$$\sum_{n=1}^{\infty} a_n$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots = \frac{1/2}{1-\frac{1}{2}}=1$$

$$\sum_{n=0}^{\infty} a_0 q^n = a_0 + a_0 q + a_0 q^2 + \dots = \frac{a_0}{1-q}$$

$$S=\sum_{k=0}^{n} a_0 q^k = a_0 + a_0 q + a_0 q^2+\dots+a_0 q^n = \frac{a_0 q^{n+1}-a_0}{q-1}$$

$qS = \dots$ konvergence $\iff |q|<1$

11 30-12:01

$\forall x\in(-1,1)$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} =: f(x) \quad \text{pro } x\in(-1,1)$$

$\boxed{f_n(x)=x^n}$ $f_0(x)\equiv 1,\ f_1(x)=x,\ f_2(x)=x^2,\dots$

$$\sum_{n=0}^{\infty} f_n(x) = f(x)$$

11 30-12:05

Pro $f_n(x)=x(1-x^2)^n$

$f_n'(0)=1 \quad \forall n$

$\lim_{n\to\infty} f_n'(0)=1$

ALE:

$\lim_{n\to\infty} f_n(0)=0$

$\Rightarrow f(x)\equiv 0$ na $[-1,1]$

$\Rightarrow f'(x)=0 \quad \forall x\in[-1,1]$

$\Rightarrow f'(0)=0 \quad ((\lim f_n(x))'=0)$

11 30-12:18

M množina, $A\subseteq M$:

χ_A .. char. fce množiny A

$$\chi_A(a)=\begin{cases}1 & a\in A\\ 0 & a\notin A\end{cases}$$

$\chi_{\mathbb{Q}}$ ... Dirichletova funkce $\mathbb{Q}\subseteq\mathbb{R}$

$$\chi_{\mathbb{Q}}(r) = \begin{cases}1 & r\in\mathbb{Q}\\ 0 & r\notin \mathbb{Q}\end{cases}$$

Buď $q_1,q_2,\dots$ uspořádání všech racionálních čísel ($\mathbb{Q}$ je spočetná)

$$\chi_i := \chi_{\{q_i\}}(r) = \begin{cases}1 & r=q_i\\ 0 & \text{jinak}\end{cases}$$

$$\chi_{\mathbb{Q}} = \sum_{n=1}^{\infty} \chi_i$$

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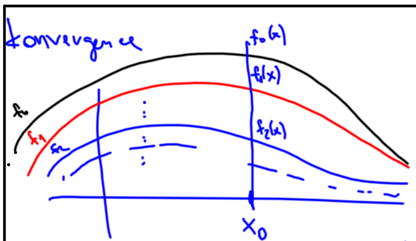
konvergence



$\forall x\in[a,b]\ \forall\varepsilon>0\ \exists N\in\mathbb{N}: \forall n\ge N: |f_n(x)-f(x)|<\varepsilon$

stejnom. konvergence:

$\forall\varepsilon>0\ \exists N\in\mathbb{N}: \forall n\ge N\ \forall x\in[a,b]: |f_n(x)-f(x)|<\varepsilon$

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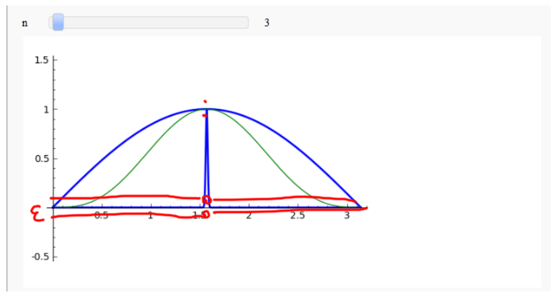

konverguje stejnoměrně?

k fci $\begin{cases}1 & \frac{\pi}{2}=x\\ 0 & \text{jinak}\end{cases}$

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Δ nerovnost:

$|a+b| \le |a| + |b|$

$\sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$ konverguje stejnoměrně (Weierstrass)

neboť $\left|\frac{\sin nx}{n^2}\right| \le \frac{1}{n^2} = a_n$

a $\sum_{n=1}^{\infty} \frac{1}{n^2}$ konverguje

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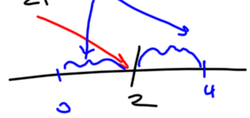
$\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-2)^n = \sum_{n=0}^{\infty} \left(\frac{-x+2}{2}\right)^n$

$q: = \frac{-x+2}{2} = -\frac{x-2}{2}$

$|x-2| < 2$



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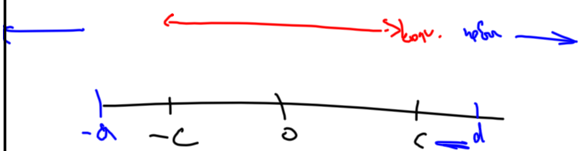
mocninná řada se středem v $x_0 = 0$



Polom. konvergence

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$1, -1, 1, -1, 1, -1, \dots$

má 2 hromadné body $1, -1$

stejně jako

$1+\left(\frac{1}{2}\right)^n, -1+\left(\frac{1}{2}\right)^n, \dots \quad n = 0, \infty$

$\limsup = 1$

$\liminf = -1$

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$\sum_{n=1}^{\infty} n \cdot x^n = \dots = x \cdot \left(\frac{x}{1-x}\right)' =$

$= x \cdot \frac{1-x+x}{(1-x)^2} = \frac{x}{(1-x)^2}$

APLIKACE: vytvořující funkce

(počet koster K_n - C_n)

$\sum_{n=0}^{\infty} C_n x^n = C(x)$

např. $C_n = C_{n-1} + C_{n-2} \Rightarrow C(x) = x \cdot C(x) + x^2 C(x)$

11 30-13:12

Domácí příklady:

$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = \dots$ integrace $= \ln(1+x) + C$

dosazením $x := 0$: $\sum 0 = 0 = \ln(1+0) + C$

$\Rightarrow C = 0$.

$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

$\Rightarrow$ alternující harmonická řada

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \dots = \ln(1+1) = \ln 2$

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