$u(x), v(x)$

$(u(x)\cdot v(x))' = u'(x)v(x) + u(x)v'(x)$

$u(x)v(x) = \int u'(x)v(x)\,dx + \int u(x)v'(x)\,dx$

$\int u'(x)v(x)\,dx = u(x)v(x) - \int u(x)v'(x)\,dx$

$\int u v'\,dx = uv - \int u'v\,dx$

$$\int x\cdot a^x\,dx = \begin{vmatrix} u = x & v' = a^x \\ u' = 1 & v = \frac{a^x}{\ln a} \end{vmatrix} = \frac{x a^x}{\ln a} - \int 1\cdot\frac{a^x}{\ln a}\,dx = \frac{x a^x}{\ln a} - \frac{1}{\ln a}\int a^x\,dx = \frac{x a^x}{\ln a} - \frac{1}{\ln a}\cdot\frac{a^x}{\ln a} + C$$

$$\int x\cdot a^x\,dx = \begin{vmatrix} u = a^x & v' = x \\ u' = a^x\cdot\ln a & v = \frac{x^2}{2} \end{vmatrix} = \frac{x^2}{2}\cdot a^x - \frac{\ln a}{2}\int x^2\cdot a^x\,dx$$

11 7-17:42

$$\int \frac{x^2}{2^x}\,dx = \int x^2\cdot\frac{1}{2^x}\,dx = \int x^2\cdot 2^{-x}\,dx = \begin{vmatrix} u = x^2 & v' = 2^{-x} \\ u' = 2x & v = -\frac{2^{-x}}{\ln 2} \end{vmatrix} =$$

$$= -\frac{x^2\cdot 2^{-x}}{\ln 2} - \int -2x\cdot\frac{2^{-x}}{\ln 2}\,dx = \frac{-x^2\cdot 2^{-x}}{\ln 2} + \frac{2}{\ln 2}\int x\cdot 2^{-x}\,dx = \begin{vmatrix} u = x & v' = 2^{-x} \\ u' = 1 & v = -\frac{2^{-x}}{\ln 2} \end{vmatrix} =$$

$$= \frac{-x^2\cdot 2^{-x}}{\ln 2} + \frac{2}{\ln 2}\cdot\left(-\frac{x2^{-x}}{\ln 2} + \frac{1}{\ln 2}\int 2^{-x}\,dx\right)$$

$$= \frac{-x^2\cdot 2^{-x}}{\ln 2} + \frac{2}{\ln 2}\cdot\left(-\frac{x2^{-x}}{\ln 2} + \frac{1}{\ln 2}\cdot\frac{-2^{-x}}{\ln 2}\right) + C$$

11 7-18:09

$$\int \ln x\,dx = \int 1\cdot\ln x\,dx = \begin{vmatrix} u = \ln x & v' = 1 \\ u' = \frac{1}{x} & v = x \end{vmatrix} = x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - x + C$$

$$\int \ln^2 x\,dx = \int \ln x\cdot\ln x\,dx = \begin{vmatrix} u = \ln x & v' = \ln x \\ u' = \frac{1}{x} & v = x\ln x - x \end{vmatrix}$$

$$= \ln x\cdot(x\ln x - x) - \int \frac{1}{x}\cdot(x\ln x - x)\,dx = \ln x\cdot(x\ln x - x) - \int \ln x - 1\,dx$$

$$\boxed{= \ln x\cdot(x\ln x - x) - (x\ln x - x) + x + C}$$

$\int 1\cdot \ln^2 x\,dx$

$$\int x\ln^2 x\,dx = \begin{vmatrix} u = x & v' = \ln^2 x \\ u' = 1 & v = \end{vmatrix}$$

$$\boxed{\int x\ln x\cdot\ln x\,dx} = \begin{vmatrix} u = x\ln x & v' = \ln x \\ u' = \ln x + 1 & v = x\ln x - x \end{vmatrix}$$

