$f(x,y) = \sqrt{|x \cdot y|}$ $[0,0] = [x_0, y_0]$

$x = y$ $\sqrt{|x \cdot x|} = |x|$

$u = (1,1)$

$$\lim_{h \to 0} \left[\frac{\sqrt{|(x_0+h)\cdot(y_0+h)|} - \sqrt{|x_0, y_0|}}{h} = \frac{\sqrt{|h^2|}}{h} \right] = \lim_{h \to 0} \frac{|h|}{h} \quad \begin{matrix} -1 \\ 1 \end{matrix}$$

$\Rightarrow$ neexistuje lim $\Rightarrow$ neexistuje derivace

10 7-16:00

$\arcsin \frac{0{,}48}{1{,}05} = 0{,}475$

$df(x,y) = f_x(x,y)\,dx + f_y(x,y)\cdot dy$

$[x_0, y_0]$

$f(x,y) \doteq f(x_0,y_0) + f_y(x_0,y_0)(y-y_0) + f_x(x_0,y_0)(x-x_0)$

$\arcsin \frac{x}{y} = f(x,y)$

$f_x(x,y) = \frac{1}{\sqrt{1-\frac{x^2}{y^2}}} \cdot \frac{1}{y}$ $f_y(x,y) = \frac{1}{\sqrt{1-\frac{x^2}{y^2}}} \cdot x \cdot (-y^{-2})$

$\arcsin \frac{0{,}48}{1{,}05} \doteq \arcsin \frac{0{,}5}{1} + \frac{1}{\sqrt{1-\frac{1}{4}}}(-0{,}02) + \frac{1}{\sqrt{1-\frac{1}{4}}} \cdot \frac{1}{2} \cdot (0{,}05) \doteq 0{,}472$

10 7-16:06

$1{,}04^{2{,}02} = 1{,}082$

$f(x,y) = x^y$ $[x_0, y_0] = [1,2]$

$f_x(x,y) = y \cdot x^{y-1}$

$f_y(x,y) = x^y \cdot \ln x$

$f(1{,}04; 2{,}02) = 1^2 + 2 \cdot 1^1 (0{,}04) + 1^2 \cdot \ln 1 (0{,}02$

$= 1 + 0{,}08 = 1{,}08$

10 7-16:10

$f(x,y) = x^2 + xy + 2y^2$ $[1,1,4]$

$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0) \cdot (y-y_0)$

$f_x(x,y) = 2x + y = 3$

$f_y(x,y) = x + 4y = 5$

$z = 4 + 3(x-1) + 5\cdot(y-1)$

$z = 4 + 3x - 3 + 5y - 5$

ρ: $3x + 5y - z - 4 = 0$

10 7-17:19

$\mathrm{arctg}\,\frac{x}{y}$ $[1, -1, -\frac{\pi}{4}]$

$f_x(x,y) = \frac{1}{1+\frac{x^2}{y^2}} \cdot \frac{1}{y} = \frac{1}{2}(-1) = -\frac{1}{2}$

$f_y(x,y) = \underbrace{\frac{1}{1+\frac{x^2}{y^2}}} \cdot \underbrace{x} \cdot \underbrace{(-y^{-2})} = \frac{1}{2}(-1) = -\frac{1}{2}$

$f(1,-1) = \mathrm{arctg}(-1) = -\frac{\pi}{4}$

$z = -\frac{\pi}{4} + \left(-\frac{1}{2}\right)(x-1) + \left(-\frac{1}{2}\right)\cdot(y+1)$

$x + y + 2z + \frac{\pi}{2} = 0$

10 7-17:25

$$T_n(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) + \frac{1}{2!}\Big(f_{xx}(x_0,y_0)(x-x_0)^2 + 2 f_{xy}(x_0,y_0)(x-x_0)(y-y_0) + f_{yy}(x_0,y_0)\cdot(y-y_0)^2\Big) + \dots + \frac{1}{n!} \cdot \sum_{i=1}^{n} \binom{n}{i} \cdot \frac{\partial^n f(x_0,y_0)}{\partial^i x \cdot \partial^{n-i} y} \cdot (x-x_0)^i (y-y_0)^{n-i}$$

10 7-17:30

$f(x,y) = \ln\sqrt{x^2+y^2}$ $[1,1]$

$f_x(x,y) = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{x^2+y^2} = \frac{1}{2}$

$f_y(x,y) = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{x^2+y^2} = \frac{1}{2}$

