$$\int_{-1}^{8} \sqrt[3]{x}\,dx = \int_{-1}^{8} x^{\frac{1}{3}}\,dx = \left[\frac{3}{4}x^{\frac{4}{3}}\right]_{-1}^{8}$$

$$= \left[\frac{3}{4}\sqrt[3]{x^4}\right]_{-1}^{8} =$$

$$= \frac{3}{4}\sqrt[3]{8^4} - \frac{3}{4}\sqrt[3]{(-1)^4} =$$

$$= \frac{3}{4}\cdot 2^4 - \frac{3}{4}\cdot 1 = \frac{3}{4}\cdot(15) = \frac{45}{4}$$

11 14-17:51

$$\int_{-1}^{1} \frac{x^5}{x+2}\,dx = \left| \begin{array}{l} t = x+2 \\ t-2 = x \\ x=-1 \Rightarrow t=1 \\ x=1 \Rightarrow t=3 \\ dx = dt \end{array} \right| =$$

$$\int_1^3 \frac{(t-2)^5}{t}\,dt$$

$$x^5 : (x+2) = x^4 - 2x^3 + 4x^2 - 8x + 16$$

$$\begin{array}{l} -(x^5+2x^4) \\ \quad -2x^4 \\ \quad -(-2x^4-4x^3) \\ \qquad 4x^3 \\ \qquad -(4x^3+8x^2) \\ \qquad\quad -8x^2 \\ \qquad\quad -(-8x^2-16x) \\ \qquad\qquad 16x \\ \qquad\qquad -(16x+32) \\ \qquad\qquad\quad -32 \end{array}$$

$$\int_{-1}^{1} \frac{x^5}{x+2}\,dx = \int_{-1}^{1} x^4 - 2x^3 + 4x^2 - 8x + 16 - \frac{32}{x+2}\,dx$$

$$\left[\frac{x^5}{5} - \frac{x^4}{2} + \frac{4x^3}{3} - 4x^2 + 16x - 32\ln|x+2|\right]_{-1}^{1}$$

$$= \frac{1}{5} - \frac{1}{2} + \frac{4}{3} - 4 + 16 - 32\ln 3 -$$
$$\left(-\frac{1}{5} - \frac{1}{2} - \frac{4}{3} - 4 - 16\right)$$

11 14-18:08

$a > 0$

$$\int_0^a \frac{x}{x^3+a^3}\,dx = \int_0^a \frac{x}{(x+a)(x^2-ax+a^2)}\,dx$$

$$\frac{x}{(x+a)(x^2-ax+a^2)} = \frac{A}{x+a} + \frac{Bx+C}{x^2-ax+a^2}$$

$$x = A(x^2-ax+a^2) + Bx(x+a) + C(x+a)$$

$$0 = A+B \quad \Rightarrow A = -B$$
$$1 = -Aa + Ba + C$$
$$0 = Aa^2 + Ca$$

$$1 = Ba + Ba + C$$
$$\boxed{1 = 2Ba + C} \Rightarrow 1 = 2\cdot\frac{a}{3a} + C$$
$$0 = -Ba^2 + Ca \qquad C = 1 - \frac{2}{3} = \frac{1}{3}$$

$$a = 2Ba^2 + Ca$$
$$0 = -Ba^2 + Ca$$

$$a = 3Ba^2$$
$$B = \frac{1}{3a} \qquad A = -\frac{1}{3a}$$

$$\int_0^a \frac{A}{x+a} + \frac{Bx+C}{x^2-ax+a^2}\,dx =$$

$$\int_0^a -\frac{1}{3a(x+a)} + \frac{-\frac{1}{3a}x + \frac{1}{3}}{x^2-ax+a^2}\,dx$$

$$-\frac{1}{3a}\int_0^a \frac{1}{x+a}\,dx - \frac{1}{3}\underbrace{\int_0^a \frac{\frac{x}{a}+1}{x^2-ax+a^2}\,dx}_{\Downarrow\ \frac{f'}{f}\text{; arctan}}$$

