



Př. 4 (4b)

řešíme soustavu  $A^T A \hat{x} = A^T b$

$$\rightarrow \underbrace{\begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ -2 & 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 & -2 \\ 1 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}}_{\begin{pmatrix} 2 & -2 & 3 \\ -2 & 2 & -3 \\ 3 & -3 & 6 \end{pmatrix}} \underbrace{\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}}_{\begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix}} = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ -2 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\rightarrow \left( \begin{array}{ccc|c} 2 & -2 & 3 & -1 \\ -2 & 2 & -3 & 1 \\ 3 & -3 & 6 & -4 \end{array} \right) \sim \left( \begin{array}{ccc|c} 2 & -2 & 3 & -1 \\ 0 & 0 & 3/2 & -5/2 \\ 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow \begin{array}{l} \hat{z} = -5/3 \\ \hat{y} = 1 \\ \hat{x} = \frac{-1 + 5 + 2t}{2} = 2 + t \end{array}$$

$$\Rightarrow \hat{x} = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -5/3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, t \in \mathbb{R}$$

$$\|A\hat{x} - b\| \dots t=0 \rightarrow \hat{x} = \begin{pmatrix} 2 \\ 0 \\ -5/3 \end{pmatrix} \rightarrow A\hat{x} = \begin{pmatrix} -1 & 1 & -2 \\ 1 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ -5/3 \end{pmatrix} = \begin{pmatrix} 4/3 \\ 1/3 \\ 5/3 \end{pmatrix}$$

$$\Rightarrow \left\| \begin{pmatrix} 4/3 \\ 1/3 \\ 5/3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 1/3 \\ 1/3 \\ -1/3 \end{pmatrix} \right\| = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}} = \sqrt{\frac{1}{3}} = \underline{\underline{\frac{1}{\sqrt{3}}}}$$