Euler:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

Def: $e^z$ pro $z \in \mathbb{C}$

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$e^{ix} = \sum_{n=0}^{\infty} \frac{(ix)^n}{n!} = \cos x + i \sin x$$

$$e^{a+ib} = e^a \cdot e^{ib} = e^a \cdot (\cos b + i \sin b)$$

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$$f(x) = e^{-\frac{1}{x^2}}$$

$$f'(x) = e^{-\frac{1}{x^2}} \cdot (-x^{-2})' = e^{-\frac{1}{x^2}} \cdot (+2x^{-3})$$

$$f'_+(0) = \lim_{x \to 0^+} e^{-\frac{1}{x^2}} \cdot \frac{2}{x^3} = \dots = 0$$

$\vdots$

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Př: $\sqrt{e}$

$e^{\frac{1}{2}}$ : $f(x) = e^x$ $x_0 = 0$ $f(\frac{1}{2}) = e^{\frac{1}{2}}$

$$T_{f,5}(x) = \sum_{n=0}^{5} \frac{x^n}{n!} \Rightarrow T_{f,5}(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

$$= T_{f,5}\left(\tfrac{1}{2}\right) = 1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{48} + \frac{1}{16 \cdot 4!} + \frac{1}{32 \cdot 5!} \approx 1{,}6487$$

Jinak: $f(x) = \sqrt{x}$ $x_0 = 4$ $f(e) = \sqrt{x}$

$$T_{f,5}(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \dots$$

$$= 2 + \frac{1}{4} \cdot \frac{1}{1!}(x-4) + \dots + \dots (x-4)^5$$

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h=sqrt(x);

j=h.taylor(x,4,5)

j
    7/131072*(x - 4)^5 - 5/16384*(x - 4)^4 + 1/512*(x - 4)^3 - 1/64*(x -
    4)^2 + 1/4*x + 1

j(x=e).n()
    1.64878082444922
```

aprox. $\sqrt{e}$

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$$\left|\frac{a_{n+1}}{a_n}\right| \le q < 1$$

$\forall n \in \mathbb{N}$: zbytek

$$|R_k| = |a_{k+1} + a_{k+2} + \dots| \le$$

$$\le |a_{k+1}| + |a_{k+2}| + \dots$$

$$\le q \cdot |a_k| + q^2 |a_k| + \dots$$

$$= |a_k| (q + q^2 + \dots) = |a_k| \frac{q}{1-q}$$

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$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

**Příklad**

Vypočtěte limitu

$$\lim_{x \to \infty} \left(x - x^2 \ln\left(1 + \frac{1}{x}\right)\right).$$

$$\ln\left(1 + \frac{1}{x}\right) = \frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \frac{1}{4x^4} + \dots$$

$$x^2 \ln\left(1 + \frac{1}{x}\right) = x - \frac{1}{2} + \frac{1}{3x} - \frac{1}{4x^2} + \dots$$

$$x - x^2 \ln\left(1 + \frac{1}{x}\right) = \frac{1}{2} - \frac{1}{3x} + \frac{1}{4x^2} - \dots \to \frac{1}{2}$$

$$x \to \infty$$

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$$f(x) = \frac{1}{1+x^4} = \frac{1}{1-\underbrace{(-x^4)}_{q}} = 1+q+q^2+q^3+\dots$$
$$= 1 - x^4 + x^8 - x^{12} + \dots$$

$$= \sum_{n=0}^{\infty} (-x^4)^n \qquad |x^4| < 1$$

$$\int_0^{1/2} f(x)\,dx = \int_0^{1/2} \sum_{n=0}^{\infty} (-x^4)^n dx = \sum_{n=0}^{\infty} \int_0^{1/2} (-x^4)^n dx =$$

$$= \sum_{n=0}^{\infty} (-1)^n \left[ \frac{x^{4n+1}}{4n+1} \right]_0^{1/2} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(4n+1)\cdot 2^{4n+1}}$$

$$|R_n| < a_{n+1} = (-1)^{n+1} \frac{1}{(4n+5)\cdot 2^{4n+5}}$$

Chceme: $\frac{1}{(4n+5)2^{4n+5}} < \frac{1}{10^4}$

$$10^4 < (4n+5)\,2^{4n+5}$$
$$10^4 > 9 \cdot 2^9 \quad n=1$$
$$10^4 < 13 \cdot 2^{13} \quad \underline{\underline{n=2}}$$

Proto $\int_0^{1/2} \frac{dx}{1+x^4} \approx \frac{1}{2} - \frac{1}{5\cdot 2^5} + \frac{1}{9 \cdot 2^9}$

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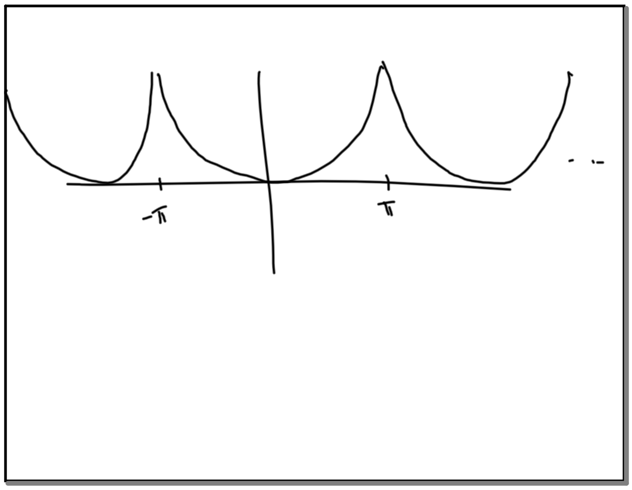


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