① $f(x) = \dfrac{x^3+2x^2+x+2}{x^4+x^3+2x^2} = \dfrac{x^3+2x^2+x+2}{x^2(x^2+x+2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+x+2}$

$$x^3+2x^2+x+2 = Ax(x^2+x+2) + B(x^2+x+2) + (Cx+D)x^2$$

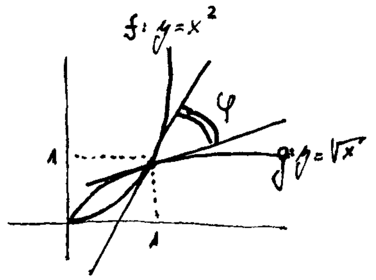
$$\left.\begin{aligned}
x^3 &: \quad A + C = 1 \\
x^2 &: \quad A + B + D = 2 \\
x &: \quad 2A + B = 1 \\
1 &: \quad 2B = 2 \Rightarrow B = 1 \\
& A + C = 1 \qquad \Rightarrow C = D = 1 \\
& A + D = 1 \\
& 2A = 0 \Rightarrow A = 0
\end{aligned}\right\} \Rightarrow f(x) = \frac{1}{x^2} + \frac{x+1}{x^2+x+2}$$

② $\lim\limits_{x \to 1} \left(\dfrac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}} = \left(\dfrac{2}{3}\right)^{\lim\limits_{x\to 1} \frac{1-\sqrt{x}}{1-x}} = \left(\dfrac{2}{3}\right)^{\lim\limits_{x\to 1} \frac{(1-\sqrt{x})}{1-x} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}}} =$

$\frac{1+x}{2+x}$ je spojitá v 1

$$= \left(\frac{2}{3}\right)^{\lim\limits_{x\to 1} \frac{1-x}{(1-x)(1+\sqrt{x})}} = \left(\frac{2}{3}\right)^{\lim\limits_{x\to 1} \frac{1}{1+\sqrt{x}}} = \left(\frac{2}{3}\right)^{\frac{1}{2}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$$

③



$x^2 = \sqrt{x} \wedge x \in (0,\infty) \iff x = 1$

$f(1) = g(1) = 1$

$t_f: \; f'(x) = 2x$

$f'(1) = 2$

$\Rightarrow t_f: \; y = 2x + q$

$\Rightarrow s_{t_f} = (1,2)$

$t_g: \; g'(x) = \dfrac{1}{2\sqrt{x}}$

$g'(1) = \dfrac{1}{2}$

$t_g: \; y = \dfrac{1}{2}x + p$

$\Rightarrow s_{t_g} = (2,1)$

$$\cos\varphi = \frac{4}{5}$$

$$\varphi = \arccos\frac{4}{5}$$

④ $f(x) = \ln \dfrac{e^x}{x^2+1}$

$$f'(x) = \frac{x^2+1}{e^x} \cdot \frac{e^x(x^2+1) - e^x \cdot 2x}{(x^2+1)^2} = \frac{(x-1)^2}{x^2+1}$$

$\mathcal{D}(e^x) = \mathbb{R}$, navíc $e^x > 0$ pro $\forall x \in \mathbb{R}$

$\mathcal{D}(x^2+1) = \mathbb{R}$

$\mathcal{D}\left(\frac{e^x}{x^2+1}\right) = \mathbb{R}$, protože $x^2+1 > 0$

$\dfrac{e^x}{x^2+1} > 0$ pro $\forall x \in \mathbb{R} \Rightarrow \mathcal{D}(f) = \mathbb{R}$

2222

① $f(x) = \dfrac{x^3+4x^2-x}{x^4-1} = \dfrac{x^3+4x^2-x}{(x^2+1)(x-1)(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{Cx+D}{x^2+1}$

$$x^3+4x^2-x = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$$
$$= A(x^3+x^2+x+1) + B(x^3-x^2+x-1) + (Cx+D)(x^2-1)$$

x^3: $A+B+C=1$

x^2: $A-B+D=4$

x: $A+B-C=-1$

1: $A-B-D=0$

$2C=2 \Rightarrow C=1$

$2D=4 \Rightarrow D=2$

$A+B=0$

$A-B=2$

$2A=2 \Rightarrow A=1$

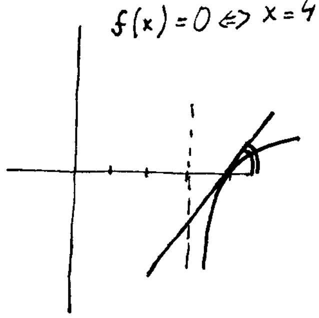
$B=-1$

$$f(x) = \frac{1}{x-1} - \frac{1}{x+1} + \frac{x+2}{x^2+1}$$

② $\lim\limits_{x\to 4} \left(\dfrac{x+2}{x-1}\right)^{\frac{x-4}{\sqrt{x}-2}} = 2^{\lim\limits_{x\to 4}\frac{x-4}{\sqrt{x}-2}} = 2^{\lim\limits_{x\to 4}\frac{x-4}{\sqrt{x}-2}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}} = 2^{\lim\limits_{x\to 4}\sqrt{x}+2} = 16$

③ $y = \ln(x-3)$

$f(x)=0 \Leftrightarrow x=4$



$f'(x) = \dfrac{1}{x-3}$

$f'(4) = 1$

$t_f: y = x + q$

$s_{t_f} = (1,1)$

$s_x = (1,0)$

$\cos\varphi = \dfrac{1}{\sqrt{2}\cdot 1} = \dfrac{\sqrt{2}}{2}$

$\varphi = \dfrac{\pi}{4}$

④ $f(x) = \ln \dfrac{(x+1)^2}{\sqrt{(2x+1)^3}}$

$f'(x) = \dfrac{\sqrt{(2x+1)^3}}{(x+1)^2} \cdot \dfrac{2(x+1)\sqrt{(2x+1)^3} - 2(x+1)^2\sqrt{2x+1}\cdot\frac{3}{2}}{(2x+1)^3} = \dfrac{\sqrt{(2x+1)^3}\cdot(x+1)\sqrt{2x+1}\,(4x+2-3x-1)}{(x+1)^2\cdot(2x+1)^3} =$

$= \dfrac{x-1}{(x+1)(2x+1)}$

$D(f) = \left(-\frac{1}{2}, \infty\right)$

$D(f)$: $(x+1)^2 \geq 0 \Rightarrow x \neq -1$

$2x+1 > 0 \Rightarrow x > -\frac{1}{2}$