Př. 1 (4 body)

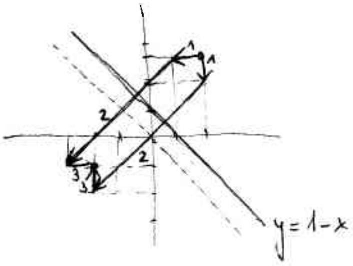
A… průnik $p: x = 3+2t,\ y = t$, $q: 2x - y = 3$

$\rightarrow 2(3+2t) - t = 3 \rightarrow 3t = -3 \Rightarrow t = -1 \rightarrow \left.\begin{matrix} x = 3 + 2\cdot(-1) = 1 \\ y = -1 \end{matrix}\right\}$ $\underline{A = [1, -1]}$

B… kolmá projekce $[-1, 2]$ na osu x → vynulujme y-ovou souřadnici

$\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \rightarrow \underline{B = [-1, 0]}$

C… zrcadlení $[2, 3]$ podle $y = 1 - x$



→ lze řešit různými způsoby, ale u všech budeme zrcadlit podle přímky $y = -x$ (prochází počátkem)

$\xrightarrow{1}$ posunutí přímky do počátku ⇒ od x-ové nebo y-ové osy odečteme 1

$\xrightarrow{2}$ zrcadlení podle $y = -x$ ($\alpha = 3\pi/4$)

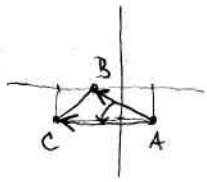
$\xrightarrow{3}$ posunutí zpět

$$\Rightarrow \begin{pmatrix} \cos 3\pi/2 & \sin 3\pi/2 \\ \sin 3\pi/2 & -\cos 3\pi/2 \end{pmatrix} \cdot \begin{pmatrix} 2-1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

nebo: −||− $\cdot \begin{pmatrix} 2 \\ 3-1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$\rightarrow \underline{C = [-2, -1]}$

→ obsah: $\vec{AB} = B - A = (-2, 1)$
$\vec{AC} = C - A = (-3, 0)$



$\Rightarrow \begin{array}{cc} \vec{AB} & \vec{AC} \\ \downarrow & \downarrow \end{array}$ $\begin{vmatrix} -2 & -3 \\ 1 & 0 \end{vmatrix} \cdot \frac{1}{2} = \frac{3}{2} = \underline{\underline{S_{ABC}}}$

Př. 2 (4 body)

$f(x) = x - 4,\ g(x) = 2x + 5$

$(f \circ g)(x) = f(g(x)) = (2x+5) - 4 = \underline{2x + 1}$

$(g \circ f)(x) = g(f(x)) = 2\cdot(x-4) + 5 = \underline{2x - 3}$

$(f \circ g)^{-1}(x) \longrightarrow x = 2\cdot y + 1 \Rightarrow y = (f\circ g)^{-1} = \underline{\underline{\frac{x-1}{2}}}$

$(f^{-1} \circ g^{-1})(x) \rightarrow \left.\begin{matrix} f^{-1}(x) = x + 4 \\ g^{-1}(x) = \frac{x-5}{2} \end{matrix}\right\}$ $(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x)) = \frac{x-5}{2} + 4 = \underline{\underline{\frac{x+3}{2}}}$

Př. 3 (4 body)

$$\begin{pmatrix} a & 1 & -2 & | & 1 \\ 1 & -1 & 1 & | & 0 \\ 0 & 1+a & -1 & | & b \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 & | & 0 \\ a & 1 & -2 & | & 1 \\ 0 & 1+a & -1 & | & b \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 1+a & -2-a & | & 1 \\ 0 & 1+a & -1 & | & b \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 1+a & -2-a & | & 1 \\ 0 & 0 & 1+a & | & b-1 \end{pmatrix}$$

→ žádné → $1+a=0 \wedge b-1 \neq 0 \Rightarrow \underline{a=-1 \wedge b \neq 1}$ → dostaneme řádek $(0\ 0\ 0 \mid \underset{\neq 0}{c})$

→ jediné → $\underline{a \neq -1}$

$$\rightarrow z = \frac{b-1}{1+a}$$

$$(1+a)y + (-2-a)z = 1 \rightarrow y = \frac{1}{1+a} + \frac{(2+a)(b-1)}{(1+a)^2} = \frac{\overset{b+b}{2b}+ab-1}{(1+a)^2} = \frac{b-1}{(1+a)^2} + \frac{b}{1+a}$$

$$x-y+z=0 \rightarrow x = y-z = \frac{2b+ab-1}{(1+a)^2} - \frac{b-1}{1+a} = \frac{2b+ab-1-(b+ab-1-a)}{(1+a)^2} = \frac{a+b}{(1+a)^2}$$

$$\Rightarrow \left\{\left(\frac{a+b}{(1+a)^2}, \frac{2b+ab-1}{(1+a)^2}, \frac{b-1}{1+a}\right)\right\}$$

→ nekonečně mnoho → $\underline{a=-1 \wedge b=1}$

$$\rightarrow \begin{pmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 0 & -1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \quad \left.\begin{array}{l} \rightarrow y = t \\ z = -1 \\ x = y - z = t+1 \end{array}\right\} \quad \{(t+1, t, -1), t \in \mathbb{R}\}$$

Př. 4 (3 body)

$$A^* = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} = \begin{pmatrix} 2 & 4 & -1 \\ 1 & 2 & 2 \\ 2 & -1 & -1 \end{pmatrix}$$

$$A_{11} = (-1)^2 \begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} = 2 \qquad A_{21} = (-1)^3 \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = 4 \qquad A_{31} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1$$

$$A_{12} = (-1)^3 \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} = 1 \qquad A_{22} = (-1)^4 \begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} = 2 \qquad A_{32} = (-1)^5 \begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} = 2$$

$$A_{13} = (-1)^4 \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} = 2 \qquad A_{23} = (-1)^5 \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = -1 \qquad A_{33} = (-1)^6 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$A^{-1} = \frac{A^*}{|A|} = \frac{1}{5}\begin{pmatrix} 2 & 4 & -1 \\ 1 & 2 & 2 \\ 2 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 2/5 & 4/5 & -1/5 \\ 1/5 & 2/5 & 2/5 \\ 2/5 & -1/5 & -1/5 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ -1 & 2 & 0 \end{vmatrix} = 1 + 4 = 5$$