]$ $\quad f(A_6)=\frac{12}{e^4}$

11 10-19:12

(ii) dolní půlkružnice

$x^2+y^2=4 \Rightarrow y=-\sqrt{4-x^2}$

$\Downarrow$ $y^2=4-x^2 \Rightarrow y=\pm\sqrt{4-x^2}$

$y=x^2$

$x=\pm\sqrt{y}$

$x^2=1$ $\quad x=\pm 1$

$e^{-(x^2+(-\sqrt{4-x^2})^2)}(2x^2+3(-\sqrt{4-x^2})^2) = f(x,-\sqrt{4-x^2})$

$e^{-4}(12-x^2)$

$f'(x,-\sqrt{4-x^2}) = -2x e^{-4}$

$-2x e^{-4}=0 \iff x=0$

$A_7=[0,-2]$ $\quad f(A_7)=\frac{12}{e^4}$ $\quad y=-\sqrt{4-0}=-2$

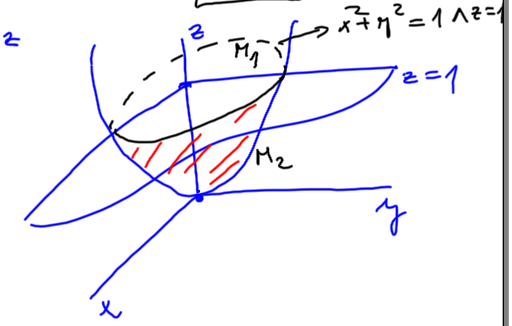
11 10-19:16

(104) $f(x,y,z) = x+2y+3z$

$M: \underbrace{x^2+y^2 \leq z}\underbrace{\leq 1}$

$x^2+y^2 \leq z$

$z\leq 1$




11 10-19:25

STAC. BODY

$f_x=1$ $\quad f_y=2$ $\quad f_z=3$ $\quad A_1=[1,2,3]\notin M$

HRANICE:

$M_1$... kruh v $z=1$, $x^2+y^2=1$

$f(x,y,1)=x+2y+3$

$\left.\begin{array}{l} f_x(x,y,1)=1 \\ f'_y(x,y,1)=2 \end{array}\right\} \notin M_1$

hraniční kružnice: $\begin{cases} k_1: y=\sqrt{1-x^2} \\ k_2: y=-\sqrt{1-x^2} \end{cases}$

11 10-19:29

$k_1$: $(z=1!)$

$$f(x, \sqrt{1-x^2}, 1) = x + 2\sqrt{1-x^2} + 3$$

$$f'(x, \sqrt{1-x^2}, 1) = 1 - \frac{2x}{\sqrt{1-x^2}}$$

$$1 - \frac{2x}{\sqrt{1-x^2}} = 0$$

$$\frac{2x}{\sqrt{1-x^2}} = 1 \quad /^2$$

$$4x^2 = 1 - x^2$$

$$x = \pm \frac{1}{\sqrt{5}} \Rightarrow 2k! \Rightarrow$$

$$\Rightarrow x = \frac{1}{\sqrt{5}} \Rightarrow y = \sqrt{1 - \frac{1}{5}} = \frac{2}{\sqrt{5}} \quad z = 1$$

$$A_2 = \left[\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 1\right] \quad \boxed{f(A_2) = 3 + \sqrt{5}} \text{ MAX}$$

11 10-19:34

$k_2$: $(z=1!)$

$$f(x, -\sqrt{1-x^2}, 1) = x - 2\sqrt{1-x^2} + 3$$

$$f'(x, -\sqrt{1-x^2}, 1) = 1 + \frac{2x}{\sqrt{1-x^2}}$$

$$1 + \frac{2x}{\sqrt{1-x^2}} = 0$$

$$x = \pm \frac{1}{\sqrt{5}} \Rightarrow 2k \Rightarrow$$

$$\Rightarrow x = -\frac{1}{\sqrt{5}} \Rightarrow y = -\frac{2\sqrt{5}}{5} \quad z = 1$$

$$A_3 = \left[-\frac{1}{\sqrt{5}}, -\frac{2\sqrt{5}}{5}, 1\right] \quad f(A_3) = 3 - \sqrt{5}$$

11 10-19:37

$M_2$: $x^2 + y^2 = z$ PARABOLOID

$$f(x, y, x^2+y^2) = x + 2y + 3x^2 + 3y^2$$

$$f'_x(x, y, x^2+y^2) = 1 + 6x$$

$$f'_y(x, y, x^2+y^2) = 2 + 6y$$

$$\left.\begin{aligned} 1 + 6x &= 0 \\ 2 + 6y &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= -\tfrac{1}{6} \\ y &= -\tfrac{1}{3} \end{aligned} \qquad z = \frac{5}{36}$$

$$A_4 = \left[-\frac{1}{6}, -\frac{1}{3}, \frac{5}{36}\right] \quad \boxed{f(A_4) = -\frac{5}{12}}$$

MIN

11 10-19:40

$f(x_1, x_2, \dots, x_n)$ SYLVESTROVO KRITÉRIUM

$$\begin{pmatrix} f_{x_1 x_1} & f_{x_1 x_2} & \cdots & f_{x_1 x_n} \\ f_{x_1 x_2} & \ddots & & \\ & & \ddots & \\ f_{x_1 x_n} & & & f_{x_n x_n} \end{pmatrix}$$

MIN: $+ + + + \dots +$ min.

