Intensional Logic

Lecture 8

Syntactic formalisms for natural language parsing

FI MU autumn 2011

Outline

- Overview
- Intension vs. extension
- Intensional semantics and possible worlds
- Montague's Intensional Logic
- First-order Intensional Logic

Logics

- Standard Logics
  - Propositional logic
  - First-order predicate logic
  - Higher-order predicate logic
- Non-standard Logics
  - Categorical logic, **Combinatory logic**, Conditional logic, Constructive logic, Cumulative logic, Deontic logic, Dynamic logic, Epistemic logic, Erotetic logic, Free logic, Fuzzy logic, Infinitary logic, **Intensional logic**, Intuitionistic logic, Linear logic, Many-valued logic, Modal logic, Non-monotonic logic, Paraconsistent logic, Partial logic, Prohairetic logic, Quantum logic, Relevant logic, Stoic logic, Substance logic, Substructural logic, Temporal (tense) logic

Overview in Intensional Logic (LI)

- Approach to predicate logic extending first-order logic
  - Quantifiers that range over the individuals of a universe (extensions)
  - Additional quantifiers that range over terms that may have such individuals as their value (intensions)

- The distinction between extensional and intensional entities is parallel to the distinction between

comprehension and denotation by the Port-Royal

connotation and denotation by J.S.Mill.

sense and reference by Frege

intension and extension by Carnap.

The essential dichotomy is that between

what a term means and **what it denotes**.

- logical analysis can penetrate into varying depths of the language:

  sentences regarded as atomic, or splitting them to predicates applied to individual terms, or even revealing such fine logical structures like modal, temporal, dynamic, epistemic ones.

- Different types of intensional phenomena in IL :
  - modal logic→probability logic of arithmetic and dynamic logic of computation
  - epistemic logic→ knowledge and belief
  - deontic logic→ obligation and permission
  - temporal logic→tense

Intensional logic

- Theory of logical systems, where there are expressions, whose extension is not uniquely defined by the extensions of their subexpressions, but by their intensions.

  ⇒ The truth value of logically composed expressions is not simply a logical function of the truth values of their subexpressions.

Examples: Sentences containing “*always*” indicate an intensional context.

In general: “indexical expressions”

“*The president of the Republic of France is always the first state representative*.”

and “*Nicolas Sarkozy is president of the Republic of France*.”

$\overset{?}{\Rightarrow}$ “Nicolas Sarkozy is <u>always</u> the first state representative.”

- Intensional logics are systems that distinguish an expression's **intension**

  (i.e. **sense, meaning**) from its **extension** (**reference, denotation**).

- **Reason**: Capture the logical "behavior" of intensional expressions, which create contexts which violate standard principles of logic, most notably the law of substitution of identities:

  "From $a = b$ and $P(a)$ it follows that $P(b)$"

  ⇒ Intensional logics provide an analysis of meaning.

Another example: "*obviously*"

A: *Scott is the author of Waverley.*

B: *Obviously Scott is Scott.*

but *obviously Scott* is **<u>not</u>** (necessarily) the author of Waverley.

<u>Modality</u>:

If *A* and *B* are both true, "*it is necessary that A*" may have a truth value different from "*it is necessary that B*".

Example

If *A* ⇌ "7 = the number of world miracles" (A is a contingent truth),

and *B* ⇌ "7 = 7". (⇒ Modal Logic)

Examples of Intensional Logics

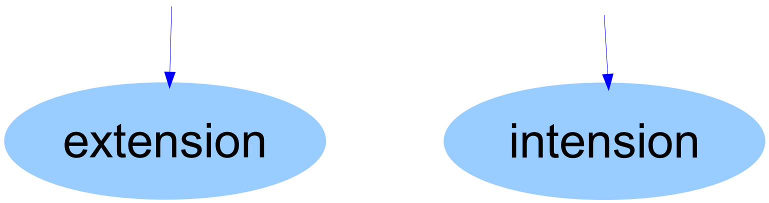
- Modal logic (ontic, deontic, epistemic)
- Temporal logic (mellontic)
- Relevance logic, logic of "strict implication", "logic of entailment"
- Counterfactual conditionals

Intensional contexts are signaled by, e.g.

