$$\int \frac{\sqrt{x}-2\sqrt[3]{x^2}+1}{\sqrt[4]{x}}\,dx = \int\frac{\sqrt{x}}{\sqrt[4]{x}}\,dx + \int\frac{-2\cdot\sqrt[3]{x^2}}{\sqrt[4]{x}}\,dx + \int\frac{1}{\sqrt[4]{x}}\,dx$$

$$= \int \frac{x^{\frac12}}{x^{\frac14}}\,dx - 2\int\frac{x^{\frac23}}{x^{\frac14}}\,dx + \int x^{-\frac14}\,dx =$$

$$= \int x^{\frac14}\,dx - 2\int x^{\frac{5}{12}}\,dx + \frac{x^{\frac34}}{\frac34} + C =$$

$$= \frac{x^{\frac54}}{\frac54} - 2\frac{x^{\frac{17}{12}}}{\frac{17}{12}} + \frac43\sqrt[4]{x^3} + C$$

$$= \frac45\sqrt[4]{x^5} - 2\cdot\frac{12}{17}\sqrt[12]{x^{17}} + \frac43\sqrt[4]{x^3} + C$$

10 31-17:59

$$\int 1+\sin x+\cos x\,dx = \int 1\,dx + \int \sin x\,dx + \int \cos x\,dx = x - \cos x + \sin x + C$$

10 31-18:14

$$\int\frac{2^{x+1}-5^{x-1}}{10^x}\,dx = \int\frac{2^{x+1}}{10^x}\,dx - \int\frac{5^{x-1}}{10^x}\,dx$$

$$= 2\int\left(\frac{2}{10}\right)^x dx - \frac15\int\left(\frac{5}{10}\right)^x dx =$$

$$= 2\int\left(\frac15\right)^x dx - \frac15\int\left(\frac12\right)^x dx =$$

$$= \frac{2\left(\frac15\right)^x}{\ln\left(\frac15\right)} - \frac{\frac15\left(\frac12\right)^x}{\ln\left(\frac12\right)} + C$$

$$= \frac{\frac{2}{5^x}}{-\ln 5} + \frac{\frac{1}{5\cdot 2^x}}{\ln 2} = \underline{-\frac{2}{5^x\ln 5} + \frac{1}{5\cdot 2^x\ln 2} + C}$$

10 31-18:15

$$\int\frac{2x+1}{x^2+1}\,dx = \underline{\int\frac{2x}{x^2+1}\,dx} + \int\frac{1}{x^2+1}\,dx$$

$$= \ln(x^2+1) + \operatorname{arctg} x + C$$

10 31-18:21

$$\int\frac{x}{x+1}\,dx = \int\frac{x+1-1}{x+1}\,dx =$$

$$\int\frac{x+1}{x+1}\,dx - \int\frac{1}{x+1}\,dx =$$

$$x - \ln|x+1| + C$$

$$\int\frac{x}{x+1}\,dx = \left|\begin{array}{l} x+1 = t \\ dx = dt \\ x = t-1 \end{array}\right| =$$

$$\int\frac{t-1}{t}\,dt = \int\frac{t}{t}\,dt - \int\frac1t\,dt$$

$$= t - \ln|t| = \underline{x+1-\ln|x+1| + C}$$

10 31-18:22

$$\int \operatorname{tg} x\,dx = \int\frac{\sin x}{\cos x}\,dx =$$

$$\int\frac{(-1)\sin x}{(-1)\cos x}\,dx = -\int\frac{-\sin x}{\cos x}\,dx$$

$$= -\ln|\cos x| + C$$

10 31-18:27

$$\int \frac{\sqrt{x^2+2+x^{-2}}}{x}\,dx = \int \frac{\sqrt{(x+x^{-1})^2}}{x}\,dx$$

$$= \int \frac{x+x^{-1}}{x}\,dx =$$

$$= \int 1\,dx + \int \frac{1}{x^2}\,dx =$$

$$x - \frac{1}{x} + C$$

$$\int \frac{\cos 2x}{\sin^2 x}\,dx = \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x}\,dx$$