MAX: $- + - + - + - \dots$ max

11 10-19:46

(117) $\int_3^4 \left( \int_x^{2x} \frac{1}{x^2 - 3x + 2} \, dy \right) dx =$

$$= \int_3^4 \left[ \frac{1 \cdot y}{x^2 - 3x + 2} \right]_x^{2x} dx = \int_3^4 \frac{2x - x}{x^2 - 3x + 2} \, dx = \int_3^4 \frac{x}{x^2 - 3x + 2} \, dx$$

$$x^2 - 3x + 2 = (x-1)(x-2)$$

$$\frac{x}{x^2 - 3x + 2} = \frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2) + B(x-1)}{(x-1)(x-2)}$$

11 10-19:50

$$A(x-2) + B(x-1) = x$$

$$Ax - 2A + Bx - B = x$$

$$\left.\begin{aligned} x^0&: -2A - B = 0 \\ x^1&: A + B = 1 \end{aligned}\right\} \oplus$$

$$-A = 1 \Rightarrow A = -1 \Rightarrow B = 2$$

$$= \int_3^4 \frac{-1}{x-1} + \frac{2}{x-2} \, dx = \left[ -\ln(x-1) + 2\ln(x-2) \right]_3^4$$

$$= -\ln 3 + 2\ln 2 - (-\ln 2 + 2\ln 1) = 3\ln 2 - \ln 3$$

11 10-19:54

(128) $\iint_A 2(x^2+y^2)\,dA$  A: $1 \le x^2+y^2 \le 4$, $y \ge |x|$

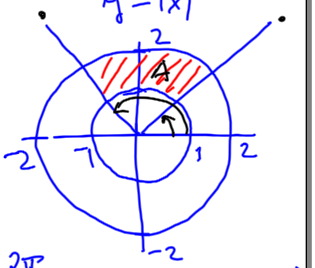
MEZE:

$r \in \langle 1, 2 \rangle$

$\varphi \in \langle \frac{\pi}{4}, \frac{3\pi}{4} \rangle$

$x = r\cos\varphi$

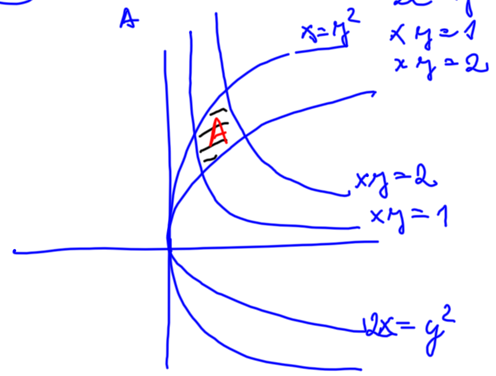
$y = r\sin\varphi$




$$\int_1^2 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} r \cdot \left(2\cdot(r^2\cos^2\varphi + r^2\sin^2\varphi)\right) d\varphi\, dr$$

11 10-20:01

(127) $I = \iint_A \sqrt{xy}\,dx\,dy$  A: $x = y^2$, $2x = y^2$, $xy = 1$, $xy = 2$




11 10-20:07

TRANS: $u = x\cdot y$, $v = \frac{y^2}{x}$

$y^2 = x$
$y^2 = 2x$
$xy = 1$
$xy = 2$

MEZE: $y^2 = x \Rightarrow 1 = \frac{y^2}{x} \to v$

$y^2 = 2x \Rightarrow 2 = \frac{y^2}{x} \to v$  $v \in \langle 1, 2 \rangle$

$u <$ $xy = 1$, $xy = 2$  $u \in \langle 1, 2 \rangle$

INT.FCE: $\sqrt{xy} = \sqrt{u}$

11 10-20:10

$J(x,y)$

$\left(\text{tr}\,(x,y) \to (u,v)\right) \circ \left(\text{tr}\,(u,v) \to (x,y)\right) = id$

$J(x,y) \cdot J(u,v) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$|J(x,y)| \cdot |J(u,v)| = 1$

