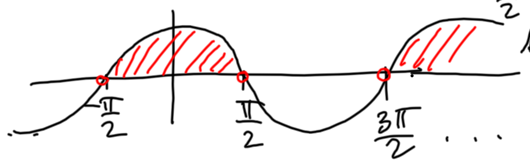
PŘ.: Určete definiční obor fce $f(x)=\log_2\sqrt{\cos x}$

z odmocniny $\cos x \geq 0$

a z logaritmu $\sqrt{\cos x} > 0$

$\Big\}\ \cos x > 0$

$D_f = \left(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi\right)$ $k \in \mathbb{Z}$



10 19-18:56

PŘ 4.2 Pomocí Lagr. interpolace proložte polynom body

$A_0=[-1,10]$, $A_1=[1,4]$, $A_2=[4,25]$

$$L_n(x)=\sum_{i=0}^{n} y_i \cdot l_i(x)$$

kde

$$l_i(x)=\frac{(x-x_0)(x-x_1)\cdot\ldots\cdot(x-x_{i-1})(x-x_{i+1})\cdot\ldots\cdot(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdot\ldots\cdot(x_i-x_{i-1})(x_i-x_{i+1})\cdot\ldots\cdot(x_i-x_n)}$$

$$l_0(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}=\frac{(x-1)(x-4)}{(-1-1)(-1-4)}=$$
$$=\frac{x^2-5x+4}{10}$$

$$l_1(x)=\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}=\frac{(x+1)(x-4)}{(1+1)(1-4)}=\frac{-x^2+3x+4}{6}$$

$$l_2(x)=\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}=\frac{(x+1)(x-1)}{(4+1)(4-1)}=\frac{x^2-1}{15}$$

$$L_2(x)=\cancel{10}\cdot\frac{x^2-5x+4}{\cancel{10}}+\overset{2}{\cancel{4}}\cdot\frac{-x^2+3x+4}{\underset{3}{\cancel{6}}}+\overset{5}{\cancel{25}}\frac{x^2-1}{\underset{3}{\cancel{15}}}$$

$$=x^2-5x+4-\frac{2x^2}{3}-2x+\frac{8}{3}+\frac{5}{3}x^2-\frac{5}{3}=$$

$$=2x^2-7x+5$$

10 19-19:10

PŘ.: Vypočtěte limitu posloupnosti $a^2-b^2=(a+b)(a-b)$

$$\lim_{n\to\infty} n\left(\sqrt{n^2+1}-n\right)=$$

$$=\lim_{n\to\infty} n\cdot\frac{\left(\sqrt{n^2+1}-n\right)\cdot\left(\sqrt{n^2+1}+n\right)}{\sqrt{n^2+1}+n}=$$

$$=\lim_{n\to\infty}\frac{n\cdot(\cancel{n^2}+1-\cancel{n^2})}{\sqrt{n^2+1}+n}=$$

$$=\lim_{n\to\infty}\frac{n}{\sqrt{n^2+1}+n}=$$

$$=\lim_{n\to\infty}\frac{\cancel{n}\cdot 1}{\cancel{n}\left(\sqrt{1+\underbrace{\tfrac{1}{n^2}}_{\to 0}}+1\right)}=\frac{1}{2}$$