$$= \int \frac{\cos^2 x}{\sin^2 x}\,dx - \int 1\,dx$$

$$= \int \frac{1-\sin^2 x}{\sin^2 x} - x + C$$

$$= \int \frac{1}{\sin^2 x}\,dx - \int 1\,dx - x + C$$

$$= -\operatorname{cotg} x - 2x + C$$

10 31-18:29

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|$$

$$\int \frac{f'(x)}{f(x)}\,dx = \left| \begin{array}{l} f(x) = t \\ f'(x)\,dx = dt \\ dx = \frac{dt}{f'(x)} \end{array} \right| =$$

$$\int \frac{\cancel{f'(x)}}{t}\,\frac{dt}{\cancel{f'(x)}} = \int \frac{1}{t}\,dt =$$

$$\ln|t| + C = \underline{\ln|f(x)| + C}$$

10 31-18:35

$$\int \frac{1}{\cos^2 2x}\,dx = \left| \begin{array}{l} 2x = t \\ 2dx = dt \\ dx = \frac{dt}{2} \end{array} \right| =$$

$$\int \frac{1}{\cos^2 t}\cdot\frac{dt}{2} = \frac{1}{2}\int \frac{1}{\cos^2 t}\,dt =$$

$$= \frac{1}{2}\operatorname{tg} t + C = \frac{1}{2}\operatorname{tg} 2x + C$$

$$\underline{\int f(x)\,dx = g(x) + C}$$

$$\int f(ax+b)\,dx = \frac{1}{a}\,g(ax+b) + C$$

$$\int f(ax+b)\,dx = \left| \begin{array}{l} ax+b = t \\ a\,dx = dt \\ dx = \frac{dt}{a} \end{array} \right| =$$

$$\int f(t)\,\frac{dt}{a} = \frac{1}{a}\int f(t)\,dt =$$

$$= \frac{1}{a}\,g(t) + C$$

$$= \frac{1}{a}\,g(ax+b) + C$$

$$\int (2x+3)^5\,dx = \left| \begin{array}{l} 2x+3 = t \\ 2dx = dt \\ dx = \frac{dt}{2} \end{array} \right| =$$

$$= \frac{1}{2}\cdot\frac{(2x+3)^6}{6} + C$$

$$= \int t^5\,\frac{dt}{2} = \frac{1}{2}\int t^5\,dt =$$

$$= \frac{1}{2}\,\frac{t^6}{6} = \frac{1}{2}\cdot\frac{(2x+3)^6}{6} + C$$

10 31-18:37

$$\int \frac{e^{3x}+1}{e^x+1}\,dx = \int \frac{(e^x+1)(e^{2x}-e^x+1)}{e^x+1}\,dx$$

$$= \int e^{2x} - e^x + 1\,dx =$$

$$= \frac{1}{2}e^{2x} - e^x + x + C$$

10 31-18:45

$$\int \frac{x}{(1+2x)^5}\,dx = \left| \begin{array}{l} 1+2x = t \\ 2dx = dt \\ dx = \frac{dt}{2} \\ x = \frac{t-1}{2} \end{array} \right| =$$

$$= \int \frac{\frac{t-1}{2}}{t^5}\,\frac{dt}{2} = \frac{1}{4}\int \frac{t-1}{t^5}\,dt$$

$$= \frac{1}{4}\int t^{-4}\,dx - \frac{1}{4}\int t^{-5}\,dt =$$

$$= \frac{1}{4}\cdot\frac{t^{-3}}{-3} - \frac{1}{4}\cdot\frac{t^{-4}}{-4} =$$

$$= -\frac{1}{12t^3} + \frac{1}{16t^4} + C = -\frac{1}{12(1+2x)^3} +$$

$$\frac{1}{16(1+2x)^4} + C$$

10 31-18:47

$$\int \frac{x}{\sqrt{x-1}}\,dx = \left| \begin{array}{l} \sqrt{x-1} = t \\ \frac{1}{2}\cdot\frac{1}{\sqrt{x-1}}\,dx = dt \\ dx = 2\sqrt{x-1}\,dt \\ \quad = 2t\,dt \\ x-1 = t^2 \\ x = t^2+1 \end{array} \right|$$

