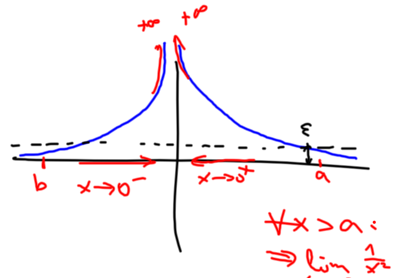
posloupnost p_n

$p: \mathbb{N} \to \mathbb{R}$

$p_2 = p(2)$

má smysl mluvit pouze o limitě v „nekonečnu"

10 5-12:01



$\forall x > a: \frac{1}{x^2} < \varepsilon$

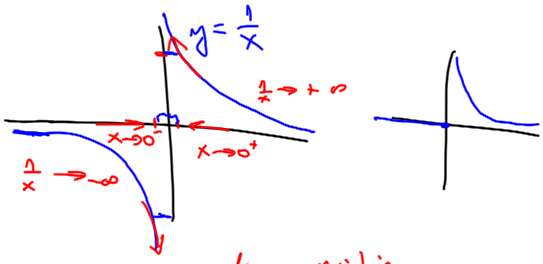
$\Rightarrow \lim_{x \to \infty} \frac{1}{x^2} = 0$

analogicky: $\forall x < b: \frac{1}{x^2} < \varepsilon$

$\Rightarrow \lim_{x \to -\infty} \frac{1}{x^2} = 0$

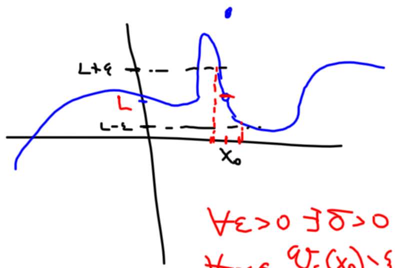
$a(\varepsilon)$

10 5-12:15



$\lim_{x \to 0}$ neexistuje

10 5-12:19



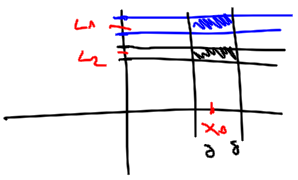
$\forall \varepsilon > 0\ \exists \delta > 0: \forall x \in U_\delta(x_0) \setminus \{x_0\}:$

$f(x) \in (L - \varepsilon, L + \varepsilon)$

10 5-12:24

$f(x)$ má v x_0 jedinou limitu. (≤ 1)

Dk sporem: $L_1, L_2,\ L_1 \neq L_2$



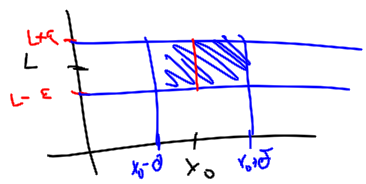
$O_\varepsilon(L_1) \cap O_\varepsilon(L_2) = \emptyset$

$\varepsilon = \frac{|L_1 - L_2|}{3}$

10 5-12:39

$f(x)$ vlastní limitu $L \in \mathbb{R}$ v x_0

$\Rightarrow$ ohraničená v (ryzím) okolí x_0



zřejmé je v $(x_0 - \delta, x_0 + \delta) \setminus \{x_0\}$ ohr.

10 5-12:42

$\lim_{x\to x_0} f(x) = L$, $\lim_{x\to x_0} g(x) = M$.

$\overset{?}{\Rightarrow} \lim_{x\to x_0} (f(x)+g(x)) = L+M$

$\boxed{|a+b| \leq |a|+|b| \quad \triangle\text{-nerovnost}}$

Dk. buď $\varepsilon>0$ lib. chceme najít $U_\delta(x_0)$

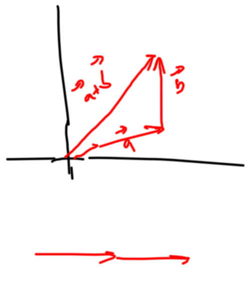
$|f(x)+g(x)-(L+M)| = |(f(x)-L)+(g(x)-M)|$

z předp. víme že $\forall \varepsilon^*>0\ \exists \delta$: $|f(x)-L|<\varepsilon^*$, $|g(x)-M|<\varepsilon^*$ $\forall x \in U_\delta(x_0)$

