$$\lim_{t\to 0} \frac{1}{t}\left(f(x+tv) - f(x)\right) =$$

$$= \lim_{t\to 0} \frac{1}{t}\Big(\underbrace{df(x)(tv)}_{= df(x)(v)} + \tau(tv)\Big)$$

$$\lim_{t\to 0} \frac{\tau(tv)}{t} = \|v\| \cdot \underbrace{\lim_{t\to 0} \frac{\tau(tv)}{\|tv\|}}_{=0} = 0$$

$$\lim_{v\to 0} \frac{\tau(v)}{\|v\|} = 0$$

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$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} =$$

$$= \binom{n}{0} y^n + \binom{n}{1} x y^{n-1} + \dots + \binom{n}{n-1} x^{n-1} y + \binom{n}{n} x^n$$

$$(x+y)^n = (x+y)\cdots(x+y) = \sum \binom{n}{k} x^k y^{n-k}$$

10 6-16:03

Přibližnou hodnotu vypočteme pomocí Taylorova polynomu 2. stupně funkce $f(x,y) = e^{x^3+y}$ v bodě $[0,0]$ s diferencemi $v = (\xi,\eta) = (0,05; -0,02)$.

Parciální derivace jsou:

$\frac{\partial f}{\partial x} = e^{x^3+y} \cdot 3x^2$, $\frac{\partial f}{\partial y} = e^{x^3+y}$, $\frac{\partial^2 f}{\partial x^2} = e^{x^3+y} \cdot (3x^2 \cdot 3x^2 + 6x)$, $\frac{\partial^2 f}{\partial xy} = e^{x^3+y} \cdot 3x^2$, $\frac{\partial^2 f}{\partial y^2} = e^{x^3+y}$.

Pak

$$T_2(0+\xi, 0+\eta) =$$

$$= f(0,0) + df(0,0)\cdot(\xi,\eta) + (\xi,\eta)\cdot d^2 f(0,0) \cdot \begin{pmatrix} \xi \\ \eta \end{pmatrix} =$$

$$= 1 + \eta + \eta^2.$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial y^2} \end{pmatrix}$$

$$(\xi,\eta)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} = (0,\eta)\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \eta^2$$

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kvadr. forma 2 prom.

$$(x,y)\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+cy & bx+dy \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= ax^2 + cxy + bxy + dy^2 = ax^2 + (b+c)xy + dy^2$$

$$(x,y) \longmapsto ax^2 + (b+c)xy + dy^2 \in R$$

$$ax^2 + 2bxy + cy^2$$

má matici $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$

indefinitní: $x^2 - y^2$

$(2,1) \mapsto 3$

$(1,2) \mapsto -3$

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