- "*It is necessary, that. . .* " (ontic modal logic)
- "*Thou shalt. . .* " (deontic modal logic)
- "*x believes, that. . .* " (epistemic modal logic)
- "*Always. . .* " (temporal logic)
- "*. . . follows from. . .* " (logic of entailment, relevance logic)
- "*If Rattle had been conducting, I would have gone to the concert.*" (counterfactual conditionals)

- Intensional logic

  - attempts to study both designation and meaning



  - investigate the relationships between them.

Intension vs. Extension

- Extension

  Semantic values can be attributed to expressions in basic categories:

  the reference of an individual name (the "designated" object named by that) is called its extension; and as for sentences, their truth value is called intension.

- Intension

  any property or quality connoted by a word, phrase or other symbol. In the case of a word, it is often implied by the word's definition.

Intension is analogous to the signified, extension to the referent. The intension thus links the signifier to the sign's extension. Without intension of some sort, words can have no meaning.

- More about Intensions

Def. Of Intension in Possible World:

The intension of an expression A is the function which assigns to every possible world the extension of A in that world.

I.e., the intension of an expression determines its extensions in all possible worlds, and vice versa. Two expressions have the same intension iff they have the same extension in any possible world.

More precisely:

Assumption: For a set of possible worlds I there is one and the same unique domain of individuals U.

The intension of a nominator a is the function which determines for each possible world $i \in I$ the object which a denotes.

We interpret proper names as standard names, i.e. they denote the same object in all possible worlds:

Example:

"*Barack Obama*" vs. "*The president of the United States*"

- The intension of a predicator p is the function which assigns for each possible world $i \in I$ the extension of p in i.
- The intension of a sentence A is the function which assigns for each possible world $i \in I$ the truth value of A in i.

Natural language semantics: Toward intensional semantics

- Extensional semantics that models the meaning of sentences based on the extensions of linguistic expressions is limited and cannot handle these intensional constructions.

- What is common in these intensional constructions is that they call for a consideration of extensions that an expression may have in circumstances other than the one in which it is evaluated. This is called an INTENSION of a linguistic expression.

- In order to get at the intensions, we need to consider alternative ways in which the world might have been, alternative sets of circumstances, or POSSIBLE WORLDS.

- The framework that models the meaning of sentences based on the intensions of linguistic expressions is called INTENSIONAL SEMANTICS or POSSIBLE-WORLDS SEMANTICS.

Intensions and Possible Worlds

- Traditional viewpoint:

  An extensional interpretation (as for FOL) fixes the truth values of sentences in our world.

- **<u>Carnap's idea</u>**:

  Generalize interpretations in a way that they fix the truth values of sentences in all "*possible worlds*".

<u>**Possible world**</u>:

Possible states (state of affairs) and state combinations s.t. every complete state description describes a possible world.

- "Complete" wrt. the class of objects and the class of predicators: For each object or system of objects, resp., and for each predicator it is determined whether the predicator applies or not.

- The notion of "*possible world*" has been introduced by Leibniz; in modern logic it is used as a model theoretic term. Possible worlds are constituents of model structures used to interpret a language, in particular modal concepts — necessity and possibility.

- Possible Worlds and Intensions

Possible worlds are possible circumstances in which some (or all) events or states are different from what they in fact are in the actual circumstance.

w1	*w2*	*w3*
Mat and Sue are funny	Sue, Pete and John are funny	Mat and Pete are funny
Sue is tall	Sue is tall	Sue is not tall
Stevenson is the pres. of MU	Anderson is the pres. of MU	Anderson is the pres. of MU

- *W* is a set of all possible worlds.

  W = {*w1, w2, w3, ...*}

- The intension of a sentence S: <u>proposition</u> (e.g., *Mat is funny*)

  The set of possible worlds in which it is true.

  Function from possible worlds to truth values.

$$P = \begin{pmatrix} w1 \rightarrow 1 \\ w2 \rightarrow 0 \\ w3 \rightarrow 1 \\ w4 \rightarrow 0 \\ . \\ . \end{pmatrix}$$

- Possible Worlds and Intensions (cont.)

