3D Structural Alignment
and
Tunnel Computation

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Contents

- Two Worlds
  - 3D Structural Alignment (on the small scale)
  - Tunnel Computation
- The Worlds Collide?

3D Alignment

- **Large scale** (proteins)
  - Global (classification, evolutionary links)
  - Local (compare smaller sub-structures)
- **Small scale**

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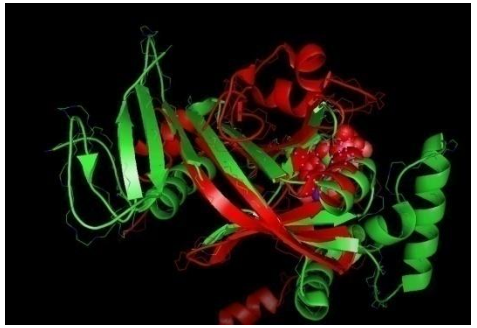
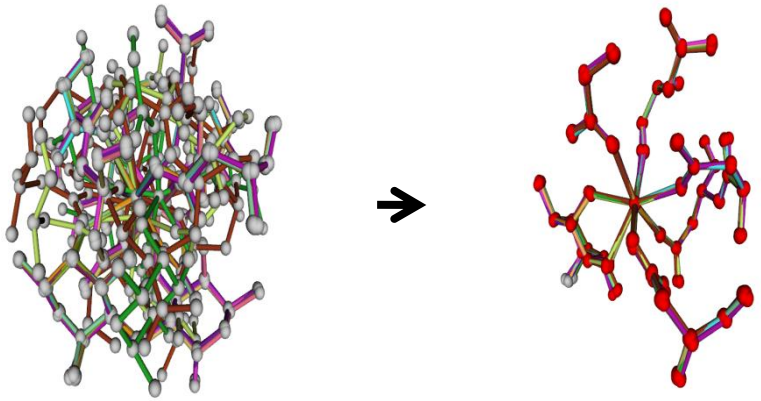
Global FTFTALILLAVAV
       F--TAL-LLA-AV

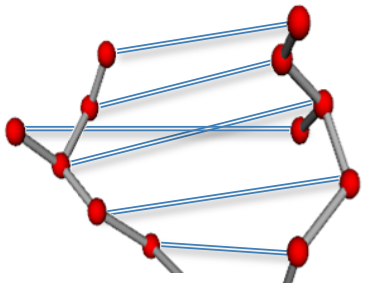
Local  FTFTALILL-AVAV
       --FTAL-LLAAV--
```





3D Alignment

- What we need?
  - Know which atoms/amino acids/… belong together



  - Find a transformation that fits the corresponding pairs

$$\mathrm{RMSD}(A, B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \|A_i - B_i\|^2}$$

The Transformation Part

- Horn K.P.: Closed-form solution of absolute orientation using unit quaternions, Journal of the Optical Society of America, 1986.
- Karney C.F.F.: *Quaternions in molecular modeling*. Journal of Molecular Graphics and Modelling, 2007.

$$\mathrm{RMSD}(T(A), B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \|R(A_i) + t - B_i\|^2}$$

Translation component

Rotation component

Stays the same

Maximize this

$$\sum_{i=1}^{n} \|R(A_i) - B_i\|^2 = \sum_{i=1}^{n} \|R(A_i)\|^2 + \sum_{i=1}^{n} \|B_i\|^2 - 2\sum_{i=1}^{n} R(A_i) \cdot B_i$$

$$q^T \left(\sum_{i=1}^{n} \hat{\mathbf{A}}_i^T \mathbf{B}_i \right) q$$

