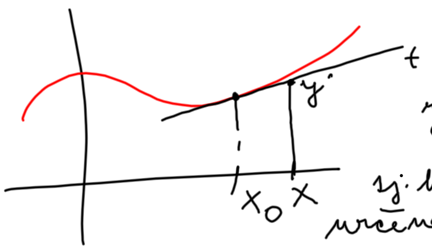
ODEČTĚTE HODNOTU NA TEČNĚ



y ... hodnota na tečně

y_i: hodnota určená pomocí diferenciálu

$$y = f(x) = f(x_0) + \underbrace{f'(x_0)(x-x_0)}_{\text{DIFENCIÁL}}$$

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27.3 $\lim\limits_{x\to 1} \dfrac{x^m - x}{x^m - 1} = \left\|\dfrac{0}{0}\right\| \overset{L'H}{=} \lim\limits_{x\to 1} \dfrac{m x^{m-1} - 1}{m\cdot x^{m-1}} =$

$= \dfrac{m\cdot 1^{m-1} - 1}{m\cdot 1^{m-1}} = \dfrac{m-1}{m} \qquad m \in N$

27.6 $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\mathrm{tg}\, 3x}{\mathrm{tg}\, x} = \left\|\dfrac{\infty}{\infty}\right\| \overset{L'H}{=} \lim\limits_{x\to \frac{\pi}{2}} \dfrac{\frac{1\cdot 3}{\cos^2 3x}}{\frac{1}{\cos^2 x}} =$

$= \lim\limits_{x\to \frac{\pi}{2}} \dfrac{3\cos^2 x}{\cos^2 3x} = \left\|\dfrac{0}{0}\right\| \overset{L'H}{=} \lim\limits_{x\to \frac{\pi}{2}} \dfrac{\cancel{3}\cdot 2\cdot \cos x \cdot \sin x}{2\cdot \cancel{3} \cos 3x \cdot \sin 3x} =$

$= \lim\limits_{x\to \frac{\pi}{2}} \dfrac{\sin 2x}{\sin 6x} = \left|\dfrac{0}{0}\right| = \lim\limits_{x\to \frac{\pi}{2}} \dfrac{2\cos 2x}{6\cos 6x} =$

$= \dfrac{2\cdot(-1)}{6\cdot(-1)} = \underline{\underline{\dfrac{1}{3}}}$

27.8 $\lim\limits_{x\to 0^+} x^a \ln x = \|0\cdot\infty\| = \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x^a}} =$ $\quad x^{-a}$

$= \left|\dfrac{\infty}{\infty}\right| \overset{L'H}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{(-a)\cdot x^{(-a-1)}} = \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{\frac{(-a)}{x^{a+1}}} =$

$= \lim\limits_{x\to 0^+} \dfrac{x^a}{-a} = \underline{\underline{0}} \qquad a \in R$

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PŘ 30 Určete pomocí TP hodnotu ln 3.

$e \doteq 2{,}71828\ldots = x_0 \ldots$ STŘED

$(x - x_0) \ \ldots\ldots\ (3-e)$

$f(x) = \ln x \qquad f(e) = \ln e = 1$

$f'(x) = \dfrac{1}{x} \qquad f'(e) = \dfrac{1}{e}$

$f''(x) = \dfrac{-1}{x^2} \qquad f''(e) = -\dfrac{1}{e^2}$

$f'''(x) = \dfrac{2}{x^3} \qquad f'''(e) = \dfrac{2}{e^3}$

$f^{(4)}(x) = \dfrac{-6}{x^4} \qquad f^{(4)}(e) = -\dfrac{6}{e^4}$

$T_e^4(x) = f(x_0) + \dfrac{f'(x_0)}{1!}(x-x_0) + \dfrac{f''(x_0)}{2!}(x-x_0)^2 +$
$+ \dfrac{f'''(x_0)}{3!}(x-x_0)^3 + \dfrac{f^{(4)}(x_0)}{4!}(x-x_0)^4 =$

$T_e^4(3) = 1 + \dfrac{1}{e}(3-e) - \dfrac{\frac{1}{e^2}}{2!}(3-e)^2 + \dfrac{\frac{2}{e^3}}{3!}(3-e)^3 -$
$- \dfrac{\frac{6}{e^4}}{4!}(3-e)^4 = 1 + \dfrac{3-e}{e} - \dfrac{1}{2e^2}(3-e)^2 +$
$+ \dfrac{1}{3e^3}(3-e)^3 - \dfrac{1}{4\cdot e^4}(3-e)^4$

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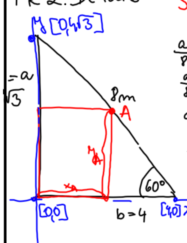
PŘ 2. DEMO č. 6: $S_{\square} = \max$?



