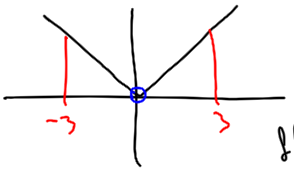


$f(x)=|x|$

$f(3)=f(-3)$

$\nexists c\in(-3,3): f'(c)=0$

$$f'(x_0)=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}=\lim\frac{k-k}{x-x_0}=0$$

10 19-11:52

max. nastává v bodě $c\in(a,b)$

dokážeme, že $f'(c)=0$

Kdyby $f'(c)>0$, ex. $U(c)$ na němž je derivace >0.

$$0<f'_+(c)=\lim_{h\to 0_+}\frac{f(c+h)-f(c)}{h}$$

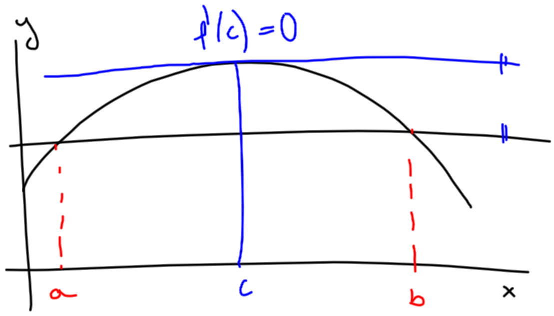
na tomto okolí je $f(c+h)>f(c)$, což je spor.

Analogicky

$$0<f'_-(c)=\lim_{h\to 0_-}\frac{f(c+h)-f(c)}{h}\Rightarrow f(c+h)<f(c)$$

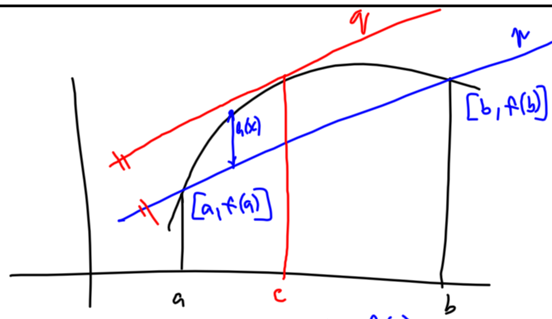
(spor s min.)

10 19-12:04



$f(a)=f(b)$

10 19-12:08

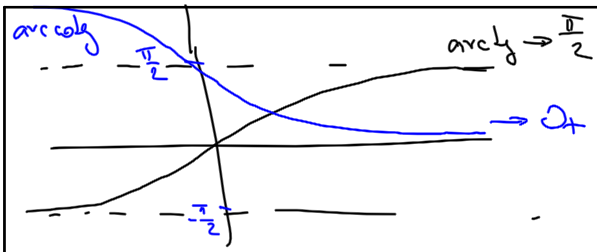


p má směrnici $\dfrac{f(b)-f(a)}{b-a}$;

q má směrnici $f'(c)$

$$p:\quad y-f(a)=\frac{f(b)-f(a)}{b-a}\cdot(x-a)$$

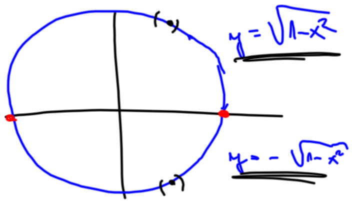
10 19-12:10



$$(\operatorname{arctg} x)'=\frac{1}{1+x^2}$$

$$(\operatorname{arccotg} x)'=-\frac{1}{1+x^2}$$

10 19-12:27



$x^2+y^2=1$ implicitně zadaná funkce.

Př: $5x^3y^2-xy^3+y-x^2+1=0$

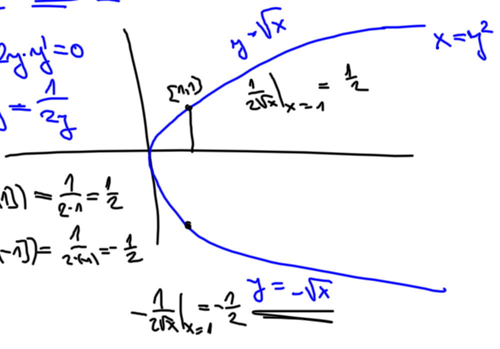
$5\cdot 3x^2y^2+5\cdot x^3\cdot 2y\cdot y'-\dots=0$

10 19-12:36

$x - y^2 = 0 \quad [x - y(x)^2 = 0]$

$1 - 2y \cdot y' = 0$

$y' = \frac{1}{2y}$



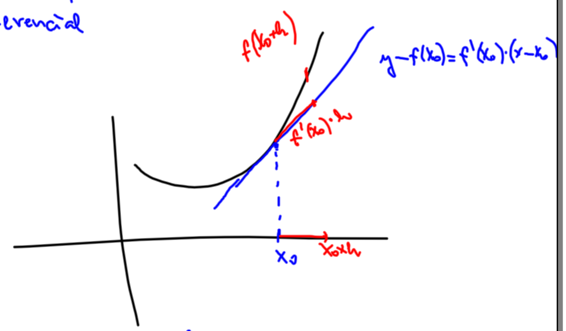
$y'([1,1]) = \frac{1}{2\cdot 1} = \frac{1}{2}$