The intension of a VP: <u>property</u> (e.g., *is funny*)

Function from possible worlds to sets of individuals

$$\left[\!\left[\textit{is funny} \right]\!\right] = \begin{pmatrix} w1 \rightarrow \{\text{Mat, Sue}\} \\ w2 \rightarrow \{\text{Sue, Pete, John}\} \\ w3 \rightarrow \{\text{Mat, Pete}\} \\ w4 \rightarrow \{\text{Sue, John, Pete}\} \\ . \\ . \end{pmatrix}$$

- The intension of a NP: individual concept (e.g., *the president of MU*)

  Function from possible worlds to individuals

$$\left[\!\left[\textit{the president of MU} \right]\!\right] = \begin{pmatrix} w1 \rightarrow \text{Stevenson} \\ w2 \rightarrow \text{Anderson} \\ w3 \rightarrow \text{Anderson} \\ w4 \rightarrow \text{Pete} \\ . \\ . \end{pmatrix}$$

- Possible Worlds, Tense and Intensions

➢ To incorporate tense, we need to consider possible worlds at different times. That is, we will want to consider not just possible worlds, but possible world-time pairs (circumstances).

$$W = \{<w_1, i_1>, <w_1, i_2>, \ldots, <w_2, i_5>, \ldots, <w_3, i_4>, \ldots\}$$

- Proposition: a set of possible world-time pairs, or a function from possible world-time pairs to truth values.

$$[\![\textit{Mat is funny}]\!] = \begin{pmatrix} < w_1, i_1 > \rightarrow 1 \\ < w_1, i_2 > \rightarrow 0 \\ < w_2, i_1 > \rightarrow 1 \\ < w_3, i_3 > \rightarrow 0 \\ \vdots \end{pmatrix}$$

- Possible Worlds, Tense and Intensions (cont.)

- Property: a function from possible world-time pairs to sets of individuals.

[*is funny*] =

$< w_1, i_1 > \rightarrow$ {Mat, Sue}
$< w_1, i_2 > \rightarrow$ {Sue, Pete, John}
$< w_2, i_1 > \rightarrow$ {Mat, Pete}
$< w_3, i_3 > \rightarrow$ {Sue, John, Pete}
.
.

- Individual concept: a function from possible world-time pairs to individuals.

[*the president of SFU*] =

$< w_1, i_1 > \rightarrow$ Stevenson
$< w_1, i_2 > \rightarrow$ Anderson
$< w_2, i_1 > \rightarrow$ Anderson
$< w_3, i_3 > \rightarrow$ Pete
.
.

• Possible Worlds, Tense and Intensions (cont.)

➢ Structural properties of propositions in terms of set-theoretic operations

1. *p* entails *q* $=_{\mathrm{df}} p \subseteq q$

2. *p* is equivalent to *q* $=_{\mathrm{df}} p = q$

3. *p* and *q* are contradictory $=_{\mathrm{df}} p \cap q =$; (there is no world-time pair in which *p* and *q* are both true)

4. $\neg p =_{\mathrm{df}} \{< w, i > \in W : < w, i > \notin p\}$ (the world-time pairs in which *p* is not true)

5. $p \wedge q =_{\mathrm{df}} p \cap q = \{< w, i > \in W : < w, i > \in p$ and $< w, i > \in q\}$

6. $p \vee q =_{\mathrm{df}} \mathrm{p} \cup \mathrm{q} = \{< w, i > \in W : < w, i > \in p$ or $< w, i > \in q\}$

7. *p* is possible $=_{\mathrm{df}} p \neq \varnothing$ (there is at least one world-time pair in which *p* is true

8. *p* is necessary $=_{\mathrm{df}} p = W$ (there is no world-time pair in which *p* is false)

Montague's Intensional Logic

- Montague, R. (1930-1970)
- IL lies at the heart of a Montague style natural language semantics:

  representation of properties, relations and propositions

  → possible worlds

- Montague semantics

: developed as a powerful tool for model-theoretic treatments of the semantics of English and other natural languages

: had a profound influence in linguistics

: intensional phenomena as at the core of the way language worlds

- In Montague semantics,

: intensionality is a ubiquitous phenomenon, there are not only operators, but also intensional verbs, adjectives, adverbs, and so on.

