$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt[3]{n}}$$

$$\cancel{(r \ (r))} \quad 0$$

$$r = \lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \lim \frac{\frac{1}{\sqrt[3]{n}}}{\frac{1}{\sqrt[3]{n+1}}} = \lim_{n\to\infty} \frac{\sqrt[3]{n+1}}{\sqrt[3]{n}} = 1$$

$$x \in (-1, 1)$$

$$x = 1: \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$$

$$\sqrt[3]{n} < n \quad \forall n > 1, n \in \mathbb{N}$$

$$\frac{1}{\sqrt[3]{n}} > \frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ DIV} \overset{SK}{\Longrightarrow} \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$$

$x = -1$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}} \quad .. \sum (-1)^n a_n \quad K$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} = 0$$

$$\sum \frac{x^n}{\sqrt[3]{n}} \quad K \text{ pro } x \in \langle -1, 1)$$

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$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n 5^{n-1}}$$

$$\cancel{(r \ (\tfrac{r}{2}))} \quad \cdot \frac{1}{n 5^{n-1}}$$

$$r = \lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \lim_{n\to\infty} \frac{\frac{1}{n 5^{n-1}}}{\frac{1}{(n+1)\cdot 5^n}} = \qquad \sum a_n (x-s)^n$$

$$\lim_{n\to\infty} \frac{5(n+1)}{n} = 5$$

$$x \in (-3, 7)$$

$$x = 7: \sum_{n=1}^{\infty} \frac{5^n}{n 5^{n-1}} = \sum_{n=1}^{\infty} \frac{5}{n} \quad D$$

$$x = -3 \quad \sum_{n=1}^{\infty} \frac{(-5)^n}{n (5)^n} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot \cancel{5^n}}{n \cdot \cancel{5^n}} =$$

$$= \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n} \quad K$$

$$\sum \frac{(x-2)^n}{n \cdot 5^n} \quad K \text{ pro } x \in \langle -3, 7)$$

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$$\left(\sum f_n(x)\right)' = \sum f'(n) \qquad x = OK$$

$$\left(\sum_{n=1}^{\infty} x^n\right)' = \sum (x^n)'$$

$$= \sum n \cdot x^{n-1}$$

$$\int \left(\sum f_n(x)\right) dx = \sum \left(\int f_n(x) dx\right) \quad x \in OK$$

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n} \qquad \int x^{n-1} dx = \frac{x^n}{n} \qquad |x| < 1$$

$$= \int \left(\sum_{n=1}^{\infty} x^{n-1}\right) dx \qquad \sum_{n=1}^{\infty} \frac{x^n}{n} \quad x = \frac{1}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n} \qquad \int x^{n-1} dx = \frac{x^n}{n}$$

$$\sum \int x^{n-1} dx = \int \overbrace{\sum_{n=1}^{\infty} x^{n-1}} dx = \int \frac{1}{1-x} dx$$

$$\sum \frac{x^n}{n} \qquad = -\ln|1-x|$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln|1-x|$$

$$x = \frac{1}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n} = -\ln\left|1 - \frac{1}{3}\right|$$

$$= -\ln\left(\frac{2}{3}\right) = \ln \frac{3}{2}$$

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$$\sum_{n=0}^{\infty} (-1)^n \cdot (2n+1) \left(\frac{1}{5}\right)^{2n} \qquad |x|^2 < 1$$

$$\sum (-1)^n \cdot (2n+1) x^{2n}$$

$$\left(x^{2n+1}\right)' = (2n+1) x^{2n}$$

$$\sum_{n=0}^{\infty} x^{2n+1} = x + x^3 + x^5 + x^7 + \ldots$$

$$\sum_{n=0}^{\infty} (-1)^n \left(x^{2n+1}\right)' = \left(\sum_{n=0}^{\infty} (-1)^n x^{2n+1}\right)' = \left(\frac{x}{1+x^2}\right)'$$

$$\sum_{n=0}^{\infty} (-1)^n (2n+1) x^{2n} = \qquad = \frac{1+x^2 - x \cdot 2x}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2}$$

$$\sum_{n=0}^{\infty} (-1)^n (2n+1) x^{2n} = \frac{1-x^2}{(1+x^2)^2}$$

$$x = \frac{1}{5}$$

$$\frac{1 - \frac{1}{25}}{\left(1 + \frac{1}{25}\right)^2} = \frac{\frac{24}{25}}{\left(\frac{26}{25}\right)^2} = \frac{25 \cdot 24}{26^2}$$

