$$\{a_n\}_{n=1}^{\infty}$$

$$\sum_{n=1}^{\infty} a_n = \underbrace{a_1 + a_2 + \ldots}$$

$$S_1 = a_1$$

$$S_n = S_{n-1} + a_n = a_1 + \ldots + a_{n-1} + a_n$$

$$\sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} S_n$$

$$S = 1 + \tfrac{1}{2} + \tfrac{1}{4} + \ldots$$

$$2S = 2 + 1 + \tfrac{1}{2} + \tfrac{1}{4} + \ldots$$

$$2S = 2 + S$$

$$\boxed{S = 2}$$

$$S = 1 + 2 + 4 + 8 + 16 \ldots$$

$$2S = 2 + 4 + 8 + 16 + \ldots$$

$$S = 1 + (\ \ )$$

$$S = 1 + 2S$$

$$S = -1$$

!

11 28-17:57

$$\{aq^n\}_{n=0}^{\infty}$$

$$\sum_{n=0}^{\infty} aq^n \ \ldots k \Leftrightarrow |q| < 1$$

$$\sum_{n=0}^{\infty} aq^n = \frac{a}{1-q}$$

$$\{a \cdot q^n\}_{n=0}^{\infty}$$

$$S_0 = a$$

$$S_1 = a \cdot q$$

$$S_2 = a \cdot q^2$$

$$S_n = a + aq + \ldots + a \cdot q^n = a(1 + \ldots + q^n) = a \cdot \left(\frac{1-q^{n+1}}{1-q}\right)$$

$$\lim_{n \to \infty} a \cdot \left(\frac{1-q^{n+1}}{1-q}\right) = a \cdot \frac{1}{1-q} = \frac{a}{1-q}$$

11 28-18:11

$$\sum_{n=1}^{\infty} \frac{3^{2n} + (-3)^n}{3^{3n}} = \sum_{n=1}^{\infty} \frac{3^{2n}}{3^{3n}} + \sum_{n=1}^{\infty} \frac{(-3)^n}{3^{3n}} =$$

$$= \sum_{n=1}^{\infty} \frac{9^n}{27^n} + \sum_{n=1}^{\infty} \frac{(-3)^n}{27^n} =$$

$$= \sum_{n=1}^{\infty} \left(\frac{9}{27}\right)^n + \sum_{n=1}^{\infty} \left(\frac{-3}{27}\right)^n =$$

$$= \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n}_{a = \frac{1}{3},\ q = \frac{1}{3}} + \underbrace{\sum_{n=1}^{\infty} \left(-\frac{1}{9}\right)^n}_{a = -\frac{1}{9},\ q = \left(-\frac{1}{9}\right)} =$$

$$= \frac{\frac{1}{3}}{1 - \frac{1}{3}} + \frac{-\frac{1}{9}}{1 + \frac{1}{9}} = \frac{\frac{1}{3}}{\frac{2}{3}} + \frac{-\frac{1}{9}}{\frac{10}{9}} =$$

$$= \frac{1}{2} + \left(-\frac{1}{10}\right) = \frac{1}{2} - \frac{1}{10} = \frac{2}{5}$$

11 28-18:16

$$\boxed{\sum_{n=1}^{\infty} \frac{n}{2^n}}$$

$$S_1 = \frac{1}{2}$$

$$S_2 = \frac{1}{2} + \frac{2}{4}$$

$$S_3 = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} \ldots$$

$$S_n = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \ldots + \frac{n}{2^n}$$

$$\frac{S_n}{2} = \frac{1}{4} + \frac{2}{8} + \ldots + \frac{n}{2^{n+1}}$$

$$S_n - \frac{S_n}{2} = \frac{S_n}{2} = \underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n}} - \frac{n}{2^{n+1}}$$

$$\frac{S_n}{2} = \left(\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n}\right) - \frac{n}{2^{n+1}}$$

$$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(2 \cdot \underbrace{\left(\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n}\right)} - \frac{2n}{2^{n+1}}\right)$$

$$= 2 \cdot \left(\frac{\frac{1}{2}}{\frac{1}{2}}\right) - \lim_{n \to \infty} \frac{2n}{2^{n+1}}$$

$$= 2$$

11 28-18:19

$$0{,}2\overline{15} = 0{,}2 + 0{,}015 + 0{,}00015 + 0{,}0000015 + \ldots$$

$$= 0{,}2 + 15\left(0{,}001 + 0{,}00001 + \ldots\right)$$

$$= 0{,}2 + 15\left(\frac{0{,}001}{1 - 0{,}01}\right) =$$

$$\boxed{0{,}2 + \frac{0{,}015}{0{,}99}}$$

11 28-18:25

$$\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)} = \frac{1}{4} + \ldots$$