$f_{xx}(x,y) = \frac{x^2+y^2-x(2x)}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2} = 0$

$f_{yy}(x,y) = \frac{x^2+y^2-y(2y)}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2} = 0$

$f_{xy}(x,y) = x\cdot\left(-\frac{1}{(x^2+y^2)^2}\right)\cdot 2y = \frac{-2xy}{(x^2+y^2)^2} = -\frac{2}{4} = -\frac{1}{2}$

$T_2(1,1) = \ln\sqrt{2} + \frac{1}{2}(x-1) + \frac{1}{2}(y-1) + \frac{1}{2}\left(2\cdot\left(-\frac{1}{2}\right)\right)\cdot(x-1)(y-1) = \ln\sqrt{2} + \frac{1}{2}(x-1) + \frac{1}{2}(y-1) - \frac{1}{2}(x-1)\cdot(y-1)$

10 7-17:36

$f(x,y,z) = x^{\frac{y}{z}}$ $[1,1,1]$

$x^* = (x_1, \dots, x_n)$

$h = (h_1, \dots, h_n)$

$f(x^*+h) \doteq f(x^*) + df(x^*)(h) + \frac{1}{2}d^2 f(x^*)(h) + \dots + \frac{1}{n!}d^n f(x^*)(h)$

$d^k f(x^*)(h) = \sum_{j_1+\dots+j_n=k} \frac{k!}{j_1!\cdots j_n!} \frac{\partial^k f}{\partial x_1^{j_1}\partial x_2^{j_2}\cdots\partial x_n^{j_n}}(x^*) \cdot h_1^{j_1}\cdots h_n^{j_n}$

10 7-17:46

$x^{\frac{y}{z}}$ $[1,1,1]$

$f_x = \frac{y}{z}\cdot x^{(\frac{y}{z}-1)} = 1$

$f_y = x^{\frac{y}{z}}\cdot\ln x\cdot\frac{1}{z} = 0$

$f_z = x^{\frac{y}{z}}\cdot\ln x\cdot y\left(-\frac{1}{z^2}\right) = 0$

$f_{xx} = \frac{y}{z}\left(\frac{y}{z}-1\right)\cdot x^{(\frac{y}{z}-2)} = 0$

$f_{xy} = \frac{1}{z}\cdot x^{(\frac{y}{z}-1)} + \frac{y}{z}\cdot x^{(\frac{y}{z}-1)}\cdot\ln x\cdot\frac{1}{z} = 1$

$f_{xz} = y\cdot\left(-\frac{1}{z^2}\right)\cdot x^{(\frac{y}{z}-1)} + \frac{y}{z}\cdot x^{(\frac{y}{z}-1)}\cdot\ln x\cdot y\cdot\left(-\frac{1}{z^2}\right) = -1$

$f_{yy} = \ln x\cdot\frac{1}{z}x^{\frac{y}{z}}\cdot\ln x\cdot\frac{1}{z} = 0$

$f_{yz} = \ln x\cdot x^{\frac{y}{z}}\cdot\ln x\cdot y\left(-\frac{1}{z^2}\right)\cdot\frac{1}{z} + \ln x\cdot x^{\frac{y}{z}}\left(-\frac{1}{z^2}\right) = 0$

$f_{zz} = \ln x\cdot y\, x^{\frac{y}{z}}\cdot\ln x\cdot y\cdot\left(-\frac{1}{z^2}\right)\cdot\left(-\frac{1}{z^2}\right) + \ln x\cdot y\, x^{\frac{y}{z}}\cdot\left(+2\cdot\frac{1}{z^3}\right) = 0$

$f(x,y,z) = 1 + (x-1) + \frac{1}{2!}\left(2(x-1)(y-1) + 2(-1)(x-1)(z-1)\right)$

$= 1 + (x-1) + (x-1)(y-1) - (x-1)(z-1)$

10 7-17:52

$\sin 29^\circ\cdot\operatorname{tg} 46^\circ = 0{,}50203$

$f(x,y) = \sin x\cdot\operatorname{tg} y$ $[x_0,y_0] = \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$