$$R(x) = qxq^*$$

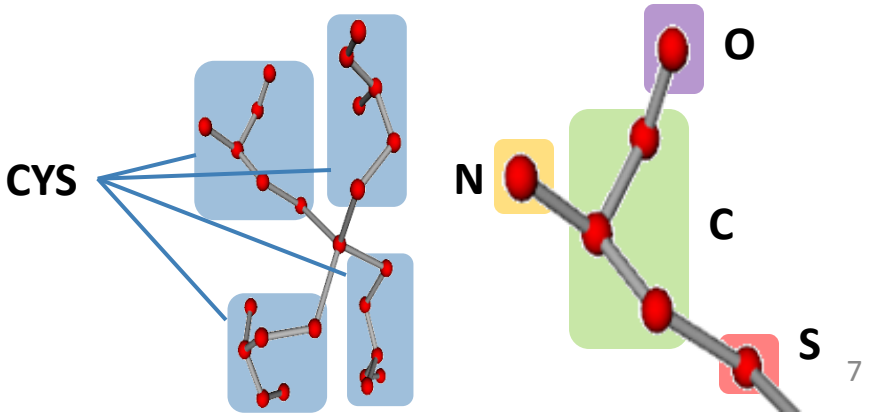
Pairing the Elements

- Large Scale
  - Usually work on the secondary structure (amino acids)
  - Combinatorial extensions, subgraph isomorphism, dynamic programming, …
- **Small Scale**
  - Usually work on 3D structure
  - Exhaustive search, pairing based on distance, subgraph isomorphism, …

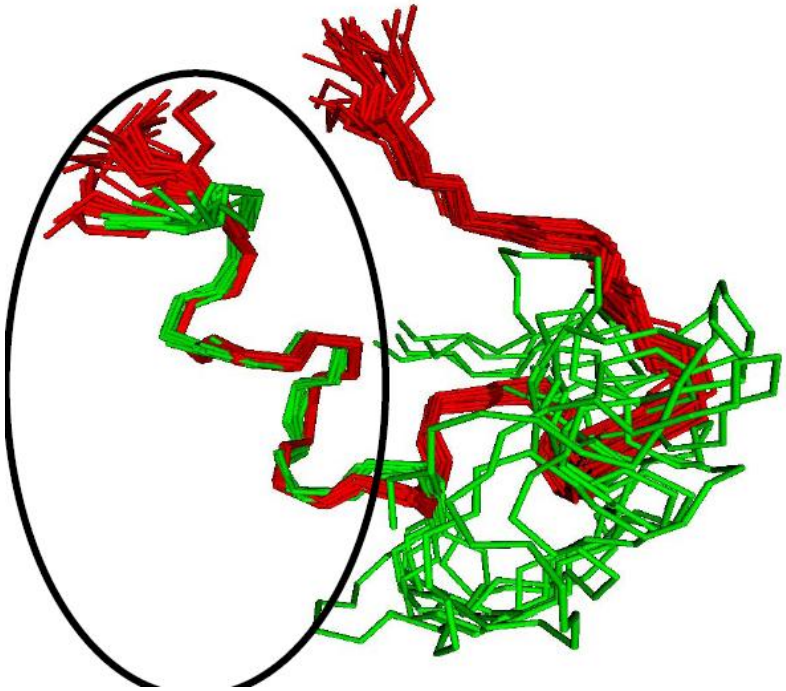
(My) Simple Algorithm

1. For each bijection f: Atoms(A) -> Atoms(B)
   - Align sequences f(A) and B using the quaternion method
2. Return the alignment that yields the lowest RMSD

- Improvement for small protein substructures:
  - Grouping of atoms



Applications



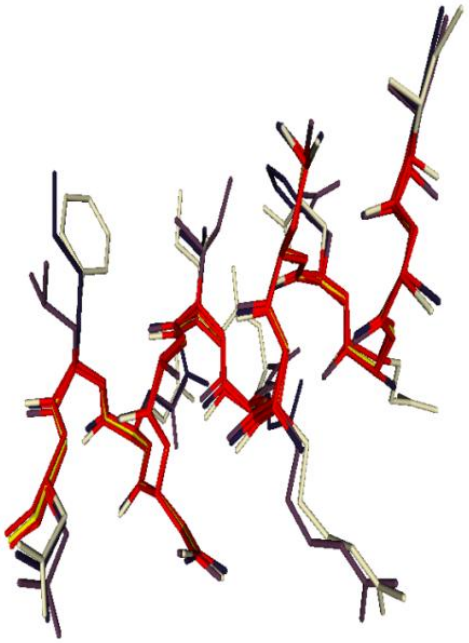
Reverse Proteins

Classification

Lectines

66 structures

56 atom on each



Apoptosis

Analysis of the

model structure

(Sort of Mine) Better Approach

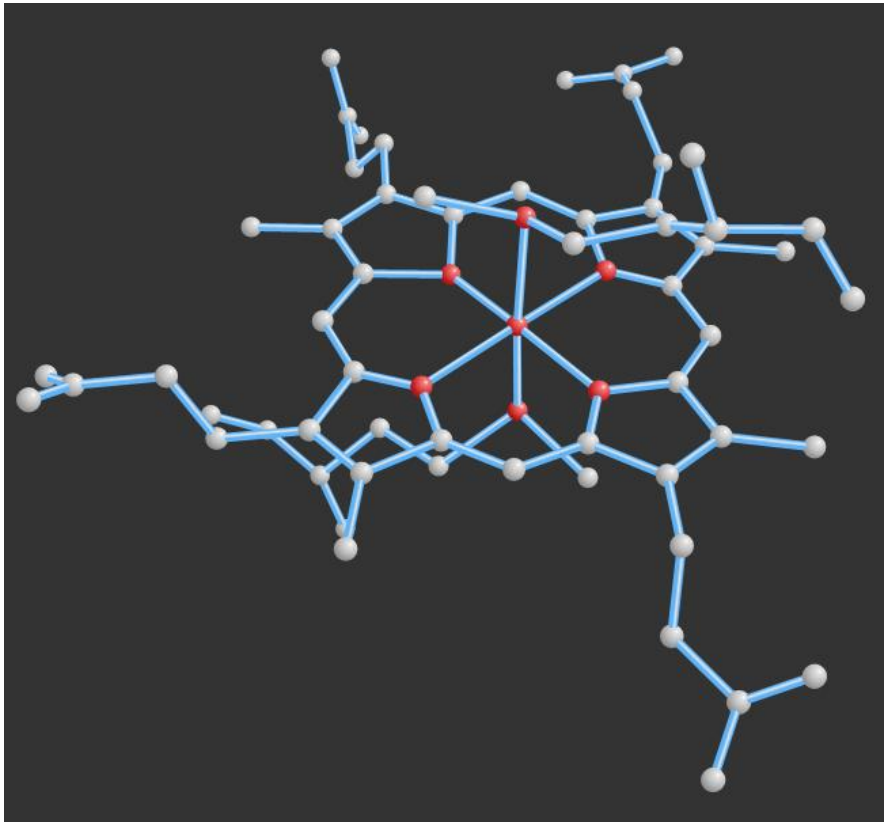
- Hoppe A., Frommel C.: *NeedleHaystack: a program for the rapid recognition of local structures in large sets of atomic coordinates,* Journal of applied crystallography, 2003.

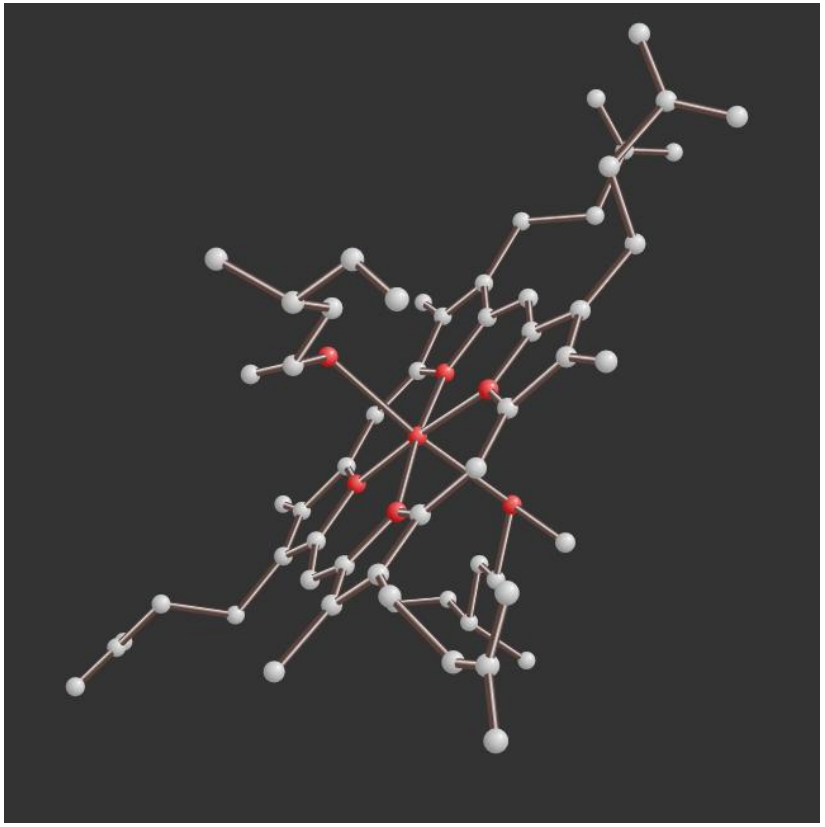
- Find sets of 3 suitable pivots in each structure
- For each pair of pivot sets
  - Align the structures using these pivots (quaternions)
    1. Match atoms based on distance (with a threshold)
    2. Align again (quaternions)
    3. (Goto 1 until a stable configuration is reached)
  - Score = f(A, B, $RMSD_M$)

M is the set of matched atoms, N is the total number of atoms, and $RMSD_M$ is the RMSD of matched atoms.