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$$\sum_{n=1}^{\infty} n \cdot \left(\frac{1}{2}\right)^n$$

$$(x^n)' = n x^{n-1} \quad / \cdot x \qquad |x| < 1$$

$$x (x^n)' = n x^n$$

$$= x \cdot \left(\sum_{n=1}^{\infty} x^n\right)' = x \cdot \left(\frac{x}{1-x}\right)'$$

$$= x \cdot \frac{1-x+x}{(1-x)^2}$$

$$= x \cdot \sum_{n=1}^{\infty} n x^{n-1}$$

$$= \frac{x}{(1-x)^2}$$

$$= \sum_{n=1}^{\infty} n \cdot x^n$$

$$x = \frac{1}{2}$$

$$\sum_{n=1}^{\infty} n \cdot \left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{\left(1 - \frac{1}{2}\right)^2} = \frac{\frac{1}{2}}{\frac{1}{4}} = 2$$

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$$\sum_{n=1}^{\infty} n^2 \cdot \left(\frac{1}{4}\right)^{n-1}$$

$$(x^n)' = n \cdot x^{n-1} \quad / \cdot x$$

$$x (x^n)' = n \cdot x^n \quad /'$$

$$\left(x (x^n)'\right)' = n^2 \cdot x^{n-1}$$

$$\left(\sum_{n=1}^{\infty} x^n\right)' = \boxed{\sum_{n=1}^{\infty} n x^{n-1}}$$

$$= \left(\frac{x}{1-x}\right)' = \frac{1-x+\cancel{x}}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$x \cdot \sum_{n=1}^{\infty} n \cdot x^{n-1} = \sum_{n=1}^{\infty} n x^n$$

$$= \frac{x}{(1-x)^2}$$

$$\left(\sum n \cdot x^n\right)' = \sum_{n=1}^{\infty} (n x^n)' = \sum_{n=1}^{\infty} n^2 x^{n-1}$$

$$= \left(\frac{x}{(1-x)^2}\right)' = \frac{(1-x)^2 + 2x(1-x)}{(1-x)^4}$$

$$= \frac{1-2x+x^2+2x-2x^2}{(1-x)^4} = \frac{1-x^2}{(1-x)^4}$$

$$\sum_{n=1}^{\infty} n^2 \left(\frac{1}{4}\right)^{n-1} = \frac{1 - \left(\frac{1}{4}\right)^2}{\left(\frac{3}{4}\right)^4}$$

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$$\sum_{n=1}^{\infty} n(n+1)\left(\frac{1}{2}\right)^n$$

$$\sum_{n=1}^{\infty} n(n+1)x^n$$

$$\left(x^{n+1}\right)' = (n+1)x^n$$

$$\left(\left(x^{n+1}\right)'\right)' = n(n+1)\cdot x^{n-1}$$

$$x\left(\left(x^{n+1}\right)'\right)' = n(n+1)x^n$$

$$\boxed{\begin{gathered}(x^n)' = n(x^{n-1}) \\ x^2\,(x^n)' = n\cdot x^{n+1} \\ \left(x^2\cdot(x^n)'\right)' = n\cdot(n+1)x^n\end{gathered}}$$

$$\left(\sum_{n=1}^{\infty} x^{n+1}\right)' = \sum_{n=1}^{\infty}\left(x^{n+1}\right)' = \sum_{n=1}^{\infty}(n+1)x^n$$

$$= \left(\frac{x^2}{1-x}\right)' = \frac{2x(1-x)+x^2}{(1-x)^2}$$

$$= \frac{2x-x^2}{(1-x)^2} =$$

$$\left(\sum_{n=1}^{\infty}(n+1)x^n\right)' = \sum_{n=1}^{\infty}\left((n+1)x^n\right)' = \sum_{n=1}^{\infty} n(n+1)x^{n-1}$$

$$= \underbrace{\frac{(2-2x)(1-x)^2+2(2x-x^2)(1-x)}{(1-x)^4}}$$

$$\sum_{n=1}^{\infty} n(n+1)x^n = \frac{x(2-2x)(1-x)^2+2x(2x-x^2)(1-x)}{(1-x)^4}$$

$$x=\frac{1}{2}$$

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$$\sum_{n=1}^{\infty}\frac{x^n}{n(n+1)}$$

$$\int x^{n-1}\,dx = \frac{x^n}{n}$$

$$\int\left(\int x^{n-1}\,dx\right)dx = \frac{x^{n+1}}{n\cdot(n+1)}$$

$$\frac{1}{x}\int\left(\int x^{n-1}\,dx\right)dx = \frac{x^n}{n(n+1)}$$

$$\int\left(\sum_{n=1}^{\infty} x^{n-1}\right)dx = \sum_{n=1}^{\infty}\int x^{n-1}\,dx = \sum_{n=1}^{\infty}\frac{x^n}{n}$$