$$\frac{1}{(3n-2)(3n+1)} = \frac{A}{3n-2} + \frac{B}{3n+1}$$

$$1 = A(3n+1) + B(3n-2)$$

$$0 = 3A + 3B \Rightarrow A + B = 0$$

$$1 = A - 2B \qquad A = 2B + 1$$

$$B + 2B + 1 = 0$$

$$3B = -1$$

$$B = -\frac{1}{3}$$

$$A = \frac{1}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)} = \sum_{n=1}^{\infty} +\frac{1}{3} \cdot \frac{1}{3n-2} + \sum_{n=1}^{\infty} -\frac{1}{3}\left(\frac{1}{3n+1}\right)$$

$$= +\frac{1}{3} + \left(+\frac{1}{3} \cdot \frac{1}{4}\right) + \left(+\frac{1}{3} \cdot \frac{1}{7}\right) + \left(+\frac{1}{3} \cdot \frac{1}{10}\right) + \left(+\frac{1}{3} \cdot \frac{1}{13}\right) \ldots +$$

$$+ \left(-\frac{1}{3} \cdot \frac{1}{4}\right) + \left(-\frac{1}{3} \cdot \frac{1}{7}\right) + \left(-\frac{1}{3} \cdot \frac{1}{10}\right) + \ldots$$

$$= \ldots = +\frac{1}{3}$$

11 28-18:29

$\sum \frac{1}{n(n+1)}$ konvergnje

NUTNÁ PODM. KONV.

$\sum_{n=1}^{\infty} a_n$ Konverg $\Rightarrow \lim_{n\to\infty} a_n = 0$

$\lim_{n\to\infty} a_n = 0$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverg., přesto $\lim_{n\to\infty} \frac{1}{n} = 0$

$\lim_{n\to\infty} a_n \neq 0 \Rightarrow$ NEK.

$\sum_{n=1}^{\infty} \frac{n^3-1}{n^3+1}$ NEKONVERG

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^3-1}{n^3+1} = 1 \neq 0$

$\sum_{n=1}^{\infty} \operatorname{arctg} \frac{2n}{3}$ NEKONV.

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \operatorname{arctg}\frac{2n}{3} = \frac{\pi}{2} \neq 0$

11 28-18:39

$x+3x^2+x^3+3x^4+x^5+3x^6\ldots = \frac{5}{3}$

i) $\underbrace{x+x^3+x^5+\ldots}+\underbrace{3x^2+3x^4+3x^6+\ldots} = \frac{5}{3}$

ii) $(x+3x^2)+(x^3+3x^4)+\ldots = \frac{5}{3}$ $|x^2|<1$

$\frac{x+3x^2}{1-x^2} = \frac{5}{3}$

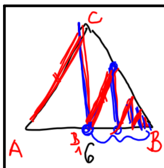
$x+3x^2 = \frac{5}{3}(1-x^2)$

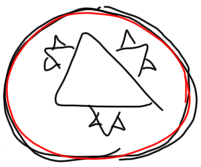
$3x+9x^2 = 5-5x^2$

$14x^2+3x-5=0$

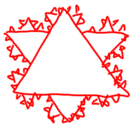
$x_{1,2} = \frac{-3\pm\sqrt{9+280}}{28} = \frac{-3\pm 17}{28} = \boxed{\begin{matrix}-\frac{5}{7} \\ \frac{1}{2}\end{matrix}}$

11 28-18:47





$\sqrt{6^2-3^2} = \sqrt{27} = 3\sqrt{3}$



$D = \frac{3\sqrt{3}}{1-\frac{1}{2}} = \frac{3\sqrt{3}}{\frac{1}{2}} = 6\sqrt{3}$

$D = \frac{6}{1-\frac{1}{2}} = 12$

$\boxed{D+D = 6\sqrt{3}+12}$

11 28-18:52

$\sum_{n=1}^{\infty} a_n, \forall n \in N: a_n > 0$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = r \begin{cases} r<1 \Rightarrow K \\ r=1 \Rightarrow ?? \\ r>1 \Rightarrow D \end{cases}$

$\sum_{n=1}^{\infty} a_n, \forall n \in N: a_n \geq 0$

$\lim_{n\to\infty} \sqrt[n]{a_n} = r \begin{cases} r<1 \Rightarrow K \\ r=1 \Rightarrow ?? \\ r>1 \Rightarrow D \end{cases}$

11 28-18:59

$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{((n+1)!)^2}{(2(n+1))!}}{\frac{(n!)^2}{(2n)!}} =$

$= \lim_{n\to\infty} \frac{\frac{(n+1)\cdot\cancel{n!}\,(n+1)\cdot\cancel{n!}}{(2n+2)(2n+1)\cdot\cancel{(2n)!}}}{\frac{\cancel{n!\cdot n!}}{\cancel{(2n)!}}} =$