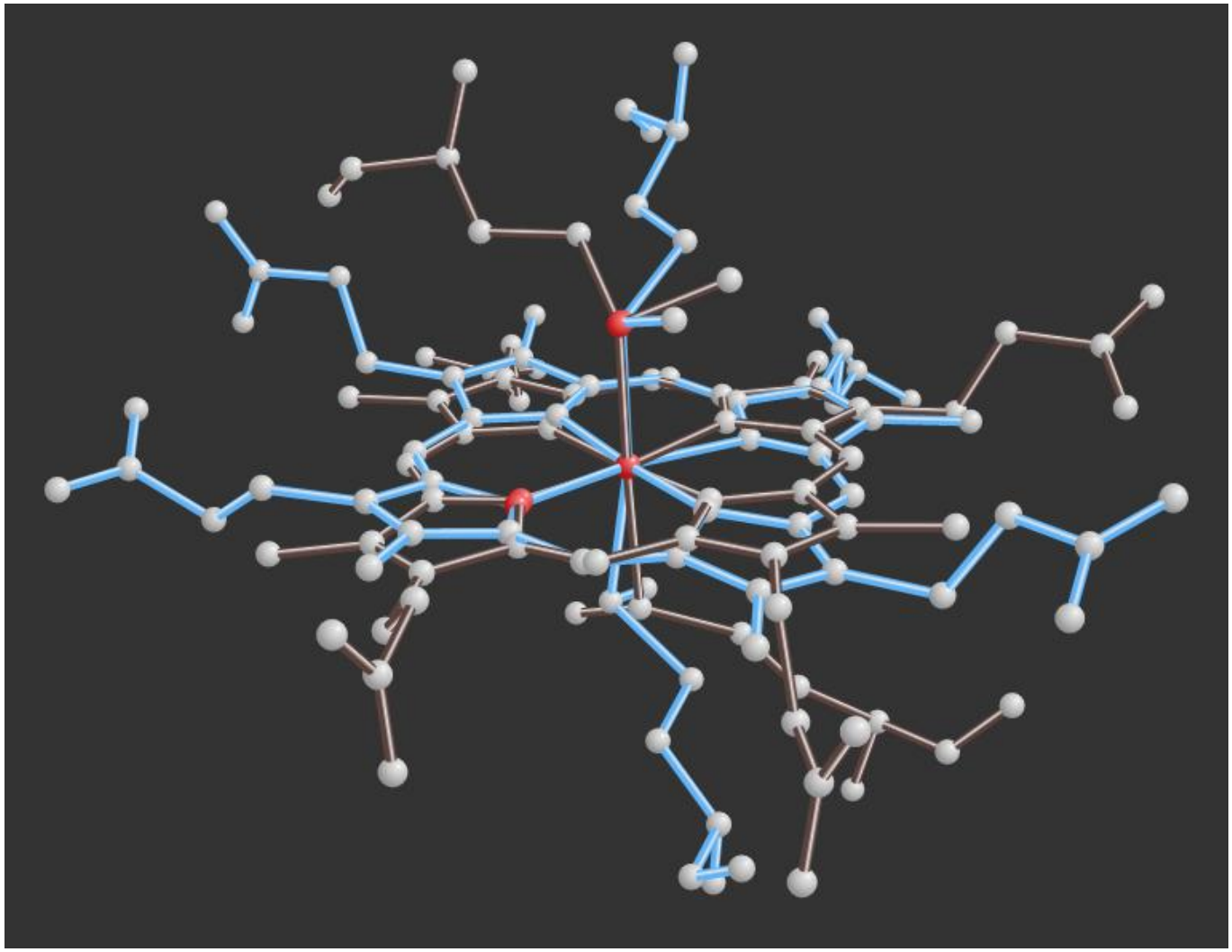
- Return the alignment with the best score

Example (FeN_4S_2)

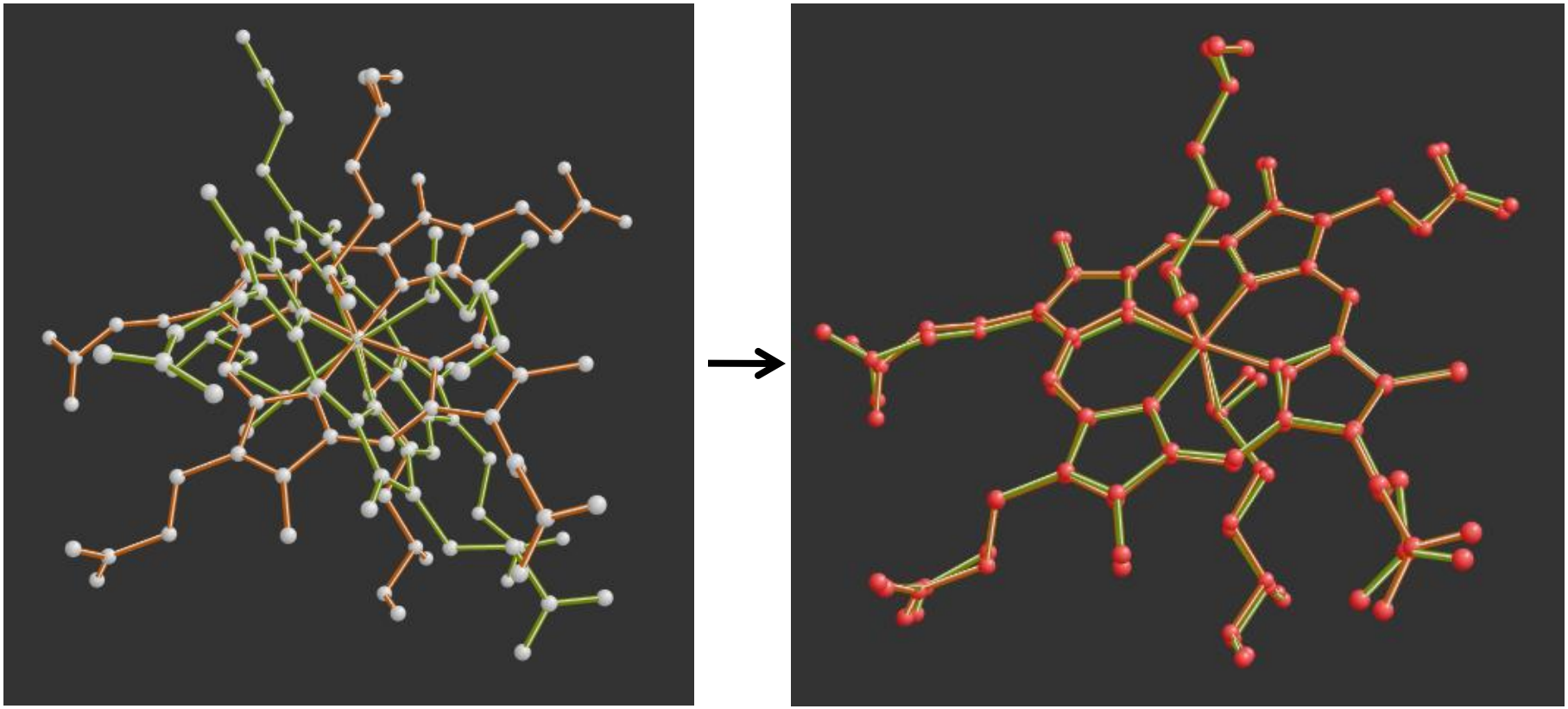




Example (FeN_4S_2)

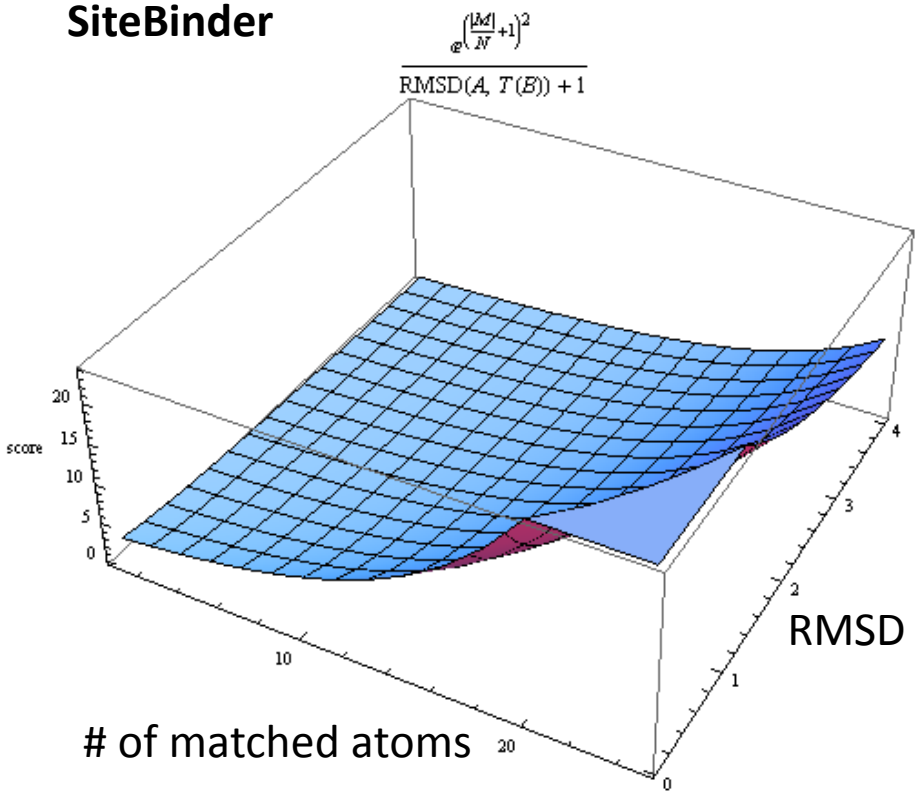


Example (FeN_4S_2)



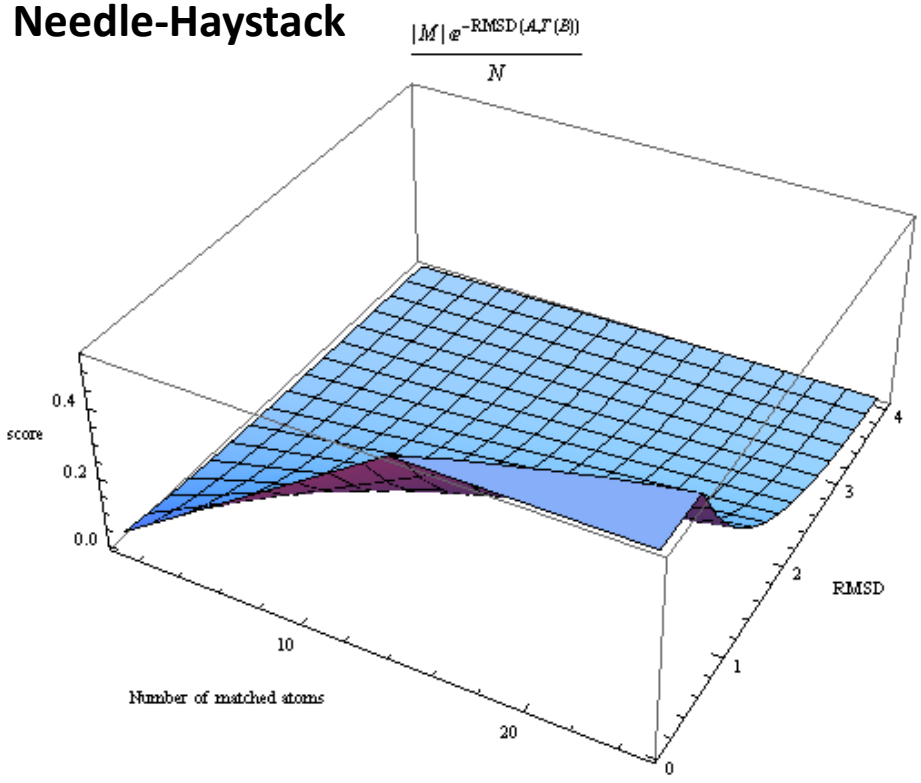
Scoring

SiteBinder



# of matched atoms

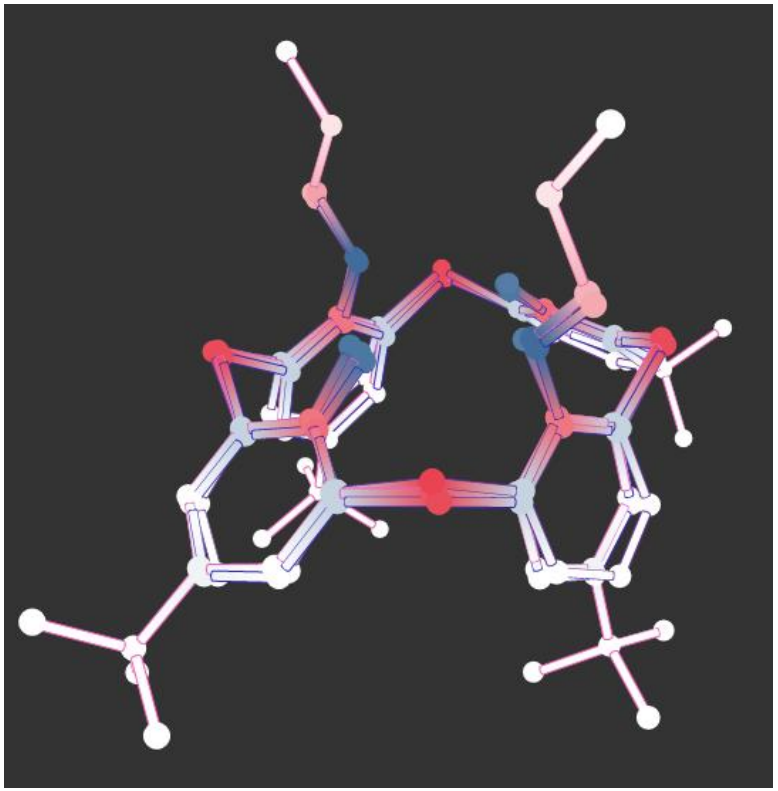
Needle-Haystack