$$-\frac{1}{3a}\left[\ln|x+a|\right]_0^a$$

11 14-18:18

$$\int_0^1 \frac{x}{(x^2+1)^2}\,dx = \left| \begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ dx = \frac{dt}{2x} \\ \begin{array}{c|c|c} x & 0 & 1 \\ \hline t & 0 & 1 \end{array} \end{array} \right| = \qquad x^2+1=t$$

$$\int_0^1 \frac{\cancel{x}}{(t+1)^2}\,\frac{dt}{2\cancel{x}} = \int_0^1 \frac{1}{(t+1)^2}\,dt =$$

$$\left| \begin{array}{l} s = t+1 \\ ds = dt \\ \begin{array}{c|c|c} t & 0 & 1 \\ \hline s & 1 & 2 \end{array} \end{array} \right| = \int_1^2 \frac{1}{s^2}\,ds =$$

$$= \int_1^2 s^{-2}\,ds =$$

$$= \left[-\frac{1}{s}\right]_1^2 = -\frac{1}{2} - \left(-\frac{1}{1}\right) = \frac{1}{2}$$

11 14-18:32

$$\int_0^1 (2^x+3^x)^2\,dx = \int_0^1 2^{2x} + 2\cdot 2^x\cdot 3^x + 3^{2x}\,dx$$

$$= \int_0^1 2^{2x}\,dx + 2\int_0^1 6^x\,dx + \int_0^1 3^{2x}\,dx$$

$$= \left[\frac{2^{2x}}{2\ln 2}\right]_0^1 + 2\left[\frac{6^x}{\ln 6}\right]_0^1 + \left[\frac{3^{2x}}{2\ln 3}\right]_0^1 =$$

$$\frac{4}{2\ln 2} - \frac{1}{2\ln 2} + 2\frac{6}{\ln 6} - 2\cdot\frac{1}{\ln 6} + \frac{9}{2\ln 3} - \frac{1}{2\ln 3}$$

$$\frac{3}{2\ln 2} + \frac{10}{\ln 6} + \frac{4}{\ln 3}$$

11 14-18:37

$$\int_1^{\sqrt{e}} \frac{1}{x\sqrt{1-\ln^2 x}}\,dx = \left| \begin{array}{l} \ln x = t \\ \frac{1}{x}dx = dt \\ dx = x\,dt \\ \begin{array}{c|c|c} x & 1 & \sqrt{e} \\ \hline t & 0 & \frac{1}{2} \end{array} \\ \ln\sqrt{e} = \ln e^{\frac{1}{2}} = \frac{1}{2}\ln e \\ = \frac{1}{2} \end{array} \right|$$

$$= \int_0^{\frac{1}{2}} \frac{1}{\cancel{x}\sqrt{1-t^2}}\,\cancel{x}\,dt = \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-t^2}}\,dt$$

$$= \left[\arcsin t\right]_0^{\frac{1}{2}} = \frac{\pi}{6}$$

11 14-18:41

$$\int_1^e \frac{1+\ln x}{x}\,dx = \left|\begin{array}{l} 1+\ln x = t \\ \frac{1}{x}dx = dt \\ dx = x\,dt \end{array}\right| =$$

x	1	e
t	1	2

$$\int_1^2 \frac{t}{x}\,x\,dt = \int_1^2 t\,dt = \left[\frac{t^2}{2}\right]_1^2 = 2-\frac{1}{2} = \frac{3}{2}$$

$$\int_0^\pi \sin^3 x\,dx \quad \left|\begin{array}{l} t = \cos x \\ dt = -\sin x\,dx \\ dx = -\frac{dt}{\sin x} \end{array}\right| =$$