Montague grammar

: developed for an important new sub-disciplines of linguistics, by using intensional logic and possible worlds semantics to give a precise semantics for constructions of natural language.

The types of Montague's IL are as follows:

Basic types: ***e*** (entities), ***t*** (truth values), ***s*** (possible world-point in time pair)

- **Functional types**: If *a*,*b* are types, then <*a,b*> is a type (the type of functions from *a*-type things to *b*-type things.)
- **Intensional types**: If *a* is any type, then <*s,a*> is a type (the type of functions from possible worlds to things (extensions) of type *a*.)

Syntactic rules of IL

The set of *meaningful expressions of type a,* called ME_a, is defined recursively as follows:

1. Every variable of type a is in ME_a.
2. Every constant of type a is in ME_a.
3. If $\alpha \in ME_a$ and u is a variable of type b, then $\lambda u\alpha \in ME_{<b, a>}$.
4. If $\alpha \in ME_{<b, a>}$ and $\beta \in ME_a$, then $\alpha(\beta) \in ME_b$.
5. If α and β are both in ME_a, then $\alpha = \beta \in ME_t$.

6.-10. If ϕ and ψ are in ME_t, then the following are also in ME_t:

6. $\neg \phi$
7. $[\phi \vee \psi]$
8. $[\phi \wedge \psi]$
9. $[\phi \rightarrow \psi]$
10. $[\phi \leftrightarrow \psi]$

11. If $\phi \in ME_t$ and *u* is a variable of any type, then $\forall u\phi \in ME_t$.

12. If $\phi \in ME_t$ and *u* is a variable of any type, then $\exists u\phi \in ME_t$.

13. If $\phi \in ME_t$, then $\Box\phi \in ME_t$.

14. If $\phi \in ME_t$, then $\mathbf{F}\phi \in ME_t$.

15. If $\phi \in ME_t$, then $\mathbf{P}\phi \in ME_t$.

16. If $\alpha \in ME_a$, then $\hat{}\alpha \in ME_{<s, a>}$.

17. If $\alpha \in ME_{<s, a>}$, then $\check{}\alpha \in ME_a$.

<u>Semantics of *IL*</u>

A. *Model*

A *model for IL*is an ordered quintuple <*A*, *W*, *T*, <, *F*> such that *A*, *W* and *T* are non-empty sets, < is a linear ordering on the set *T*, and *F* is a function that assigns to each non-logical constant of IL of type *a* an intension. The set of *possible denotations* of type *a* is defined as follows (where *a* and *b* are any types):

1. D_e is A
2. D_t is {1, 0}
3. $D_{<a, b>} = D_b^{Da}$
4. $D_{<s, a>} = D_a^{WxT}$

B. *Semantic rules of IL*

The function F will assign to each non-logical constant of IL of type *a* a member of $D_{<s, a>}$.

The semantic rules of IL define recursively for any expression a, the extension of *a with respect to model M*, w ∈ W, t *T and value assignment g*, denoted $[a]^{M,w,t,g}$, as follows:

1. If a is a non-logical constant, then $[a]^{M,w,t,g} = [F(a)](<w, t>)$.

2. If a is a variable, then $[a]^{M,w,t,g} = g(a)$.

3. If $\alpha \in ME_a$ and u is a variable of type b, then $[\lambda u\alpha]^{M,w,t,g}$ is that function h with domain D_b such that for any object x in that domain, $h(x) = [\alpha]^{M,w,t,g'}$ where g' is that value assignment exactly like g with the possible difference that $g'(u)$ is the object x.

4. If $\alpha \in ME_{<a, b>}$, and $\beta \in ME_a$, then $[\alpha(\beta)]^{M,w,t,g}$ is $[\alpha]^{M,w,t,g}([\beta]^{M,w,t,g})$.

5. If α and β are in ME_a, then $[\alpha = \beta]^{M,w,t,g}$ is 1 if and only if $[\alpha]^{M,w,t,g}$ is the same as $[\beta]^{M,w,t,g}$.

6. If $\phi \in ME_t$, then $[\neg \phi]^{M,w,t,g}$ 1 if and only if $[\phi]^{M,w,t,g}$ 0, and $[\neg \phi]^{M,w,t,g}$ 0 otherwise.