$$\frac{e^{(1+|M|/N)^2}}{1+\mathrm{RMSD}_M(A,B)}$$

M is the set of matched atoms, N is the total number of atoms, and RMSD_M is the RMSD of matched atoms.

Different Pivot Criteria (Charges)



$$\mathrm{score}^*(M,N,A,B) = R_M^2 \cdot \mathrm{score}'(M,N,A,B) = R_M^2 \cdot \frac{e^{(1+|M|/N)^2}}{1+\mathrm{RMSD}_M(A,B)}$$

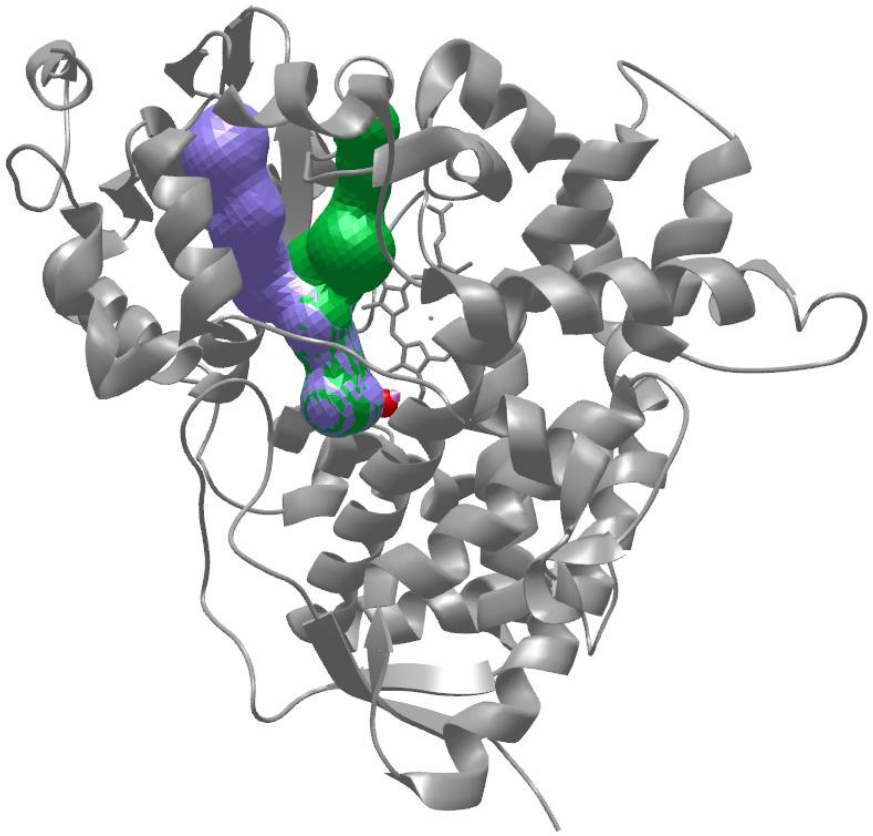
R_M^2 says how well the matched charges correlate.