$$= \int\frac{1}{1-x}\,dx = -\ln(1-x)$$

$$\int\sum_{n=1}^{\infty}\frac{x^n}{n}\,dx = \sum_{n=1}^{\infty}\int\frac{x^n}{n}\,dx = \sum_{n=1}^{\infty}\frac{x^{n+1}}{n(n+1)}$$

$$= \int -\ln(1-x)\,dx = -\int\ln(1-x)\,dx$$

$$= \begin{vmatrix} u=\ln(1-x) & v'=1 \\ u'=\frac{-1}{1-x} & v=x \end{vmatrix} =$$

$$= -x\ln(1-x) - \int\frac{x}{1-x}\,dx =$$

$$= -x\ln(1-x) + \int\frac{1-x-1}{1-x}\,dx$$

$$= -x\ln(1-x) + x + \ln|1-x|$$

$$\sum_{n=1}^{\infty}\frac{x^n}{n(n+1)} = -\ln(1-x)+1+\frac{\ln|1-x|}{x}$$

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$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$$

$$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$$

$$e^{-x^2} \qquad t=-x^2$$

$$e^{-x^2} = \sum_{n=0}^{\infty}\frac{(-x^2)^{2n}}{n!}$$

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$$\operatorname{arctg}\frac{\sqrt{3}}{3}$$

$$\int\frac{1}{1+x^2}\,dx = \operatorname{arctg} x$$

$$\int 1-x^2+x^4-x^6\dots\,dx$$

$$\int\sum_{n=0}^{\infty}(-1)^n x^{2n}\,dx =$$

$$\sum_{n=0}^{\infty}(-1)^n\cdot\int x^{2n}\,dx = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$$

$$\operatorname{arctg} x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$$

$$\operatorname{arctg}\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3} - \frac{\left(\frac{\sqrt{3}}{3}\right)^3}{3} + \frac{\left(\frac{\sqrt{3}}{3}\right)^5}{5} - \frac{\left(\frac{\sqrt{3}}{3}\right)^7}{7} + \dots + \frac{\left(\frac{\sqrt{3}}{3}\right)^{13}}{13}$$

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$$e^2$$

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}\dots$$

$$e^2 = 1+\frac{4}{2}+\frac{8}{6}+\frac{16}{24}+\frac{32}{120}+\dots+\frac{2^9}{9!}$$

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$$\lim_{x\to\infty}\left(\frac{1}{x^2}-x^2\ln\left(1+\frac{1}{x^2}\right)\right)$$

$$\ln\left(1+\frac{1}{x^2}\right)$$

$$\ln(1+t) = \int\frac{1}{1+t}\,dt \qquad a_0=1 \quad q=-t$$

$$= \int 1-t+t^2+\dots\,dt$$

$$= \int\sum_{n=0}^{\infty}(-1)^n\cdot t^n$$

$$= \sum_{n=0}^{\infty}(-1)^n\int t^n\,dt$$

$$= \sum_{n=0}^{\infty}(-1)^n\frac{t^{n+1}}{n+1}$$

$$\ln(1+t) = \sum_{n=0}^{\infty}(-1)^n\frac{t^{n+1}}{n+1}$$

$$\ln\left(1+\frac{1}{x^2}\right) = \sum_{n=0}^{\infty}(-1)^n\cdot\frac{\left(\frac{1}{x^2}\right)^{n+1}}{n+1}$$

$$= \frac{1}{x^2}-\frac{1}{2(x^2)^2}+\frac{1}{3(x^2)^3}+\dots$$

$$\lim_{x\to\infty}\left(\frac{1}{x^2}-x^2\ln\left(1+\frac{1}{x^2}\right)\right) =$$

$$\lim_{x\to\infty}\left(\frac{1}{x^2}-x^2\cdot\left(\frac{1}{x^2}-\frac{1}{2(x^2)^2}+\frac{1}{3(x^2)^3}+\dots\right)\right)$$

$$\lim_{x\to\infty}\left(\overset{0}{\frac{1}{x^2}}-1+\overset{0}{\frac{1}{2x^2}}-\overset{0}{\frac{1}{3(x^2)^2}}+\dots\right) = -1$$

$$\lim_{x\to\infty}\left(\frac{1}{x}-x\left(\ln\left(1+\frac{1}{x}\right)\right)\right)$$

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