$$x\ln x(x\ln x - x) - \int(\ln x + 1)(x\ln x - x)\,dx$$

$$= x\ln^2 x - x^2 - \int(x\ln^2 x - x\ln x + x\ln x - x)\,dx$$

$$= x\ln^2 x - x^2 - \int x\ln^2 x - x\,dx =$$

$$= x\ln^2 x - x^2 - \int x\ln^2 x\,dx + \frac{x^2}{2}$$

$$\int x\ln^2 x = x\ln^2 x - x^2 + \frac{x^2}{2} - \int x\ln^2 x\,dx$$

$$2\int x\ln^2 x = x\ln^2 x - \frac{x^2}{2}$$

$$\int x\ln^2 x = \frac{x}{2}\ln^2 x - \frac{x^2}{4}$$

11 7-18:14

7) $$\int e^x\sin x\,dx = \begin{vmatrix} u = e^x & v' = \sin x \\ u' = e^x & v = -\cos x \end{vmatrix} =$$

$$= -e^x\cos x + \int e^x\cos x\,dx = \begin{vmatrix} u = e^x & v' = \cos x \\ u' = e^x & v = \sin x \end{vmatrix} =$$

$$= -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx$$

$$\Rightarrow 2\int e^x\sin x\,dx = e^x(\sin x - \cos x)$$

$$\int e^x\sin x\,dx = \frac{e^x}{2}(\sin x - \cos x) + C$$

11 7-18:26

$$\int (x^2 - x + 2)\sin 3x\,dx = \begin{vmatrix} u = x^2 - x + 2 & v' = \sin 3x \\ u' = 2x - 1 & v = -\frac{1}{3}\cos 3x \end{vmatrix}$$

$$-\frac{1}{3}(x^2 - x + 2)\cos 3x + \frac{1}{3}\int(2x - 1)\cos 3x\,dx =$$

$$= \begin{vmatrix} u = 2x - 1 & v' = \cos 3x \\ u' = 2 & v = \frac{1}{3}\sin 3x \end{vmatrix} =$$

$$= -\frac{1}{3}(x^2 - x + 2)\cos 3x + \frac{1}{3}\left(\frac{(2x-1)}{3}\sin 3x - \frac{2}{3}\int\sin 3x\,dx\right)$$

$$= -\frac{1}{3}(x^2 - x + 2)\cos 3x + \frac{1}{9}(2x - 1)\sin 3x + \frac{2}{27}\cos 3x + C$$

11 7-18:19

$$\int \cos^4 x\,dx = \begin{vmatrix} u = \cos^3 x & v' = \cos x \\ u' = 3\cos^2 x\sin x & v = \sin x \end{vmatrix}$$

$$= \cos^3 x\sin x + 3\int\cos^2 x\sin^2 x\,dx$$

$\cos^3 x\sin x$ $= \frac{3}{4}\int\sin^2 2x\,dx$

$(\sin 2x = 2\sin x\cos x)$

$\sin^2 2x = 4\sin^2 x\cos^2 x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 1 - \sin^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$$\boxed{\sin^2 x = \frac{1 - \cos 2x}{2}}$$

$$= \cos^3 x\sin x + \frac{3}{4}\int\frac{1 - \cos 4x}{2}\,dx$$

$$= \cos^3 x\sin x + \frac{3}{8}x + \frac{3}{32}\sin 4x + C$$

11 7-18:36

$$\int x\cdot\operatorname{arctg}x\,dx = \left|\begin{matrix} u=\operatorname{arctg}x & v'=x \\ u'=\frac{1}{x^2+1} & v=\frac{x^2}{2}\end{matrix}\right| =$$

$$= \frac{x^2}{2}\operatorname{arctg}x - \frac{1}{2}\int\frac{x^2+1-1}{x^2+1}dx$$

$$= \frac{x^2}{2}\operatorname{arctg}x - \frac{1}{2}\int\frac{x^2+1}{x^2+1}dx + \frac{1}{2}\int\frac{1}{x^2+1}dx$$