3D Alignment Summary

3D structural alignment using pivot elements and improved scoring function to identify the correct result.

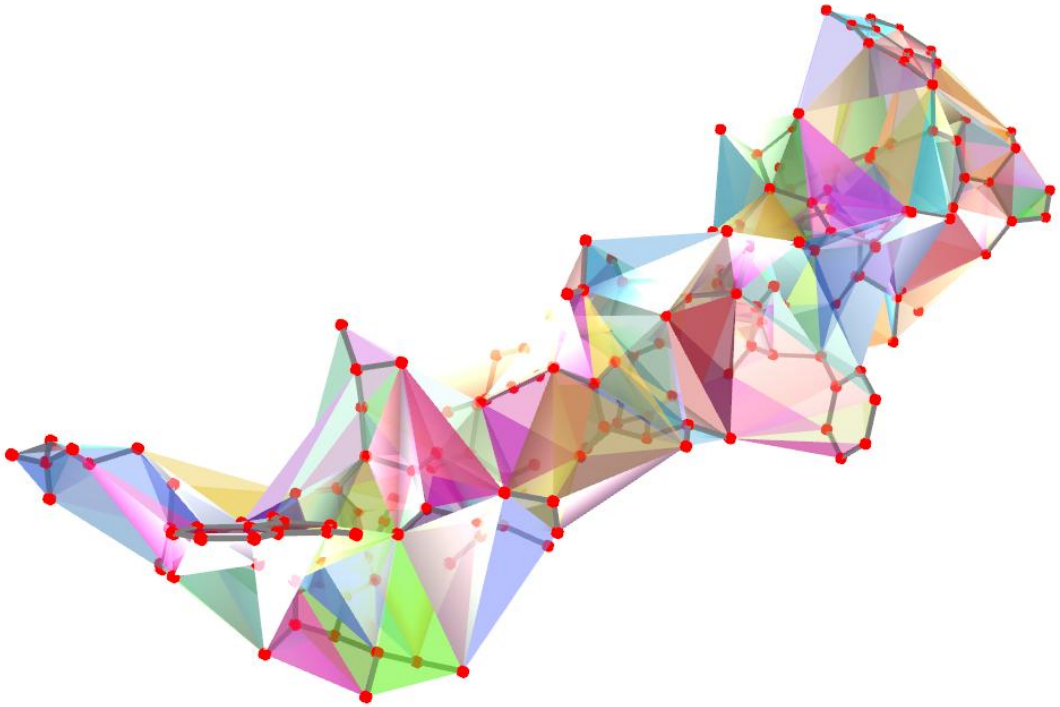
Tunnels

- **MOLE**, Caver, Hollow, MolAxis
- Static analysis vs. dynamics
- Grid/Triangulation Based

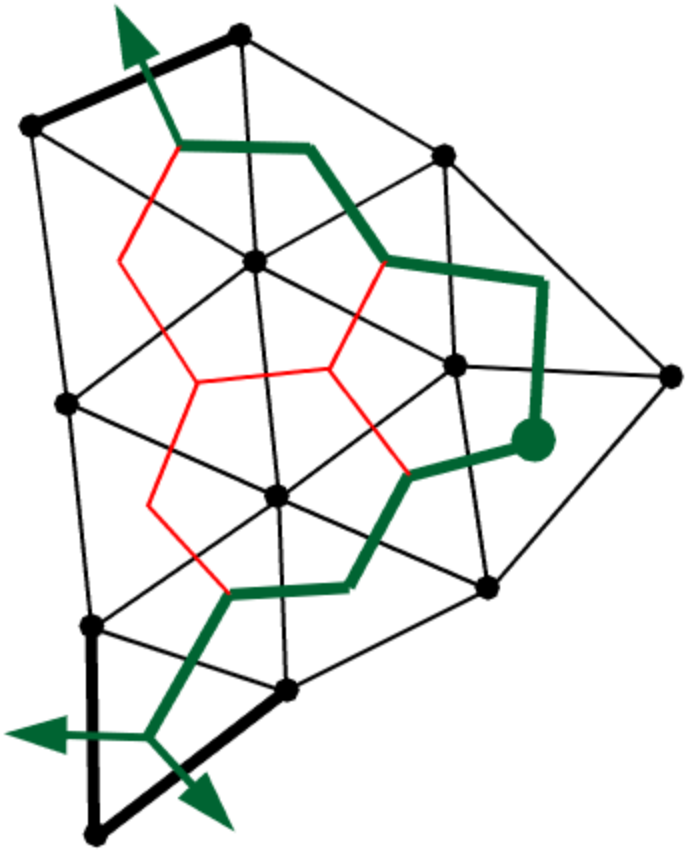


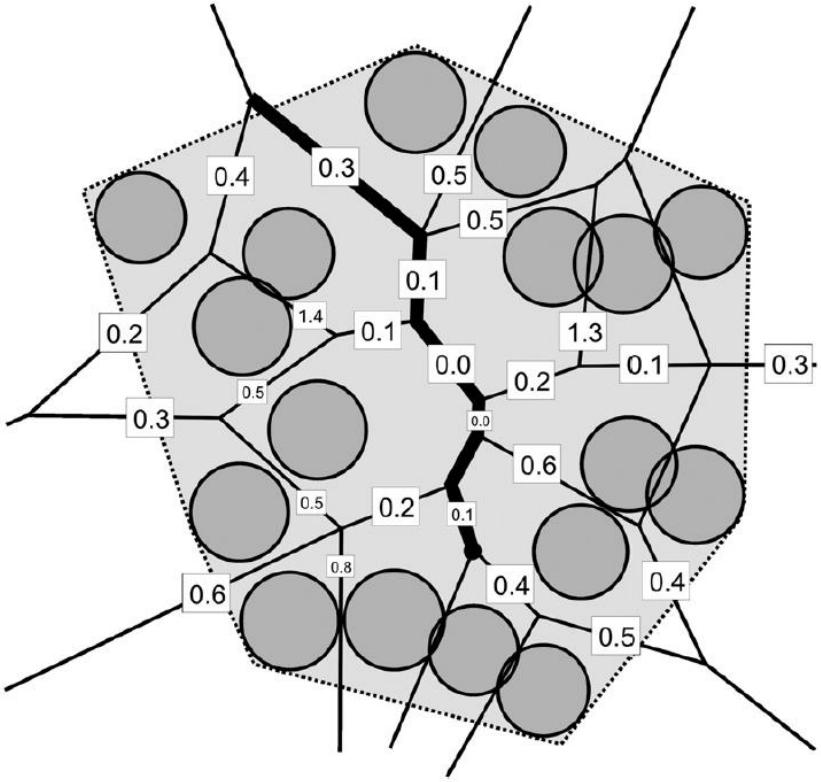
Protein (Delaunay) Triangulation

- Edelsbrunner H., Liang J.: Anatomy of protein pockets and cavities: measurement of binding site geometry and implications for ligand design, Protein Science, 1998.