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$(a+b)(a^2-ab+b^2)=a^3+b^3$

10 19-19:21

PŘ.: Rozložte na parciální zlomky

$$R(x)=\frac{x+1}{x^5+3x^3+2x}=\frac{x+1}{x(x^4+3x^2+2)}=$$

$$=\frac{x+1}{x(x^2+1)(x^2+2)}=$$

$$=\frac{A}{x}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{x^2+2}=$$

$$=\frac{A(x^2+1)(x^2+2)+(Bx+C)x(x^2+2)+(Dx+E)x(x^2+1)}{x(x^2+1)(x^2+2)}$$

$x+1=A(x^2+1)(x^2+2)+(Bx+C)x(x^2+2)+(Dx+E)x(x^2+1)$

$x+1=A(x^4+3x^2+2)+(Bx^2+Cx)(x^2+2)+(Dx^2+Ex)(x^2+1)$

$x+1=Ax^4+3Ax^2+2A+Bx^4+2Bx^2+Cx^3+2Cx+Dx^4+Dx^2+Ex^3+Ex$

$x^4: 0=A+B+D \Rightarrow D=-B-\frac{1}{2} \Rightarrow D=\frac{1}{2}$

$x^3: 0=C+E \Rightarrow C=-E \Rightarrow C=1$

$x^2: 0=3A+2B+D \Rightarrow *$

$x: 1=2C+E \Rightarrow 1=-2E+E \Rightarrow E=-1$

$x^0: 1=2A \Rightarrow A=\frac{1}{2}$

$*\quad 3\cdot\frac{1}{2}+2B-B-\frac{1}{2}=0$

$1+B=0$

$B=-1$

$$R(x)=\frac{\frac{1}{2}}{x}+\frac{-x+1}{x^2+1}+\frac{\frac{1}{2}x-1}{x^2+2}$$

(upravit zlomky)

10 19-19:29

PŘ.: Napište rovnici tečny a normály ke grafu $f(x)=x^2$, která prochází bodem $[1,1]$. (x_0, y_0)

Bod $[1,1]$ leží na grafu $f(x)$, neboť $f(1)=1^2=1$.

tečna: $y-y_0=f'(x_0)(x-x_0)$ ($y_0 = f(x_0)$)

$f'(x)=(x^2)'=2x$

$f'(1)=2\cdot 1=2$

$t: y-1=2\cdot(x-1)$

$y-1=2x-2$

$t: 2x-y-1=0$

normála: $y-y_0=-\frac{1}{f'(x_0)}(x-x_0)$

$n: y-1=-\frac{1}{2}(x-1) \quad |\cdot 2$

$2y-2=-x+1$

$n: 2y+x-1=0$

10 19-19:42

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x + \cos^2 x = 1$

$\operatorname{tg} x = \frac{\sin x}{\cos x} = \frac{1}{\operatorname{cotg} x}$

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$

$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

$\lim_{x\to 0}\frac{\sin x}{x}=1$ $\quad\cos(\arccos x)=x$

$\arccos(\cos x)=x$

$\lim_{x\to 0}\frac{\operatorname{tg} x}{x}=1$

PŘ.: $\lim_{x\to 0}\frac{\operatorname{arctg} x}{x}=\lim_{x\to 0}\frac{\operatorname{arctg} x}{\operatorname{tg}(\operatorname{arctg} x)}$

$=$ subst $[\operatorname{arctg} x = y]$ $=$

$=\lim_{y\to 0}\frac{y}{\operatorname{tg} y}=\lim_{y\to 0}\frac{1}{\frac{\operatorname{tg} y}{y}}=1$

10 19-19:49

PŘ: Zderivujte x^{x^x}

1) $(x^x)' = (e^{\ln x^x})' = (e^{x\ln x})' = e^{x\ln x}\cdot(\ln x + x\cdot\frac{1}{x})$

$= e^{x\ln x}\cdot(\ln x + 1)$

2) $(x^{x^x})' = (e^{\ln x^{x^x}})' = (e^{x^x\ln x})' =$

$= e^{x^x\ln x}\cdot\left((x^x)'\cdot\ln x + x^x\cdot(\ln x)'\right) =$

$= e^{x^x\ln x}\left(e^{x\ln x}\cdot(\ln x+1)\cdot\ln x + x^x\cdot\frac{1}{x}\right) =$

$= \underbrace{x^{x^x}\left(\left(x^x(\ln x+1)\ln x\right) + x^{x-1}\right)}$

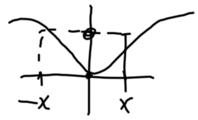
$\ln a + \ln b = \ln(ab)$

$(\ln a - \underbrace{\ln b}_{\neq 0}) = \ln\left(\frac{a}{b}\right)$

$\ln a^b = b\cdot\ln a$

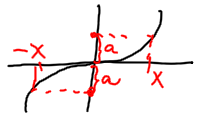
10 19-19:56

SUDÁ ... souměrná dle osy y



$f(x) = f(-x)$

LICHÁ ... souměrná dle [0,0]



$f(x) = -f(-x)$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

10 19-20:02