x	0	π
t	1	-1

$$= \int_1^{-1} \sin^3 x\left(-\frac{dt}{\sin x}\right) = -\int_{1}^{-1} \sin^2 x\,dt = -\int_1^{-1} 1-\cos^2 x\,dt = -\int_1^{-1} 1-t^2\,dt = -\left[t-\frac{t^3}{3}\right]_1^{-1} =$$

$$-\left(-1+\frac{1}{3}-1+\frac{1}{3}\right) = -\left(-2+\frac{2}{3}\right) = +\frac{4}{3}$$

$$\int_0^{\sqrt{3}} x\,\mathrm{arctg}\,x\,dx = \left|\begin{array}{ll} u = \mathrm{arctg}\,x & v' = x \\ u' = \frac{1}{1+x^2} & v = \frac{x^2}{2} \end{array}\right|$$

$$= \left[\frac{x^2}{2}\mathrm{arctg}\,x\right]_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{x^2}{1+x^2}\,dx$$

$$= \frac{\cancel{3}}{2}\cdot\frac{\pi}{\cancel{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{x^2+1-1}{1+x^2}\,dx =$$

$$= \frac{\pi}{2} - \frac{1}{2}\int_0^{\sqrt{3}} 1\,dx + \frac{1}{2}\int_0^{\sqrt{3}} \frac{1}{1+x^2}\,dx$$

$$= \frac{\pi}{2} - \frac{1}{2}[x]_0^{\sqrt{3}} + \frac{1}{2}[\mathrm{arctg}]_0^{\sqrt{3}}$$

$$= \frac{\pi}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\pi}{3} = \frac{\pi}{2} - \frac{\sqrt{3}}{2} + \frac{\pi}{6}$$

11 14-18:46

$$\int_2^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty}\left(\int_2^t \frac{1}{x^2}\,dx\right)$$

$$= \lim_{t\to\infty}\int_2^t x^{-2}\,dx =$$

$$= -\lim_{t\to\infty}\left[\frac{1}{x}\right]_2^t = -\lim_{t\to\infty}\left[\frac{1}{t}-\frac{1}{2}\right]$$

$$= -\left(-\frac{1}{2}\right) = \frac{1}{2}$$

11 14-18:59

$$\int_{\sqrt{2}}^\infty \frac{1}{x^2+2}\,dx = \lim_{t\to\infty}\int_{\sqrt{2}}^t \frac{1}{x^2+2}\,dx =$$

$$\frac{1}{2}\lim_{t\to\infty}\int_{\sqrt{2}}^t \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}\,dx = \frac{1}{2}\lim_{t\to\infty}\sqrt{2}\left[\mathrm{arctg}\,x\right]_{\sqrt{2}}^t$$

$$= \frac{1}{2}\lim_{t\to\infty}\sqrt{2}\cdot\left(\mathrm{arctg}\,t - \mathrm{arctg}\sqrt{2}\right) =$$

$$= \frac{1}{2}\cdot\sqrt{2}\left(\frac{\pi}{2} - \mathrm{arctg}\sqrt{2}\right)$$

11 14-19:04

$$\int_1^\infty \frac{\mathrm{arctg}\,x}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{\mathrm{arctg}\,x}{x^2}\,dx =$$

$$\left|\begin{array}{ll} u = \mathrm{arctg}\,x & v' = \frac{1}{x^2} \\ u' = \frac{1}{x^2+1} & v = -\frac{1}{x} \end{array}\right| =$$

$$\lim_{t\to\infty}\left(\underbrace{\left[-\frac{1}{x}\mathrm{arctg}\,x\right]_1^t} + \int_1^t \frac{1}{x(x^2+1)}\,dx\right)$$

$$= \lim_{t\to\infty}\left[-\frac{1}{t}\mathrm{arctg}\,t + \frac{\pi}{4}\right] + \lim_{t\to\infty}\int_1^t \frac{1}{x(x^2+1)}\,dx$$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + Bx^2 + Cx$$