Finding the Tunnels – The Idea





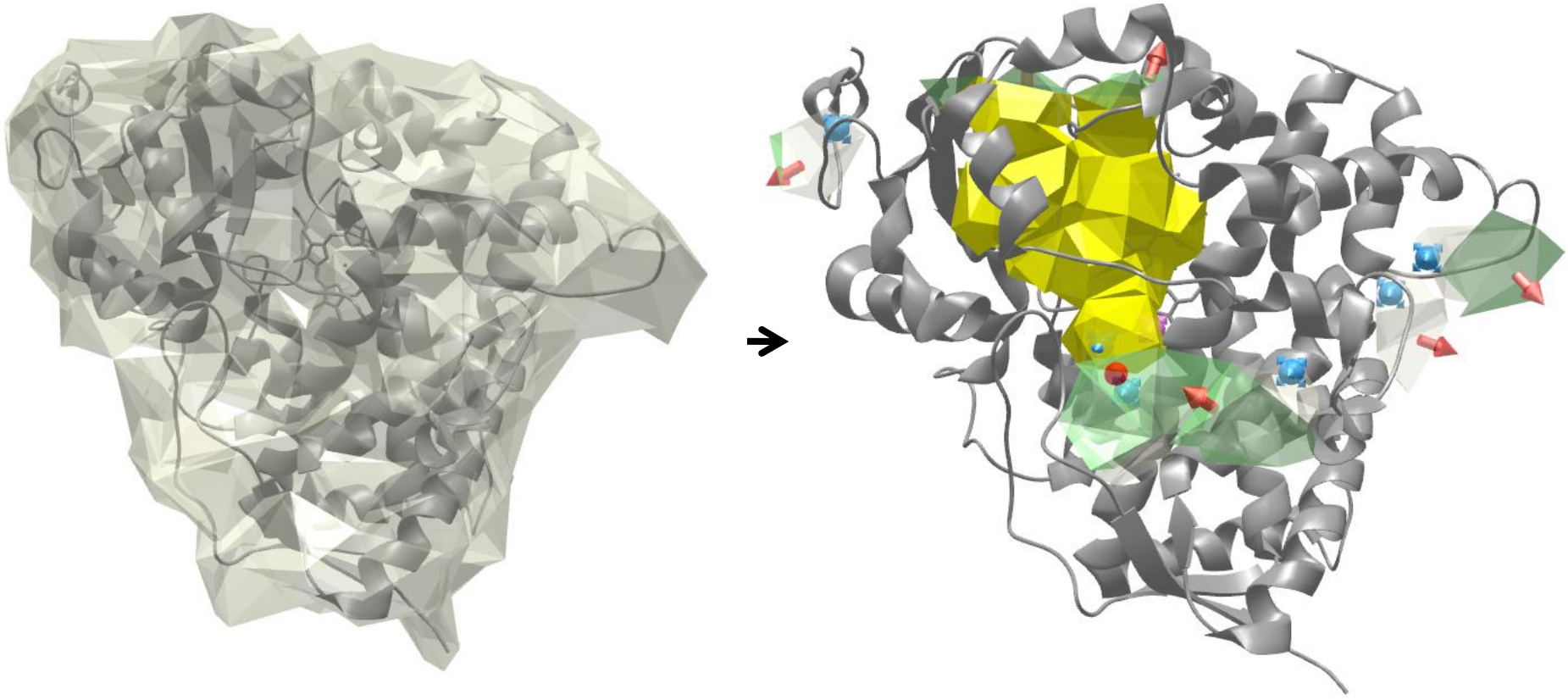
Tunnel Finding Algorithm

1. Triangulate the molecule (Voronoi diagram)
2. Specify starting point (from a database/user)
3. Do depth-first (A*) search
   - Each time a boundary tetrahedron is visited, report a tunnel
   - Edge weight ~ amount off space around the edge
4. Post-process the tunnels (remove duplicate tunnels, …)

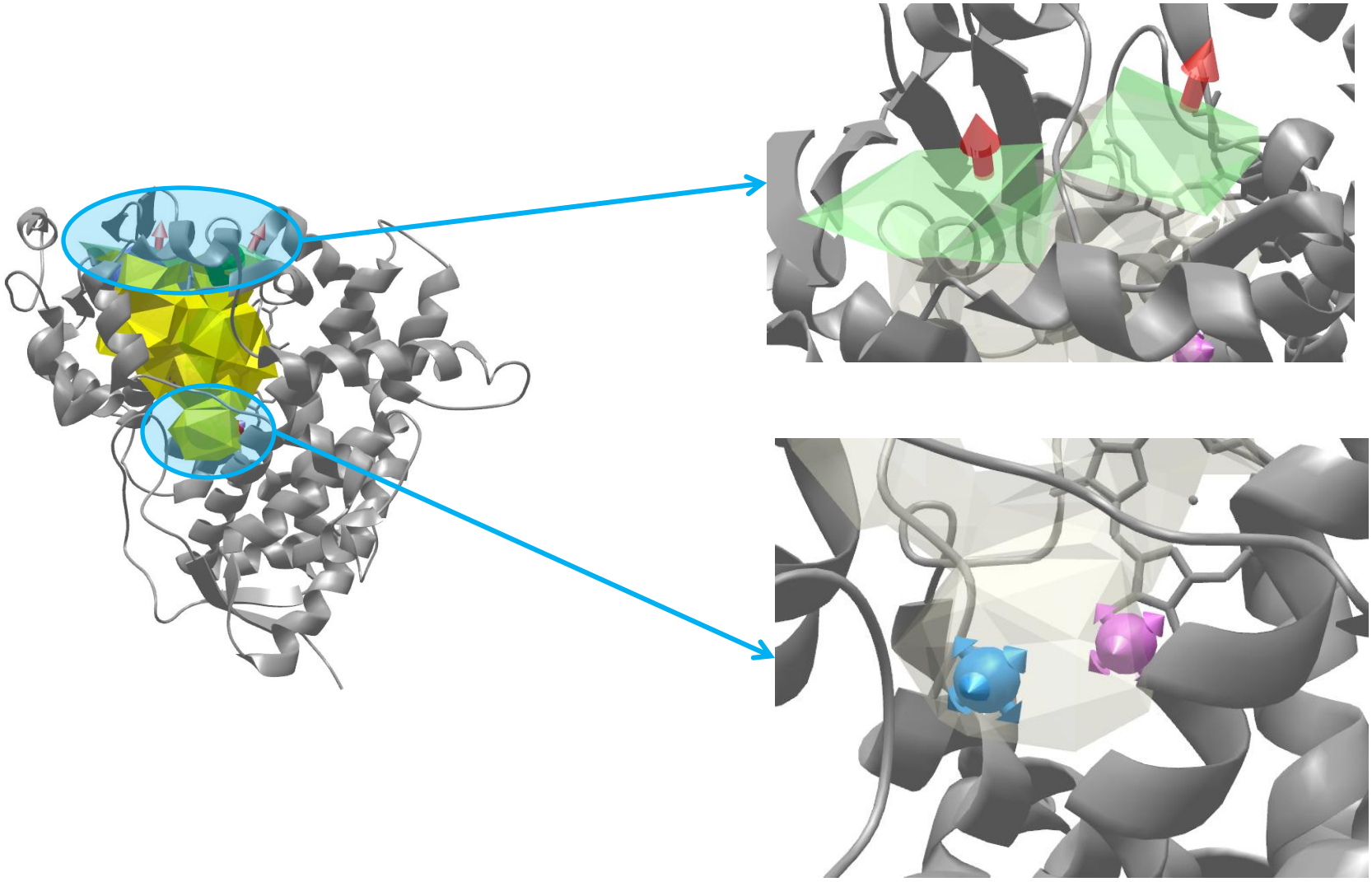
(My) Modified Approach

1. Triangulate the molecule (Voronoi diagram)
2. Identify cavities (“empty space”)
   - Remove tetrahedrons that are too big or too small
   - Find connected components in the new graph