7.-10. If ϕ and ψ are in ME_t, then $[\phi \wedge \psi]^{M,w,t,g}$ 1 if and only if both $[\phi]^{M,w,t,g}$ and $[\psi]^{M,w,t,g}$ are 1. (The definitions for the other truth-functional connectives of $[\phi \vee \psi]$, $[\phi \rightarrow \psi]$, and $[\phi \leftrightarrow \psi]$ are the usual ones, paralling this definition.)

11. If $\phi \in ME_t$ and u is a variable, then $[\forall u\phi]^{M,w,t,g}$ is 1 if and only if $[\phi]^{M,w,t,g'}$ is 1 for all g' exactly like g except possibly for the value assigned to u.

12. If $\phi \in ME_t$ and u is a variable, then $[\exists u\phi]^{M,w,t,g}$ is 1 if and only if $[\phi]^{M,w,t,g'}$ is 1 for some value assignment g' exactly like g except possibly for the value assigned to u.

13. If $\phi \in ME_t$, then $[\Box\phi]^{M,w,t,g}$ is 1 if and only if $[\phi]^{M,w',t',g}$ is 1 for all w' in W and all t' in T.

14. If $\phi \in ME_t$, then $[\mathbf{F}\phi]^{M,w,t,g}$ is 1 if and only if $[\phi]^{M,w,t',g}$ is 1 for some t' in T such that $t<t'$.

15. If $\phi \in ME_t$, then $[\mathbf{P}\phi]^{M,w,t,g}$ is 1 if and only if $[\phi]^{M,w,t',g}$ is 1 for some t' in T such that $t'<t$.

16. If $\alpha \in ME_a$, then $[\hat{}\alpha]^{M,w,t,g}$ is that function h with domain W x T such that for all $<w', t'>$ in W x T, $h(<w', t'>)$ is $[\alpha]^{M,w',t',g}$.

17. If $\alpha \in ME_{<s, a>}$, then $[\check{}\alpha]^{M,w,t,g}$ is $[\alpha]^{M,w,t,g}(<w, t>)$.

First-order Intensional Logic (FOIL)

- Fitting, M (2003, 2006)
- Extending the Intensional Logic of Montague
- Depending on a choice of the underlying propositional modal logic

- Two types of variable: x, y,... object variables

  f, g........intension variables
- Each relation symbol has a type
- Type is an n-tuple whose entities are in {O, I}
- Atomic formula: P(α1,.....αn)

  ex: $\langle \lambda x, \Phi \rangle$ (f)

  $\langle \lambda x, y.\Phi \rangle$ (f,g)

Example:

Q: predicate *is-an-important-book*

$$\langle \lambda x, Q(x)\rangle(f) \text{ is true}$$

$$\langle \lambda x, \Box Q(x)\rangle(f) \text{ is true (?)}$$

$$\Box\langle \lambda x, Q(x)\rangle(f) \text{ is true}$$

Ontology-based FOIL

- Jiang, F., Sui, Y., Cao, C. (2008)
- Ontologies are introduced to restrain the modal logic frames and models
- ontology-based first-order modal logic, which can solve the problem of rigid assignment for variables in Kripke semantics
- the language of our ontology-based first-order intensional logic is different from that of FOIL
    - do not use the mechanism of predicate abstraction
    - admit two kinds of constant symbols in their language

- axiomatization of ontology-based FOIL

1. All classical tautologies;
2. K $\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$;
3. Axioms for T, K4, S4, etc., as desired;
4. $\forall\alpha\varphi(\alpha) \rightarrow \varphi(\beta)$, where β is free for α in φ (and α and β are the same type variables);
5. $\forall\alpha(\psi \rightarrow \varphi(\alpha)) \rightarrow (\psi \rightarrow \forall\alpha\varphi(\alpha))$, where α is not free in ψ;
6. $x = x$;
7. $x = y \rightarrow (P(\ldots, x, \ldots) \equiv P(\ldots, y, \ldots))$, where P is a relation symbol of type ...,O, ...;
8. $x = y \rightarrow \Box x = y$;
9. $\neg(x = y) \rightarrow \Box\neg(x = y)$.