$f_x(x,y) = \cos x\cdot\operatorname{tg} y = \frac{\sqrt{3}}{2}$

$f_y(x,y) = \sin x\cdot\frac{1}{\cos^2 y} = 1$

$f_{xx}(x,y) = -\sin x\cdot\operatorname{tg} y = -\frac{1}{2}$

$f_{xy}(x,y) = \cos x\cdot\frac{1}{\cos^2 y} = \sqrt{3}$

$f_{yy}(x,y) = \sin x\cdot(-2)\cdot\frac{1}{\cos^3 y}\cdot(-\sin y) = 2$

$\sin 29^\circ\cdot\operatorname{tg} 46^\circ = \frac{1}{2}\cdot 1 + \frac{\sqrt{3}}{2}\cdot\left(-\frac{\pi}{180}\right) + 1\cdot\left(\frac{\pi}{180}\right) + \frac{1}{2}\left(-\frac{1}{2}\cdot\left(-\frac{\pi}{180}\right)^2 + 2\cdot\sqrt{3}\cdot\left(-\frac{\pi}{180}\right)\cdot\frac{\pi}{180} + 2\cdot\left(\frac{\pi}{180}\right)^2\right) =$

$= \frac{1}{2} + \frac{\pi}{180}\left(-\frac{\sqrt{3}}{2}+1\right) + \frac{\pi^2}{180^2}\cdot\left(-\frac{1}{4} - \sqrt{3} + 1\right) = 0{,}4973$

10 7-18:04

$\ln(x^2+y^2-1)$ $[1{,}1; 1{,}2]$ $[1,1]$

$0{,}501$

$f_x = \frac{1}{x^2+y^2-1}\cdot 2x = 2$

$f_y = \frac{1}{x^2+y^2-1}\cdot 2y = 2$

$f_{xx} = \frac{2(x^2+y^2-1) - 2x\cdot 2x}{(x^2+y^2-1)^2} = \frac{2(y^2-x^2-1)}{(x^2+y^2-1)^2} = -2$

$f_{xy} = 2x\cdot\frac{-1}{(x^2+y^2-1)^2}\cdot 2y = \frac{-4xy}{(x^2+y^2-1)^2} = -4$

$f_{yy} = \frac{2(x^2+y^2-1) - 2y(2y)}{(x^2+y^2-1)^2} = \frac{2(x^2-y^2-1)}{(x^2+y^2-1)^2} = -2$

$\ln(1{,}1^2+1{,}2^2-1) = \ln 1 + 2\cdot(0{,}1) + 2\cdot 0{,}2 +$

$+ \frac{1}{2}\left(-2\cdot 0{,}1^2 + 2\cdot(-4)\cdot 0{,}1\cdot 0{,}2 + (-2)\cdot 0{,}2^2\right) =$

$= 0{,}47$

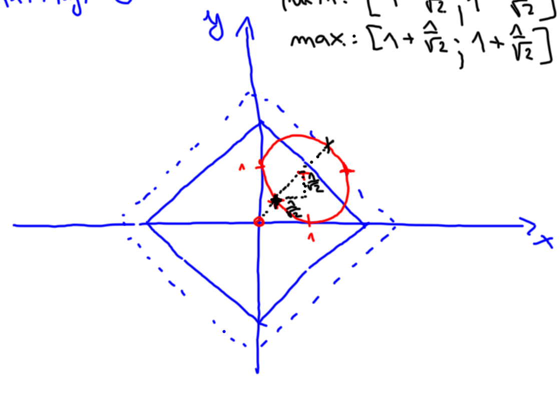
10 7-18:13

$f(x,y) = |x|+|y|$ $M: (x-1)^2+(y-1)^2 \le 1$

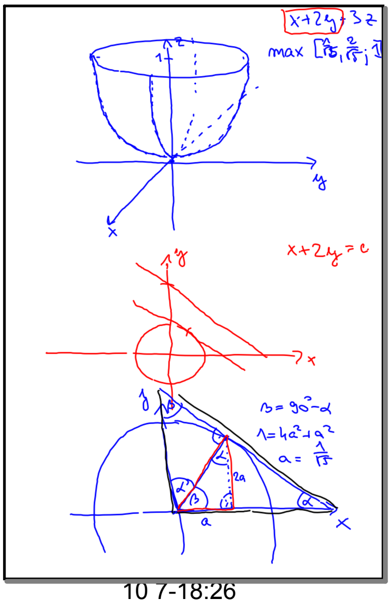
$|x|+|y| = c$

min: $\left[1-\frac{1}{\sqrt{2}}; 1-\frac{1}{\sqrt{2}}\right]$

max: $\left[1+\frac{1}{\sqrt{2}}; 1+\frac{1}{\sqrt{2}}\right]$



10 7-18:19



10 7-18:26