$\dfrac{a}{8} = \sin 60^\circ$

$\dfrac{a}{8} = \dfrac{\sqrt{3}}{2}$

$a = 4\sqrt{3}$

$\dfrac{b}{8} = \cos 60^\circ$

$\dfrac{b}{8} = \dfrac{1}{2}$

$b = 4$

Směrnice přepony $\quad -\mathrm{tg}\, 60^\circ = -\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \underline{\underline{-\sqrt{3}}}$

přepona: $y = \underline{\underline{-\sqrt{3}x + 4\sqrt{3}}}$

$S_{\square} = x\cdot(-\sqrt{3}x + 4\sqrt{3})$ maximalizuji

$= -\sqrt{3}x^2 + 4\sqrt{3}\,x$

$S'_{\square} = -2\sqrt{3}\,x + 4\sqrt{3}$

$-2\sqrt{3}x + 4\sqrt{3} = 0$

$\underline{\underline{x = 2}}$

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PŘ 5 demo č. 6:

náklady 400 000

náklady na brusky 500n

cena $\left(1200 - \dfrac{n}{10}\right)$

kde n je počet prodaných brusek maximalizujte zisk.

$-400\,000 - 500n + n\left(1200 - \dfrac{n}{10}\right) = z(n)$

$-400\,000 - 500n + 1200n - \dfrac{n^2}{10} = z(n)$

$z'(n) = -\dfrac{2n}{10} + 1200 - 500 = -\dfrac{1}{5}n + 700$

$\dfrac{n}{5} = 700$

$\underline{n = 3500}$

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$f(x) = e^{\ln(f(x))}$

$f(x)^{g(x)} = e^{\ln f(x)^{g(x)}} = e^{g(x)\ln f(x)}$

PŘ. 27.17 $\lim\limits_{x\to 0} \left(\dfrac{\mathrm{tg}\, x}{x}\right)^{\frac{1}{x^2}} = \left(\dfrac{0}{0}\right)^{\infty}$

$\boxed{\text{POZN } \lim\limits_{x\to 0} \dfrac{\mathrm{tg}\, x}{x} = 1}$

$\lim\limits_{x\to 0} e^{\frac{1}{x^2}\ln\frac{\mathrm{tg}\, x}{x}} = \lim\limits_{x\to 0} e^0 = \underline{\underline{1}}$

$\lim\limits_{x\to 0} \dfrac{1}{x^2}\cdot \ln\dfrac{\mathrm{tg}\, x}{x} = \lim\limits_{x\to 0} \dfrac{\ln\frac{\mathrm{tg}\, x}{x}}{x^2} =$

$= \lim\limits_{x\to 0} \dfrac{\dfrac{1}{\frac{\mathrm{tg}\, x}{x}}\cdot\left(\dfrac{\frac{1}{\cos^2 x}\cdot x - \mathrm{tg}\, x}{x^2}\right)}{2x} =$

$= \lim\limits_{x\to 0} \dfrac{\left(\frac{x}{\cos^2 x} - \mathrm{tg}\, x\right)\cdot 2\cancel{x}}{(\mathrm{tg}\, x)\cdot \cancel{x}} = \lim\limits_{x\to 0} \dfrac{2(x - \mathrm{tg}\, x \cos^2 x)}{\cos^2 x\cdot \mathrm{tg}\, x} =$

$= \lim\limits_{x\to 0} \dfrac{2\left(x - \frac{\sin x}{\cos x}\cos^2 x\right)}{\cos^2 x\cdot \frac{\sin x}{\cos x}} =$

$= \lim\limits_{x\to 0} \dfrac{2x - \sin 2x}{\frac{\sin 2x}{2}} \overset{L'H}{=}$

$= \lim\limits_{x\to 0} \dfrac{2 - 2\cos 2x}{\frac{\cancel{2}\cos 2x}{\cancel{2}}} = \dfrac{2-2}{1} = \underline{\underline{0}}$

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27.5 $\lim\limits_{x \to 0} \dfrac{1-\cos x^2}{x^2 \sin x^2} = \left|\dfrac{0}{0}\right|$

$= \lim\limits_{x \to 0} \dfrac{(\sin x^2)\cdot 2x}{2x \sin x^2 + x^2 \cdot (\cos x^2) \cdot 2x} =$

$= \lim\limits_{x \to 0} \dfrac{\cancel{2x} \sin x^2}{\cancel{2x}(\sin x^2 + x^2 \cos x^2)} = \left|\dfrac{0}{0}\right|$

$= \lim\limits_{x \to 0} \dfrac{2x \cos x^2}{2x \cos x^2 + 2x \cos x^2 + x^2(-\sin x^2)\cdot 2x} =$

$= \lim\limits_{x \to 0} \dfrac{\cancel{2x} \cos x^2}{\cancel{2x}(2\cos x^2 - x^2 \sin x^2)} =$

$= \underline{\dfrac{1}{2}}$

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32.1 určete dif. fce

$f'(x_0)(x - x_0)$

$f(x) = x e^x$

$f'(x) = e^x + x \cdot e^x = \underline{e^x(1+x)}$

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