3. Identify start points within cavities
   - Deep point(s) in each cavity
   - Database/user
4. Identify end points within cavities
   - Connected components of the “boundary”
5. For each pair of start and end points in the same cavity use Dijkstra’s algorithm to find the tunnel
   1. Edge weight ~ amount of space around the edge
6. Post-process the tunnels (remove duplicate tunnels, …)

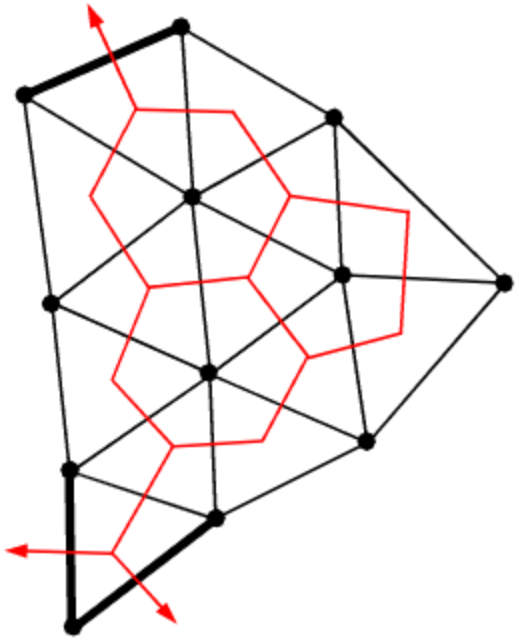
The Cavities

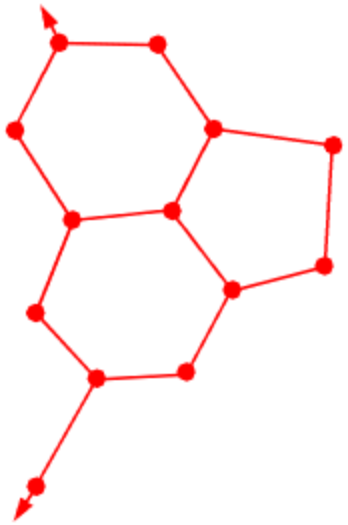


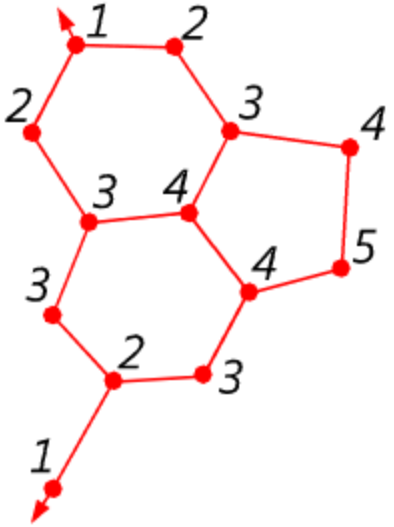
Start and End Points



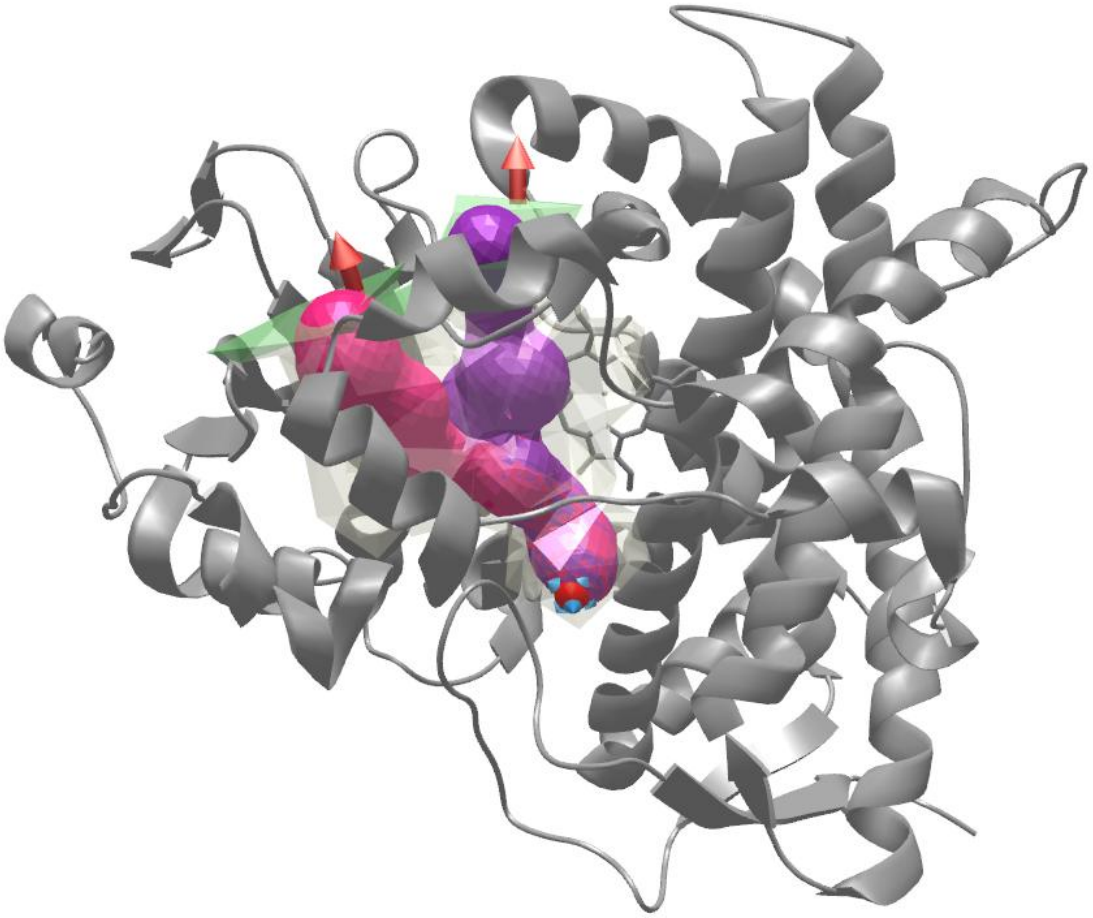
Depth Computation







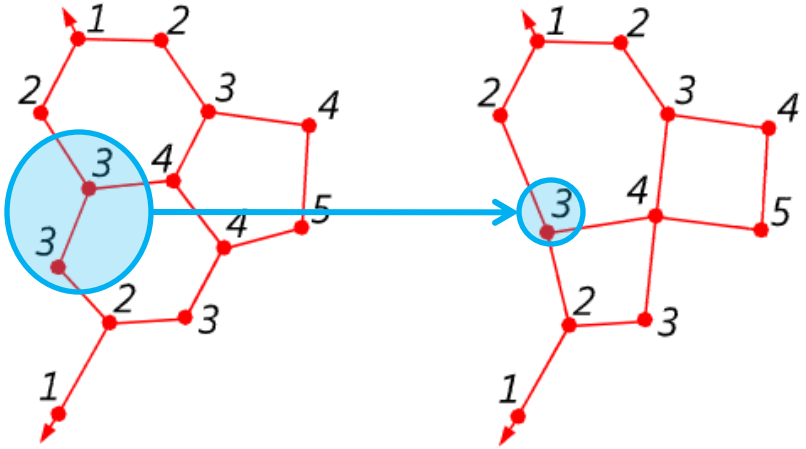
The Tunnels