$t = g(x)$

$g(x)$

$$= \int \frac{t^2+1}{t}\cdot 2t\,dt = 2\int t^2+1\,dt$$

$$= \frac{2t^3}{3} + 2t + C = \frac{2(\sqrt{x-1})^3}{3} + 2\sqrt{x-1} + C$$

$$= \frac{2}{3}(x-1)\sqrt{x-1} + 2\sqrt{x-1} + C$$

10 31-18:53

$$\int \frac{e^{2x}}{\sqrt{e^x-1}}\,dx = \left| \begin{array}{l} e^x-1 = t \\ e^x = t+1 \\ e^x dx = dt \\ dx = \frac{dt}{e^x} \end{array} \right| =$$

$$= \int \frac{e^x \cdot \cancel{e^x}}{\sqrt{t}} \frac{dt}{\cancel{e^x}} = \int \frac{t+1}{\sqrt{t}}\,dt$$

$$= \int t^{\frac{1}{2}}\,dt + \int t^{-\frac{1}{2}}\,dt =$$

$$= \frac{2}{3} t^{\frac{3}{2}} + 2 t^{\frac{1}{2}} + C$$

$$= \frac{2}{3}\sqrt{(e^x-1)^3} + 2\sqrt{(e^x-1)} + C$$

$$\int \frac{e^{2x}}{\sqrt{e^x-1}}\,dx = \left| \begin{array}{l} \sqrt{e^x-1} = t \\ \frac{1}{2}\cdot\frac{1}{t}\,dx = dt \\ dx = 2t\,dt \end{array} \right|$$

$$e^x-1 = t^2$$
$$e^x = t^2+1$$

$$\int \frac{(t^2+1)^2}{\cancel{t}} 2\cancel{t}\,dt = 2\int \frac{t^4+2t^2+1}{}\,dt =$$

$$= 2\frac{t^5}{5} + 4\frac{t^3}{3} + 2t + C$$

$$= 2\frac{\sqrt{e^x-1}^5}{5} + 4\frac{\sqrt{e^x-1}^3}{3} + 2\sqrt{e^x-1} + C$$

10 31-18:59

$$\int \frac{1}{x^2+2x+2}\,dx = \int \frac{1}{(x+1)^2-1+2}\,dx$$

$$\bigcirc^2 + \bigcirc$$

$$x^2+2x+2 = (x+1)^2 - 1 + 2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \int \frac{1}{(x+1)^2+1}\,dx = \operatorname{arctg}(x+1) + C$$

10 31-19:07

$$\int \frac{1}{x^2+4}\,dx = \int \frac{1}{4\left(\frac{x^2}{4}+1\right)}\,dx =$$

$$= \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx \qquad \left| t = \frac{x}{2} \right.$$

$$= \ldots \frac{1}{4}\cdot\frac{1}{\frac{1}{2}} = \frac{1}{4}\cdot 2$$

$$= \frac{1}{2}\operatorname{arctg}\frac{x}{2}$$

10 31-19:10

$$\int \frac{x+1}{x^2+5x+11}\,dx = \frac{1}{2}\int \frac{2x+2+3-3}{x^2+5x+11}\,dx$$

$$= \frac{1}{2}\int \frac{2x+5-3}{x^2+5x+11}\,dx =$$

$$= \frac{1}{2}\int \frac{2x+5}{x^2+5x+11}\,dx - \frac{3}{2}\int \frac{1}{x^2+5x+11}\,dx$$

