MB102 - mikro 2

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(1)

$$\lim_{x\to 1}(2-x)^{\operatorname{tg}\frac{\pi x}{2}} = \lim_{x\to 1} e^{\operatorname{tg}\frac{\pi x}{2}\cdot \ln(2-x)} = e^{\lim\limits_{x\to 1}\frac{\ln(2-x)}{\operatorname{cotg}\frac{\pi}{2}x}} =$$

$$\overset{l'H}{=} e^{\lim\limits_{x\to 1}\frac{-\frac{1}{2-x}}{-\frac{1}{\sin^2\frac{\pi}{2}x}\cdot\frac{\pi}{2}}} = e^{\frac{-1}{-\frac{\pi}{2}}} = e^{\frac{2}{\pi}}$$

(2)

$$T_{P,1}(x) = P(1) + \frac{P'(1)}{1!}(x-1) + \frac{P''(1)}{2!}(x-1)^2 + \frac{P'''(1)}{3!}(x-1)^3 + \frac{P^{(4)}(1)}{4!}(x-1)^4$$

$P(x) = x^4 - 4x^3 + 9x^2 - 10x + 12$

$P'(x) = 4x^3 - 12x^2 + 18x - 10$

$P''(x) = 12x^2 - 24x + 18$

$P'''(x) = 24x - 24$

$P^{(4)}(x) = 24$

$P(1) = 8$, $P''(1) = 6$

$P'(1) = 0$, $P'''(1) = 0$

$P^{(4)}(x) = 24$

$T_{P,1}(x) = 8 + 3(x-1)^2 + (x-1)^4$

Nejmenší možná hodnota $P(x) = 8$ pro $x = 1$.

(3) $f: y = \frac{\ln x^2}{x}$

$$f'(x) = \frac{\frac{2x}{x^2}\cdot x - \ln x^2}{x^2} = \frac{2 - \ln x^2}{x^2} = \frac{2(1-\ln|x|)}{x^2}\ ;\ f''(x) = \frac{2(-x-(1-\ln x)2x)}{x^4} = \frac{2(2\ln x - 3)}{x^3} = \frac{2(\ln x^2 - 3)}{x^3}$$

definiční obor: $x^2 > 0 \wedge x \neq 0 \Leftrightarrow x \in \mathbb{R}\setminus\{0\}$

parita: $f(-x) = -f(x)$ ... lichá funkce

nulové body: $\ln x^2 = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = 1 \vee x = -1$

znaménko: $\frac{\ln x^2}{x} > 0 \Leftrightarrow (x^2 > 1 \wedge x > 0) \vee (x^2 < 1 \wedge x < 0) \Rightarrow x > 1 \vee x < -1$

monotonie: $f'(x) = 0 \Leftrightarrow \underline{x = \pm e}$; $f'(x) > 0 \Leftrightarrow 1 > \ln|x| \Leftrightarrow \underline{|x| < e}$, $x \neq 0$

rostoucí na $(-e, 0)$ a na $(0, e)$, klesající na (e, ∞) a na $(-\infty, -e)$

$\Rightarrow$ lokální extrém v $x_0 = e$, $f(e) = \frac{2}{e}$

$f''(x) \geq 0 \Leftrightarrow (\ln x > \frac{3}{2} \wedge x > 0) \vee (\ln x^2 < 3 \wedge x < 0)$

$\Downarrow$ $x > e^{\frac{3}{2}}$ $\quad$ $\Downarrow$ $x > -e^{\frac{3}{2}}$

Tj. konvexní na $(-e^{\frac{3}{2}}, 0), (e^{\frac{3}{2}}, \infty)$, konkávní na $(-\infty, -e^{\frac{3}{2}}), (0, e^{\frac{3}{2}})$; inflexní body v $e^{\frac{3}{2}}, -e^{\frac{3}{2}}$

body: $f(1) = f(-1) = 0$, $f(e) = \frac{2}{e}$, $f(e^{\frac{3}{2}}) = \frac{3}{e^{3/2}}$

$f(-e) = -\frac{2}{e}$, $f(-e^{\frac{3}{2}}) = -\frac{3}{e^{3/2}}$

Asymptoty: bez směrnice $x_0 = 0$: $\lim\limits_{x\to 0^-} f(x) = +\infty$, $\lim\limits_{x\to 0^+} f(x) = -\infty$;

se směrnicí: $\lim\limits_{x\to\pm\infty}\frac{f(x)}{x} = \frac{\ln(x^2)}{x^2} = 0$

$\lim\limits_{x\to\pm\infty} f(x) - 0\cdot x = 0$

ASYMPTOTA SE SMĚRNICÍ $y = 0$.