$= \lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \lim_{n\to\infty} \frac{n^2+\ldots}{4n^2+\ldots} = \frac{1}{4}$

$\frac{1}{4} < 1 \Rightarrow K$

11 28-19:05

$\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n^n}$ $\left(\text{Prip}: \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e\right)$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\cdot n!}{n^n}} =$ $(n+1)\,n!$

$= \lim_{n\to\infty} \frac{\frac{2\cdot\cancel{(n+1)}}{(n+1)^n\cdot\cancel{(n+1)}}}{\frac{1}{n^n}} = \lim_{n\to\infty} \frac{2\cdot n^n}{(n+1)^n} =$

$= 2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = 2\cdot\lim_{n\to\infty}\left(\frac{1}{\frac{n+1}{n}}\right)^n$

$= 2\cdot\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^n =$

$= 2\cdot\lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{2}{e} < 1 \Rightarrow K$

11 28-19:09

$$\sum_{n=1}^{\infty} \frac{(n^2+1)\cdot 3^n}{(2n+1)!}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\dfrac{((n+1)^2+1)\cdot 3^{\cancel{n+1}}}{\cancel{(2n+3)!}\,(2n+3)(2n+2)}}{\dfrac{(n^2+1)\,\cancel{3^n}}{\cancel{(2n+1)!}}} =$$

$$= \lim_{n\to\infty} \frac{3\cdot(n^2+2n+2)}{(n^2+1)(2n+3)(2n+2)} = 0 < 1 \Rightarrow K$$

11 28-19:17

$$\sum_{n=1}^{\infty} \frac{1}{\left(3+\frac{1}{n}\right)^n}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{\left(3+\frac{1}{n}\right)^n}} =$$

$$= \lim_{n\to\infty} \frac{1}{3+\frac{1}{n}} = \frac{1}{3} < 1 \Rightarrow K$$

$$\sum_{n=1}^{\infty} \frac{n^2}{\left(2+\frac{1}{n}\right)^n} \Rightarrow K$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^2}{\left(2+\frac{1}{n}\right)^n}} =$$

$$= \lim_{n\to\infty} \frac{\sqrt[n]{n^2}}{2+\frac{1}{n}} \qquad 2+\frac{1}{n} \xrightarrow{n\to\infty} 2$$

$$\square\ \lim_{n\to\infty} \sqrt[n]{n^2} = \lim_{n\to\infty} n^{\frac{2}{n}}$$

$$= \lim_{n\to\infty} e^{\ln n^{\frac{2}{n}}} = \lim_{n\to\infty} e^{\frac{2}{n}\ln n}$$

$$= e^{\heartsuit}$$

$$\heartsuit: \lim_{n\to\infty} \frac{2}{n}\cdot\ln n = 2\cdot\lim_{n\to\infty} \frac{\ln n}{n} =$$

$$= 2\cdot\lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$$

$$\heartsuit = 0$$

$$\square = e^0 = 1$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \frac{1}{2} < 1 \quad K$$

11 28-19:21

SROVNÁVACÍ $a_n, b_n \geq 0$

- i) $\sum^{\infty} a_n, \sum^{\infty} b_n$, $a_n \geq b_n \geq 0\ \forall n \in N \wedge$
  $\sum^{\infty} b_n$ DIVERGUJE $\Rightarrow$
  $\sum a_n$ DIVERGUJE
- ii) $a_n \geq b_n \geq 0$!
  $\sum a_n$ KONVERGUJE
  $\Rightarrow \sum b_n$ konverguje

$$\sum_{n=1}^{\infty} \frac{1}{2n-1}$$

$\sum_{n=1}^{\infty} a_n$

$\sum_{n=1}^{\infty} b_n$ DIVERGUJE

$a_n \geq b_n$

$$\frac{1}{2n-1} \geq \frac{1}{2n} = \frac{1}{2}\cdot\frac{1}{n}$$

$\sum \frac{1}{n}$ Diverguje

$\boxed{\sum \frac{1}{2n}}$ Diverguje

SK

$\Rightarrow \sum \frac{1}{2n-1}$ Diverguje

$$\sum \frac{1}{n^2}$$

$\boxed{\sum \frac{1}{n(n+1)}}$ Konverguje $\boxed{\sum \frac{1}{n\cdot(n-1)}}$

$\frac{1}{n^2} \cancel{<} \frac{1}{n(n+1)}$ jinak

$\frac{1}{n^2} < \underbrace{\frac{1}{n(n-1)}}_{\boxed{\text{Konverguje}}}$

11 28-19:29

$$\sum \frac{1}{n(n-1)} = \sum \frac{A}{n} + \sum \frac{B}{n-1}$$

11 28-19:38