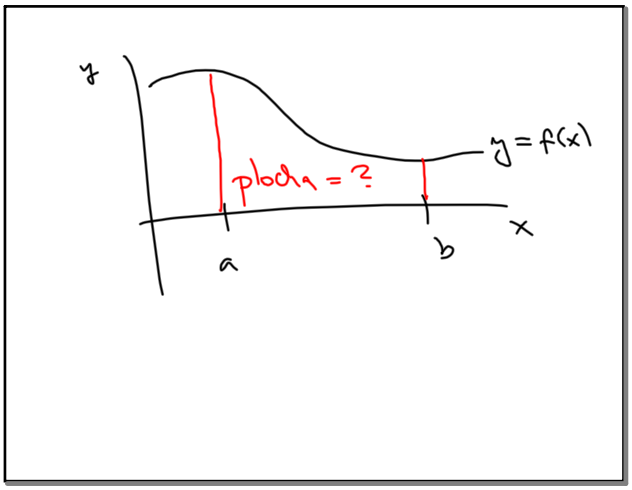
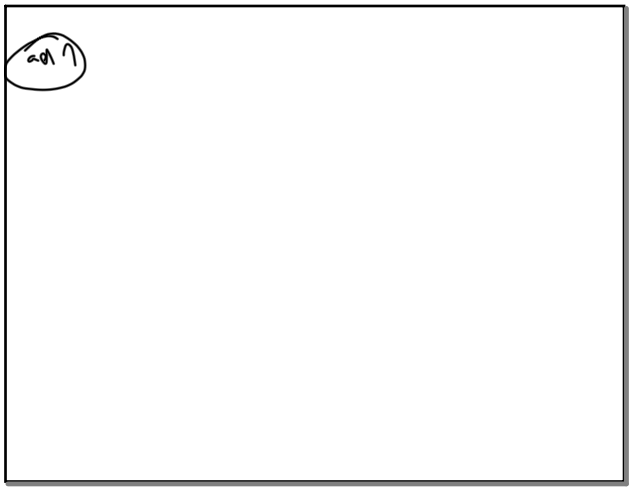
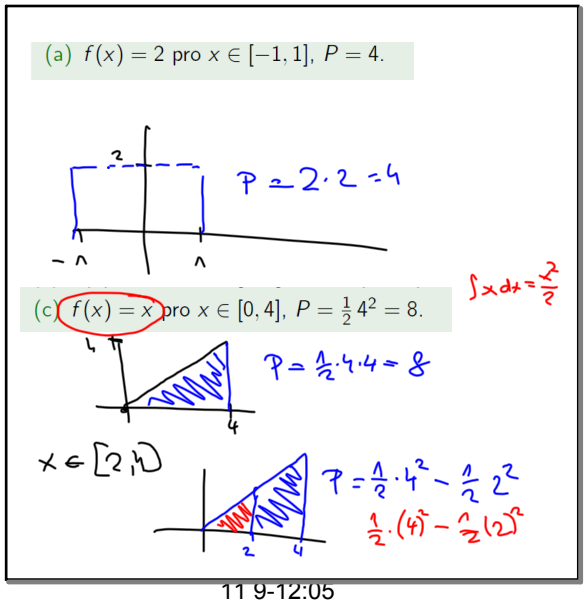