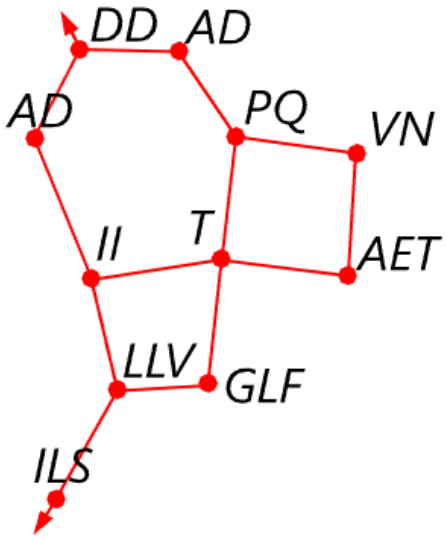


Tunnel Computation Summary

Triangulate the protein, split it into smaller graphs and use Dijkstra's algorithm to find interesting paths (= tunnels) in them.

Aligning the Tunnels/Cavities





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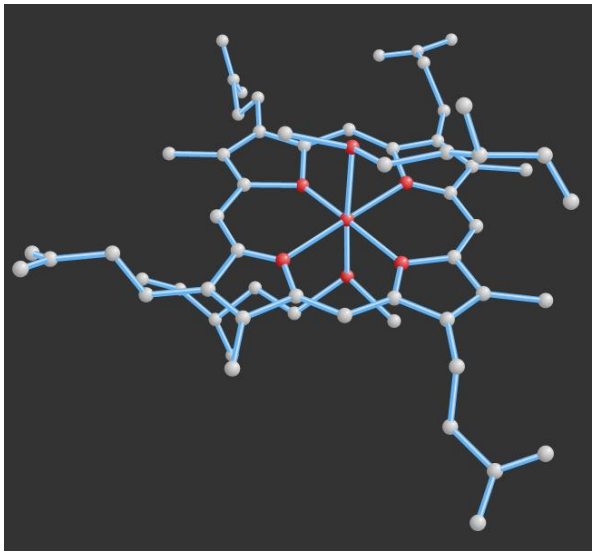
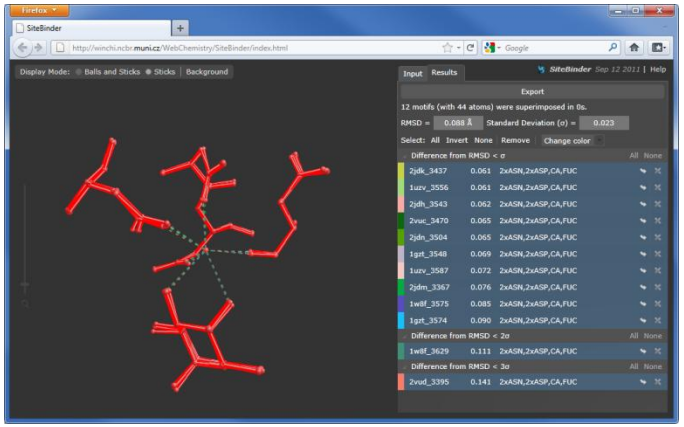


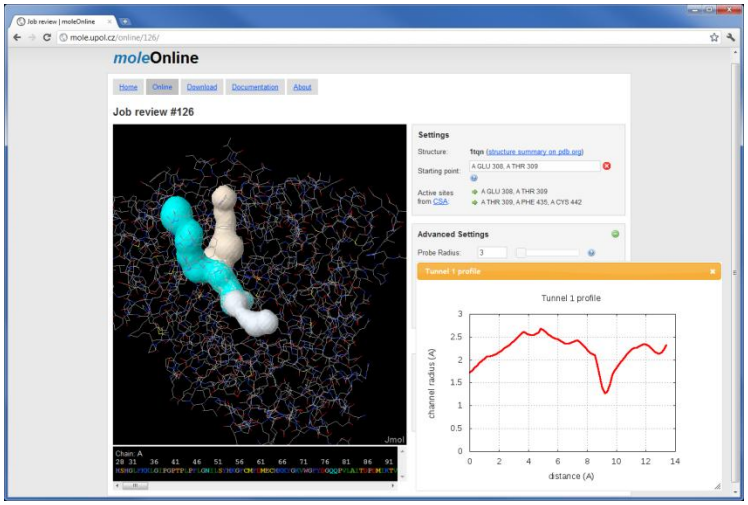
Reduced Cavity Graph

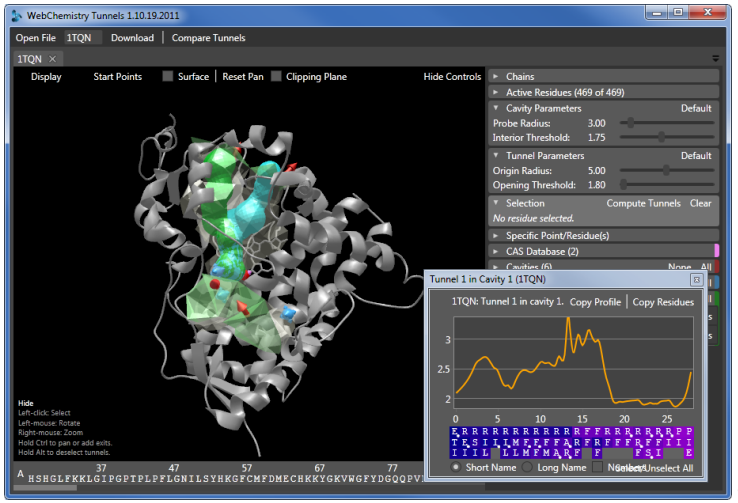
Summary

- 3D Alignment Algorithm



- Tunnel Finding Algorithm





Future Work

- Combine 3D structural alignment and tunnel computation

- (Write PhD thesis about it)

Thank you for your attention.