$y'([1,-1]) = \frac{1}{2\cdot(-1)} = -\frac{1}{2}$

$-\frac{1}{2\sqrt{x}}\Big|_{x=1} = -\frac{1}{2}$ $\quad y = -\sqrt{x}$

10 19-12:40

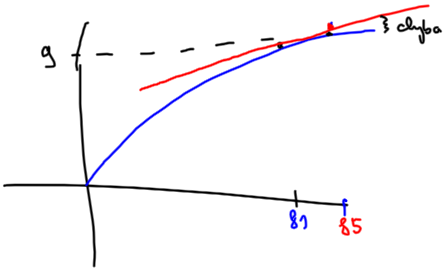
Diferenciál



$dy = f'(x_0)\cdot dx$

$df: \mathbb{R} \to \mathbb{R} \qquad h \mapsto f'(x_0)\cdot h$

10 19-12:47



$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$df: h \mapsto f'(x_0)\cdot h$

$h \mapsto \frac{1}{2\sqrt{x_0}}\cdot h$

$h \mapsto \frac{1}{18} h$

10 19-12:51

$f(x) = \cos x$

$T_6(x) = \sum_{k=0}^{6} f^{(k)}(x_0)\cdot\frac{(x-x_0)^k}{k!}$

$= \sum_{k=0}^{6} f^{(k)}(0)\,\frac{x^k}{k!}$

$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

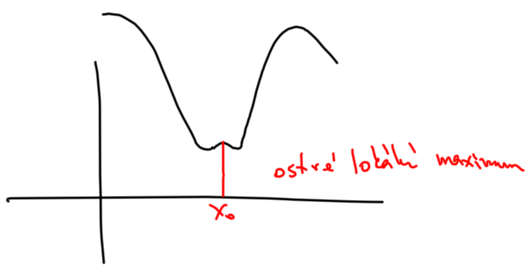
$R_6(x) = f^{(7)}(c)\cdot\frac{x^7}{7!}$

$f(x)$	$\cos x$
$f'(x)$	$-\sin x$
$f''(x)$	$-\cos x$
$f'''(x)$	$\sin x$
$f^{(4)}(x)$	$\cos x$
$f^{(5)}(x)$	$-\sin x$
$f^{(6)}(x)$	$-\cos x$
$f^{(7)}(x)$	$\sin x$

$x = \frac{\pi}{4}$ $\quad c \in \langle 0, \frac{\pi}{4}\rangle$ určitě $|\sin c| \leq 1$

$\Rightarrow |R_6(x)| \leq \frac{(\frac{\pi}{4})^7}{7!} < \frac{1}{7!} < \frac{1}{5000}$

10 19-13:10

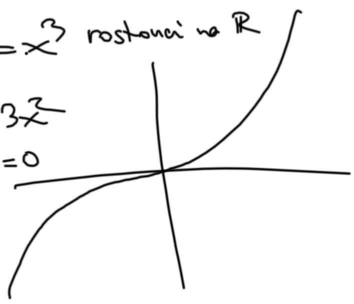


10 19-13:18

$y = x^3$ rostoucí na $\mathbb{R}$

$y' = 3x^2$

$y'(0) = 0$



10 19-13:21

lokální extrém v bodech:

— stacionární, tj. $f'(x)=0$

— $f'(x_0)$ neexistuje

$$f(x) \approx \underbrace{f'(x_0)}_{>0}\underbrace{(x-x_0)}_{>0} + f(x_0) > f(x_0)$$

$x>x_0$: $f(x)>f(x_0)$ $f'(x_0)<0$

$x<x_0$: $\exists x: f(x)<f(x_0)$

10 19-13:23

Dk (z Taylorovy věty)

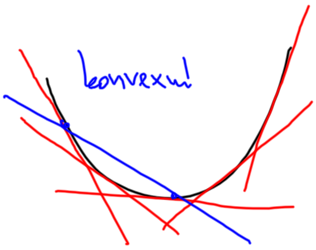
$$f(x) = f(x_0) + \cancel{f'(x_0)(x-x_0)} + \frac{\overbrace{f''(c)}^{<0}}{2!}\underbrace{(x-x_0)^2}_{>0}$$

Pokud $f'(x_0)=0 \wedge f''(x_0)<0$, pak existuje okolí bodu x_0 tak, že

$f''(x)<0 \ \forall x \in O$

$\Rightarrow f(x) < f(x_0)$ v O

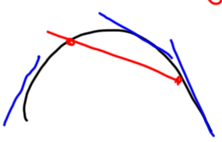
10 19-13:27



graf leží „nad tečnou"

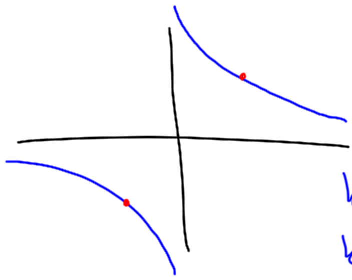
graf leží „pod sečnou"

$f'(x_0)$ se zvětšuje se zvětšujícím se x_0



konkávní

10 19-13:31



$y=\frac{1}{x}$

klesající na $(-\infty,0)$

klesající na $(0,\infty)$

NENÍ klesající na $\mathbb{R}$

či na $(-\infty,0)$
$\cup(0,\infty)$

$y'(x) = \frac{-1}{x^2} < 0 \ \forall x \in \mathbb{R}\setminus\{0\}$

10 19-13:34

$y=x^4$

$y''=12x^2$ $y''\geq 0 \ \forall x\in\mathbb{R}$

je konvexní

10 19-13:36