$$= \frac{x^2}{2}\operatorname{arctg}x - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg}x + C$$

$$\int\operatorname{arctg}x\,dx = \left|\begin{matrix} u=\operatorname{arctg}x & v'=1 \\ u'=\frac{1}{x^2+1} & v=x\end{matrix}\right| =$$

$$= x\operatorname{arctg}x - \frac{1}{2}\int\frac{2x}{x^2+1}dx$$

$$= x\operatorname{arctg}x - \frac{1}{2}\ln(x^2+1) + C$$

11 7-18:44

$$\int\frac{1}{x^3+1}dx = \int\frac{1}{(x+1)(x^2-x+1)}dx = \heartsuit$$

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$1 = A(x^2-x+1) + B(x^2+x) + C(x+1)$$

$$0 = A+B \Rightarrow A=-B$$

$$\left.\begin{matrix} 0 = -A+B+C \\ 1 = A+C \end{matrix}\right\} \quad \begin{matrix} 2B+C=0 \\ -B+C=1 \\ \hline 3B=-1 \end{matrix}$$

$$B=-\frac{1}{3} \quad A=\frac{1}{3} \quad C=\frac{2}{3}$$

$$\heartsuit = \int\frac{1}{3(x+1)}dx + \int\frac{-x+2}{3(x^2-x+1)}dx$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{3}\int\frac{x-2}{x^2-x+1}dx$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{3}\cdot\frac{1}{2}\int\frac{2x-4}{x^2-x+1}dx$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\int\frac{2x-1-3}{x^2-x+1}dx$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\int\frac{2x-1}{x^2-x+1}dx + \frac{1}{2}\int\frac{1}{x^2-x+1}dx$$

$$= \overbrace{\frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1)}^{\text{⌂}} +$$

$$+ \frac{1}{2}\int\frac{1}{(x-\frac{1}{2})^2-\frac{1}{4}+1}dx$$

$$= \text{⌂} + \frac{1}{2}\int\frac{1}{(x-\frac{1}{2})^2+\frac{3}{4}}dx =$$

$$= \text{⌂} + \frac{1}{2}\int\frac{1}{\frac{3}{4}\cdot\left(\frac{4}{3}(x-\frac{1}{2})^2+1\right)}dx =$$

$$= \text{⌂} + \frac{2}{3}\int\frac{1}{\underbrace{\left(\frac{2x-1}{\sqrt{3}}\right)}_{t}^2+1}dx$$

$$\text{⌂} +$$

$$\text{⌂} + \frac{\frac{2}{3}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right)}{\frac{2}{\sqrt{3}}}$$

11 7-18:49

$$\int\frac{1}{x^4-1}dx =$$

$$\frac{1}{x^4-1} = \frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

$$A=\frac{1}{4},\ B=-\frac{1}{4},\ C=0,\ D=-\frac{1}{2}$$

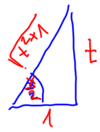
$$\int\frac{1}{x^4-1}dx = \frac{1}{4}\int\frac{1}{x-1}dx - \frac{1}{4}\int\frac{1}{x+1}dx - \frac{1}{2}\int\frac{1}{x^2+1}dx$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2}\operatorname{arctg}x$$

11 7-19:01

$$f(\sin x, \cos x)$$

$$\boxed{t = \operatorname{tg}\frac{x}{2}}$$



$$\sin\frac{x}{2} = \frac{t}{\sqrt{t^2+1}}$$

$$\cos\frac{x}{2} = \frac{1}{\sqrt{t^2+1}}$$

$$\frac{x}{2} = \operatorname{arctg}t$$

$$\frac{dx}{2} = \frac{1}{t^2+1}dt$$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\sin x = 2\cdot\frac{t}{\sqrt{t^2+1}}\cdot\frac{1}{\sqrt{t^2+1}} = \frac{2t}{t^2+1}$$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$$