$|J(x,y)| = \frac{1}{|J(u,v)|}$

$J(u,v) = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}$

$u_x = y$  $v_x = -\frac{y^2}{x^2}$

$u_y = x$  $v_y = \frac{2y}{x}$

11 10-20:15

$|J_{uv}| = \begin{vmatrix} y & x \\ -\frac{y^2}{x^2} & \frac{2y}{x} \end{vmatrix} \Rightarrow |J(u,v)| = \frac{2y^2}{x} - \left(-\frac{y^2}{x}\right) = \frac{3y^2}{x}$

$|J_{xy}| = \frac{1}{|J_{uv}|} = \frac{1}{3\frac{y^2}{x}} = \frac{1}{3v}$

$$\int_1^2\int_1^2 \frac{1}{3v}\cdot\sqrt{u}\,du\,dv = \int_1^2 \frac{1}{3v}\left[\frac{\sqrt{u^3}}{\frac{3}{2}}\right]_1^2 dv =$$

$$= \int_1^2 \frac{1}{3v}\cdot\left(\frac{2\sqrt{2}}{\frac{3}{2}} - \frac{1}{\frac{3}{2}}\right) dv = \int_1^2 \frac{4\sqrt{2}-2}{3}\cdot\frac{1}{3v}\,dv =$$

$$= \left[\frac{4\sqrt{2}-2}{3}\cdot\frac{1}{3}\ln v\right]_1^2 = \frac{4\sqrt{2}-2}{3}\cdot\frac{1}{3}\ln 2$$

11 10-20:18

(112) $[x,y] \to [\underbrace{\sqrt{x^2+y^2}}_{r}, \underbrace{\text{arctg}\,\frac{y}{x}}_{\varphi}]$  $x > 0$

$x = r\cos\varphi$

$y = r\sin\varphi$

$\sqrt{x^2+y^2} = \sqrt{r^2\cos^2\varphi + r^2\sin^2\varphi} = \sqrt{r^2(\cos^2\varphi + \sin^2\varphi)} =$

$= \sqrt{r^2} = r$

$\text{arctg}\,\frac{y}{x} = \text{arctg}\,\frac{r\sin\varphi}{r\cos\varphi} = \text{arctg}\,\frac{\sin\varphi}{\cos\varphi}$

$= \text{arctg}\,\text{tg}\,\varphi = \varphi$

11 10-20:25

$$[x,y] \longmapsto \Big[\sqrt{x^2+y^2},\ \underbrace{\pi + \operatorname{arctg}\frac{y}{x}}_{\pi+\varphi}\Big] \quad x<0$$

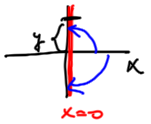
$$\sqrt{x^2+y^2} = r$$

$$\pi + \operatorname{arctg}\frac{y}{x} = \pi + \operatorname{arctg}\frac{r\sin\varphi}{r\cos\varphi} =$$

$$= \pi + \operatorname{arctg}\operatorname{tg}\varphi =$$

$$= \pi + \varphi$$

$$[0,y] \longmapsto \Big[\underset{\downarrow\, r}{y},\ \underbrace{\frac{\pi}{2}\operatorname{sgn} y}_{\varphi}\Big]$$




11 10-20:28

$$xy \xrightarrow{J} r\varphi \qquad x = r\cos\varphi$$

$$r\varphi \xrightarrow{J^{-1}} xy \qquad y = r\sin\varphi$$

$$J = \begin{pmatrix} x_r & x_\varphi \\ y_r & y_\varphi \end{pmatrix} = \begin{pmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{pmatrix}$$

$$|J| = r\cos^2\varphi + r\sin^2\varphi = r$$

$$J^{-1} \qquad |J^{-1}| = \frac{1}{r}$$

$$|J|\cdot|J^{-1}| = 1$$

11 10-20:34

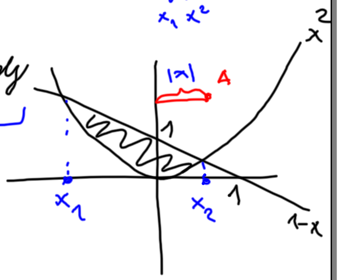
(134) $T = [x_0, y_0]$

$$\begin{cases} x_0 = \dfrac{1}{M}\displaystyle\iint_A x\,\rho(x,y)\,dx\,dy \\ y_0 = \dfrac{1}{M}\displaystyle\iint_A y\,\rho(x,y)\,dx\,dy \end{cases}$$

$$M = \int_{x_1}^{x_2}\int_{x^2}^{1-x} |x|\,dy\,dx$$

$$\begin{cases} \displaystyle\int_{0}^{x_2}\int_{x^2}^{1-x} x\,dy\,dx \\ \displaystyle\int_{x_1}^{0}\int_{x^2}^{1-x} -x\,dy\,dx \end{cases}$$

$$\underbrace{M = \iint_A \rho(x,y)\,dx\,dy}$$




$$x_1 = \frac{-1+\sqrt{5}}{2}$$

$$x_2 = \frac{-1-\sqrt{5}}{2}$$

11 10-20:38