$$= \frac{1}{2}\ln\left|x^2+5x+11\right| - \frac{3}{2}\int \frac{1}{x^2+5x+11}\,dx$$

$$= \heartsuit - \frac{3}{2}\int \frac{1}{\left(x+\frac{5}{2}\right)^2 - \frac{25}{4} + 11}\,dx =$$

$$= \heartsuit - \frac{3}{2}\int \frac{1}{\left(x+\frac{5}{2}\right)^2 + \frac{19}{4}}\,dx =$$

$$= \heartsuit - \frac{3}{2}\int \frac{1}{\frac{19}{4}\left(\frac{4}{19}\left(x+\frac{5}{2}\right)^2+1\right)}\,dx =$$

$$= \heartsuit - \frac{3}{\cancel{2}}\cdot\frac{\cancel{4}^{2}}{19}\int \frac{1}{\left(\frac{2}{\sqrt{19}}\left(x+\frac{5}{2}\right)\right)^2+1}\,dx$$

$$= \heartsuit - \frac{6}{19}\cdot \left| \begin{array}{l} \frac{2}{\sqrt{19}}\left(x+\frac{5}{2}\right) = t \\ \frac{2}{\sqrt{19}}\,dx = dt \\ dx = \frac{\sqrt{19}}{2}\,dt \end{array} \right| =$$

$$= \heartsuit - \frac{6}{19}\cdot\int \frac{1}{t^2+1}\cdot\frac{\sqrt{19}}{2}\,dt =$$

$$= \heartsuit - \frac{3\sqrt{19}}{19}\cdot\operatorname{arctg} t + C$$

$$= \frac{1}{2}\ln\left|x^2+5x+11\right| - \frac{3\sqrt{19}}{19}\operatorname{arctg}\left(\frac{2}{\sqrt{19}}\left(x+\frac{5}{2}\right)\right) + C$$

10 31-19:13

$$\int \frac{x}{16+x^4}\,dx = \left| \begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ dx = \frac{dt}{2x} \end{array} \right| =$$

$$\int \frac{\cancel{x}}{16+t^2}\cdot\frac{dt}{2\cancel{x}} = \frac{1}{2}\int \frac{1}{t^2+16}\,dt$$

$$= \frac{1}{2}\int \frac{1}{16\left(\frac{t^2}{16}+1\right)}\,dt =$$

$$= \frac{1}{32}\cdot\int \frac{1}{\left(\frac{t}{4}\right)^2+1}\,dt$$

$$= \frac{1}{8}\cdot\operatorname{arctg}\frac{t}{4} + C$$

$$= \frac{1}{8}\operatorname{arctg}\frac{x^2}{4} + C$$

10 31-19:26

$$\int 2^x\cdot\sqrt{1+2^x}\,dx = \left| \begin{array}{l} 1+2^x = t \\ 2^x \ln 2\,dx = dt \\ dx = \frac{dt}{2^x \ln 2} \end{array} \right|$$

$$= \int \cancel{2^x}\cdot\sqrt{t}\,\frac{dt}{\cancel{2^x}\ln 2} =$$

$$= \frac{1}{\ln 2}\int \sqrt{t}\,dt = \frac{1}{\ln 2}\cdot\frac{2}{3}t^{\frac{3}{2}} + C$$

$$\boxed{= \frac{2}{3\ln 2}\sqrt{1+2^x}\cdot(1+2^x) + C}$$

10 31-19:32

$$\int \frac{1}{x \cdot \ln x^2}\,dx = \left| \begin{array}{l} \ln x^2 = t \\ \frac{1}{x^2}\cdot 2x\,dx = dt \\ \frac{2}{x}\,dx = dt \\ dx = \frac{x}{2}\,dt \end{array} \right| =$$

$$= \int \frac{1}{\cancel{x} \cdot t}\,\frac{\cancel{x}}{2}\,dt =$$

$$= \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| + C$$

$$= \frac{1}{2}\ln|\ln x^2| + C$$

10 31-19:36