$\leq |f(x)-L| + |g(x)-M| < \varepsilon^* + \varepsilon^* = 2\varepsilon^*$

stačí volit $\varepsilon^* = \frac{\varepsilon}{2}$

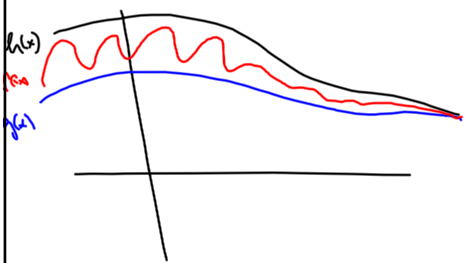
10 5-12:45



$|\vec a| + |\vec b| \geq |\vec a + \vec b|$

10 5-12:48

$x \to \infty$



$g(x) \leq f(x) \leq h(x)$

$\lim g(x) = \lim h(x) = \lim f(x)$

10 5-12:52

$-1 \leq \sin\frac{1}{x} \leq 1 \Rightarrow x\sin\frac{1}{x} \leq x \quad x>0$

$-|x| \leq x\sin\frac{1}{x} \leq |x| \qquad -x \leq x\sin\frac{1}{x} \quad x>0$

$\downarrow \qquad\qquad \downarrow$

$0 \qquad\qquad 0$

$\Rightarrow x\sin\frac{1}{x} \to 0$

10 5-12:54

$\lim_{x\to x_0} 2^{x^2} \overset{\text{Věta}}{=} 2^{\lim_{x\to x_0} x^2}$

$f(x) = 2^x$

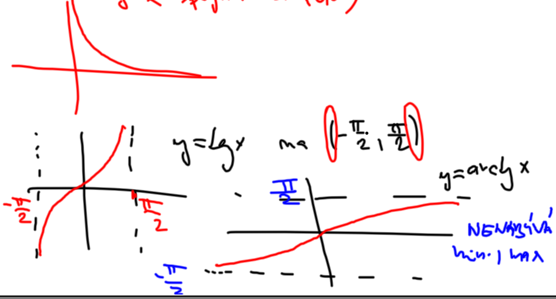
$g(x) = x^2$

10 5-13:02

Věta (Weierstrassova)

Je-li funkce $f(x)$ spojitá na intervalu $[a,b]$, tj. na uzavřeném konečném intervalu, potom je na tomto intervalu ohraničená a nabývá v něm své nejmenší a největší hodnoty.

$y=\frac{1}{x}$ spojitá na $(0,\infty)$



10 5-13:05

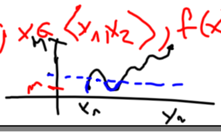
Dk Bolzanovy věty:

$f(x)$ je spojitá na $[a,b]$, označme $m := \min_{[a,b]} f(x)$, $M := \max_{[a,b]} f(x)$

Buď $h \in \langle m, M \rangle$ libovolné, $x_1 \in [a,b]: f(x_1) = m$, $x_2 \in [a,b], f(x_2) = M$

$\exists x_0: f(x_0) = h$. Předp. BÚNO, že $x_1 < x_2$:

$x_0 = \sup\{x; x \in \langle x_1, x_2 \rangle, f(x) < h\}$



Ukáže se, že $f(x_0) = h$

10 5-13:09

ryze monotónní = rostoucí nebo klesající

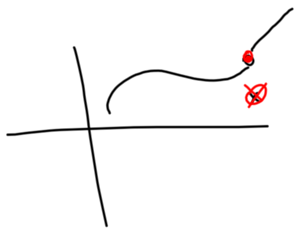
$x < y \Rightarrow f(x) < f(y) \quad \forall x, y \in D(f)$

$x < y \Rightarrow f(x) > f(y)$

$\exists x, y, x \neq y: f(x) = f(y) = z$

$f^{-1}(z) = \begin{cases} x \\ y \end{cases}$

10 5-13:16

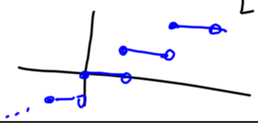


nová def.

$f^*(x_0) = \begin{cases} \lim_{x \to x_0} f(x) & x = x_0 \\ f(x) & x \neq x_0 \end{cases}$

$y = [x]$, $[x] \leq x$, $[x] \in \mathbb{Z}$

$[-\pi] = -4$, $[\pi] = 3$



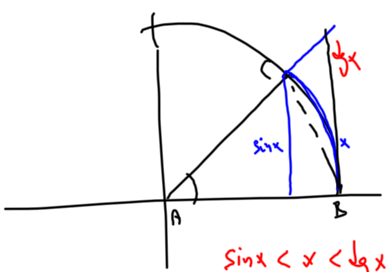
10 5-13:19

$a^{x^y} \neq (a^x)^y = a^{x \cdot y}$

$(1+b)^n = 1 + \binom{n}{1} b + \left(\binom{n}{2} b^2 + \dots + \binom{n}{n} b^n\right) > 1 + nb$; obecněji Bernoulli $b > -1$

$$\frac{\left(1+\frac{1}{n}\right)^n}{\left(1+\frac{1}{n-1}\right)^{n-1}} = \frac{\frac{(n+1)^n}{n^n}}{\frac{n^{n-1}}{(n-1)^{n-1}}} = \frac{(n^2-1)^{n-1}(n+1)}{n^{2n-1}} = \frac{n[(n-1)(n+1)](n^2-1)^{n-1}}{n^{2n}(n-1)}$$

10 5-13:32



$\sin x < x < \mathrm{tg}\, x$

10 5-13:41