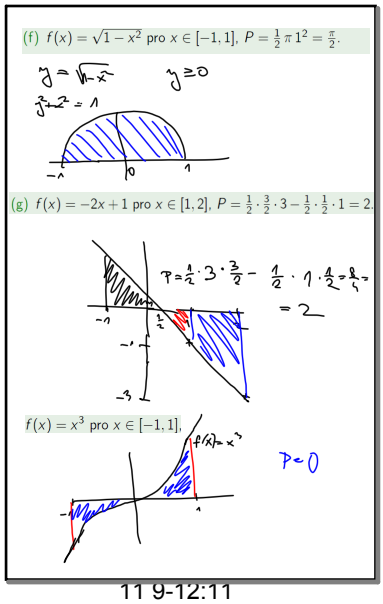
11 9-12:04



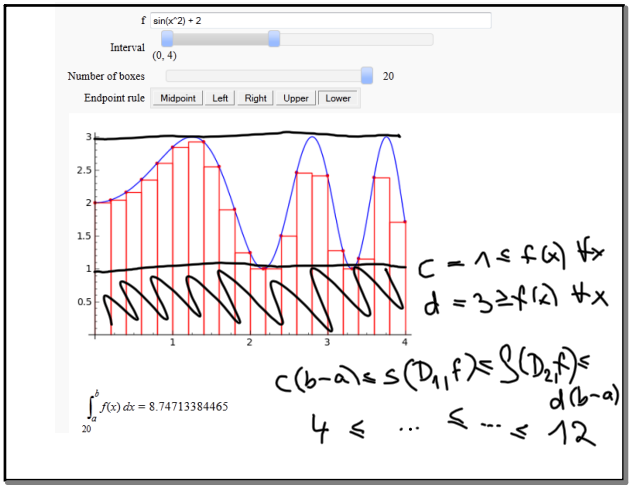
11 9-12:05



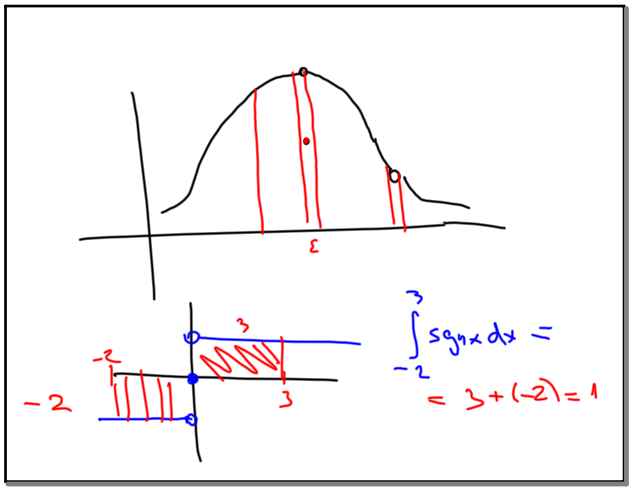
11 9-12:05



11 9-12:11



11 9-12:18



11 9-12:48

trojúhelníková nerovnost

$|a+b| \leq |a| + |b|$

$|a_1 + \dots + a_n| \leq |a_1| + \dots + |a_n|$

$\left|\sum_{i=1}^{n} a_i\right| \leq \sum_{i=1}^{n} |a_i|$

$a_i = f(x_i)$

$\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$

11 9-12:57

Pro (nespojitou) funkci

$$f(x) := \begin{cases} 5, & \text{pro } x \in [0,1), \\ 10, & \text{pro } x \in [1,2], \end{cases}$$

$F(x) = \int_0^x f(t)\,dt$

$= \int_0^x f(t)\,dt$

$x \in [0,1) \Rightarrow F(x) = \int_0^x 5\,dt = 5x$

$x \in [1,2] \Rightarrow F(x) = \int_0^1 5\,dt + \int_1^x 10\,dt = 5x + 10(x-1) = 15x - 10$

11 9-13:06

per-partes:

$\int_a^b f \cdot g'\,dx = [f g]_a^b - \int_a^b f' \cdot g\,dx =$

$= f(b) \cdot g(b) - f(a) g(a) - \int_a^b f' g\,dx$

$\int_1^e \ln x\,dx = \left|\begin{matrix} u' = 1 & u = x \\ v = \ln x & v' = \frac{1}{x} \end{matrix}\right| = [x \ln x]_1^e -$

$- \int_1^e x \cdot \frac{1}{x}\,dx = e - [x]_1^e =$

$= e - (e - 1) = 1$

$\int \ln x = x \ln x - x$

$\int_1^e \ln x = [x \ln x - x]_1^e = (e - e) - (0 - 1) =$

$= 1$

11 9-13:21

$[r\cos t, r\sin t]$

$y = \sqrt{r^2 - x^2}$

$x^2 + y^2 = r^2$

Též:

$x = r\cos t$

$y = r\sin t$

11 9-13:28

Riemannův integrál | Vlastnosti určitého integrálu | Integrál jako funkce horní meze | Aplikace určitého integrálu

Příklad

Vypočtěte obsah kruhu s poloměrem $r > 0$.

Řešení

Obsah kruhu vypočítáme např. jako dvojnásobek obsahu půlkruhu.

$$P = 2\int_{-r}^{r} \sqrt{r^2 - x^2}\,dx = \left|\begin{matrix} x = r\sin t \\ dx = (r\cos t)\,dt \\ x = -r \Rightarrow t = -\frac{\pi}{2} \\ x = r \Rightarrow t = \frac{\pi}{2} \end{matrix}\right| =$$

$$= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underbrace{\sqrt{r^2 - r^2\sin^2 t}}_{= r\cos t} \cdot \underbrace{r\cos t\,dt}_{dx} = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r^2\cos^2 t\,dt =$$

$$= 2r^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 + \cos(2t)}{2}\,dt = 2r^2\left[\frac{t}{2} + \frac{1}{2}\cdot\frac{\sin(2t)}{2}\right]_0^{\frac{\pi}{2}} =$$

$$= 2r^2\left\{\left(\frac{\pi}{4} + \underbrace{\frac{\sin\pi}{4}}_{=0}\right) - \left(-\frac{\pi}{4} + \underbrace{\frac{\sin(-\pi)}{4}}_{=0}\right)\right\} = \pi r^2.$$

$\int \cos^2 t\,dt =$

$\cos^2 t - \sin^2 t = \cos 2t$

$2\cos^2 t - 1 = \cos 2t \Rightarrow \cos^2 t = \frac{1 + \cos 2t}{2}$

$\sin^2 t = \frac{1 - \cos 2t}{2}$

Důkaz $\int f(ax)\,dx = \left|\begin{matrix} t = ax \\ dt = a\,dx \end{matrix}\right| =$

$= \frac{1}{a}\int f(t)\,dt$

11 9-13:31