$$0 = A + B \Rightarrow B = -1$$

$$0 = C$$

$$1 = A$$

$$= \frac{\pi}{4} + \lim_{t\to\infty}\int_1^t \frac{1}{x} + \frac{-1x}{x^2+1}\,dx$$

$$= \frac{\pi}{4} + \lim_{t\to\infty}\left(\left[\ln x\right]_1^t - \left[\frac{1}{2}\ln|x^2+1|\right]_1^t\right)$$

$$= \frac{\pi}{4} + \lim_{t\to\infty}\left[\ln x - \ln|x^2+1|^{\frac{1}{2}}\right]_1^t$$

$$= \frac{\pi}{4} + \lim_{t\to\infty}\left[\ln\frac{x}{\sqrt{x^2+1}}\right]_1^t =$$

$$= \frac{\pi}{4} + \lim_{t\to\infty}\left[\ln\frac{t}{\sqrt{t^2+1}} - \ln\frac{1}{\sqrt{2}}\right]$$

$$= \frac{\pi}{4} - \ln\frac{1}{\sqrt{2}} + \lim_{t\to\infty}\boxed{\ln\frac{t}{\sqrt{t^2+1}}}$$

$$= \frac{\pi}{4} - \ln\frac{1}{\sqrt{2}}$$

11 14-19:08

$$\int_{-\infty}^\infty \frac{1}{x^2-2x+5}\,dx =$$

$$\underbrace{\int_{-\infty}^0 \frac{1}{x^2-2x+5}\,dx}_{I_1} + \underbrace{\int_0^\infty \frac{1}{x^2-2x+5}\,dx}_{I_2}$$

$$I_2: \lim_{t\to\infty}\int_0^t \frac{1}{x^2-2x+5}\,dx =$$

$$= \lim_{t\to\infty}\int_0^t \frac{1}{(x-1)^2+4}\,dx =$$

$$= \frac{1}{4}\lim_{t\to\infty}\int_0^t \frac{1}{\left(\frac{x-1}{2}\right)^2+1}\,dx$$

$$= \frac{1}{4}\lim_{t\to\infty}\left[2\,\mathrm{arctg}\,\frac{x-1}{2}\right]_0^t$$

$$= \frac{1}{2}\lim_{t\to\infty}\left[\mathrm{arctg}\,\frac{t-1}{2} - \mathrm{arctg}\left(-\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\lim_{t\to\infty}\left[\mathrm{arctg}\,\frac{t-1}{2} + \mathrm{arctg}\,\frac{1}{2}\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{2} + \mathrm{arctg}\,\frac{1}{2}\right)$$

11 14-19:23

$$\int_0^1 \frac{1}{\sqrt[5]{x^4}}\,dx = \lim_{t\to0^+}\int_t^1 \frac{1}{\sqrt[5]{x^4}}\,dx$$

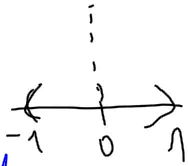
$$= \lim_{t\to0^+}\int_t^1 x^{-\frac{4}{5}}\,dx =$$

$$= \lim_{t\to0^+}\left[5\sqrt[5]{x}\right]_t^1$$

$$= 5\lim_{t\to0^+}\left(1-\sqrt[5]{t}\right) = 5$$

11 14-19:29

$$\boxed{\int_{-1}^{1} \frac{1}{\sqrt[3]{x^7}}\,dx}$$



$$\int_{-1}^{0} \frac{1}{\sqrt[3]{x^7}}\,dx + \int_{0}^{1} \frac{1}{\sqrt[3]{x^7}}\,dx =$$

$$= \underbrace{\lim_{t\to 0^-} \int_{-1}^{t} \frac{1}{\sqrt[3]{x^7}}\,dx}_{} + \underbrace{\lim_{t\to 0^+} \int_{t}^{1} \frac{1}{\sqrt[3]{x^7}}\,dx}_{}$$

11 14-19:33