$$= \frac{1}{t^2+1} - \frac{t^2}{t^2+1} = \frac{1-t^2}{1+t^2}$$

$$f(\sin(x), \cos(x))$$

$$f(-\sin x, -\cos x) = f(\sin x, \cos x)$$

$$t = \operatorname{tg}x$$

11 7-19:05

$$\int\frac{1}{1+3\sin^2x}dx =$$

$$t = \operatorname{tg}x$$

$$x = \operatorname{arctg}t$$

$$dx = \frac{1}{t^2+1}dt$$

$$\sin x = \frac{t}{\sqrt{t^2+1}}$$

$$= \int\frac{1}{1+\frac{3}{t^2+1}}\cdot\frac{1}{t^2+1}dt = \int\frac{1}{\frac{t^2+4}{t^2+1}}\cdot\frac{1}{t^2+1}dt$$

$$= \int\frac{\cancel{t^2+1}}{t^2+4}\cdot\frac{1}{\cancel{t^2+1}}dt = \int\frac{1}{t^2+4}dt$$

$$= \frac{1}{4}\int\frac{1}{(\frac{t}{2})^2+1}dt = \frac{1}{2}\operatorname{arctg}\frac{t}{2} + C$$

$$= \frac{1}{2}\operatorname{arctg}\frac{\operatorname{tg}x}{2} + C$$

$$f(-\sin x, \cos x) = -f(\sin x, \cos x)$$

$$t = \cos x$$

$$f(\sin x, -\cos x) = -f(\sin x, \cos x)$$

$$\boxed{t = \sin x}$$

$$\int\frac{\cos x}{\sin^2x}dx = \int\frac{\cancel{\cos x}}{t^2}\cdot\frac{dt}{\cancel{\cos x}} = \int\frac{1}{t^2}dt =$$

$$\boxed{t = \sin x}$$

$$dt = \cos x\,dx$$

$$dx = \frac{dt}{\cos x}$$

$$= -\frac{1}{t} + C = -\frac{1}{\sin x} + C$$

11 7-19:13

$$\int\operatorname{tg}^3x\,dx = \int\frac{\sin^3x}{\cos^3x}dx$$

$$t = \operatorname{tg}x$$

$$x = \operatorname{arctg}t$$

$$dx = \frac{1}{t^2+1}dt$$

$$\sin x = \frac{t}{\sqrt{t^2+1}}$$

$$\cos x = \frac{1}{\sqrt{t^2+1}}$$

$$= \int\frac{\frac{t}{\cancel{\sqrt{t^2+1}}}\cdot\frac{t^2}{\cancel{t^2+1}}}{\frac{1}{\cancel{\sqrt{t^2+1}}}\cdot\frac{1}{\cancel{t^2+1}}}\cdot\frac{1}{t^2+1}dt$$

$$= \int\frac{t^3+t-t}{t^2+1}dt = \int\frac{t(t^2+1)-t}{t^2+1}dt$$

$$= \int t\,dt - \frac{1}{2}\int\frac{2t}{t^2+1}dt$$

$$= \frac{t^2}{2} - \frac{1}{2}\ln|t^2+1| + C$$

$$= \frac{\operatorname{tg}^2x}{2} - \frac{1}{2}\ln(\operatorname{tg}^2x+1) + C$$

11 7-19:24

$\int \sin^3 x \cos^5 x \, dx =$

$= \int (1-\cos^2 x)\sin x \cdot \cos^5 x \, dx =$

$= \int \sin x \cos^5 x - \sin x \cos^7 x \, dx$

$= \left| \begin{array}{l} t = \cos x \\ dt = -\sin x \, dx \end{array} \right|$

$dx = \frac{dt}{-\sin x}$

$= \int (\sin x \; t^5 - \sin x \; t^7) \frac{dt}{-\sin x} =$

$= \int t^7 - t^5 \, dt = \frac{t^8}{8} - \frac{t^6}{6} + C$

$= \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + C$

11 7-19:31