MB102 - vnitro 2

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(1) $\lim_{x\to 2}(x-1)^{\cotg \pi x} = \left|1^{\infty}\right| = \lim_{x\to 2} e^{\cotg \pi x \cdot \ln(x-1)} =$

$= e^{\lim_{x\to 2}\frac{\ln(x-1)}{\tg \pi x}} = \left|\frac{0}{0}\right| \stackrel{l'H}{=} e^{\lim_{x\to 2}\frac{\frac{+1}{x-1}}{\frac{1}{\cos^2 \pi x}\cdot\pi}} = \underline{\underline{e^{\frac{+1}{\pi}}}}$

(2) $P(x) = x^4 - 8x^3 + 26x^2 - 40x + 27$

$T_{P,2}(x) = P(2) + \frac{P'(2)}{1!}(x-2) + \frac{P''(2)}{2!}(x-2)^2 + \frac{P'''(2)}{3!}(x-2)^3 + \frac{P^{(4)}(2)}{4!}(x-2)^4$

$P'(x) = 4x^3 - 24x^2 + 52x - 40 \qquad P'(2) = 0 \quad P(2) = 3$

$P''(x) = 12x^2 - 48x + 52 \qquad P''(2) = 4$

$P'''(x) = 24x - 48 \qquad P'''(2) = 0$

$P^{(4)}(x) = 24 \qquad P^{(4)}(2) = 24$

$T_{P,2}(x) = (x-2)^4 + 2(x-2)^2 + 3 > 0 \quad \forall x \in \mathbb{R}$, proto nemá žádný reálný kořen.

(3) $f:\ y = \frac{x}{e^{x^2}} \qquad f'(x) = \frac{e^{x^2} - x\cdot 2x\cdot e^{x^2}}{e^{2x^2}} = \frac{1-2x^2}{e^{x^2}}, \quad f''(x) = \left[(1-2x^2)e^{-x^2}\right]' = e^{-x^2}\cdot 2x(2x^2-3)$

def. obor $\mathbb{R}$, parita: $f(-x) = -f(x)$. lichá funkce ; nulové body $x = 0$, $f(x) > 0 \iff x > 0$

monotonie a lok. extrémy: $f'(x) > 0 \iff 1-2x^2 > 0 \iff x^2 < \frac{1}{2} \iff x \in \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

klesající na $\left(-\infty, -\frac{\sqrt{2}}{2}\right)$ a $\left(\frac{\sqrt{2}}{2}, \infty\right)$; rostoucí na $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\Rightarrow$ lok. minimum v $-\frac{\sqrt{2}}{2}$; lok. maximum v $\frac{\sqrt{2}}{2}$.

konvexnost: $f''(x) > 0 \iff 2x(2x^2-3) > 0 \iff x > 0 \wedge x^2 > \frac{3}{2} \vee x < 0 \wedge x^2 < \frac{3}{2} \iff x > \sqrt{\frac{3}{2}} \vee x \in \left(-\sqrt{\frac{3}{2}}, 0\right)$.

inflexní body $-\sqrt{\frac{3}{2}}, 0, \sqrt{\frac{3}{2}}$

konvexní na $\left(-\sqrt{\frac{3}{2}}, 0\right)$ a $\left(\sqrt{\frac{3}{2}}, \infty\right)$, konkávní na $\left(-\infty, -\sqrt{\frac{3}{2}}\right)$, $\left(0, \sqrt{\frac{3}{2}}\right)$.

asymptoty b.s. nemá, se směrnicí: $\lim_{x\to\pm\infty}\frac{f(x)}{x} = \lim_{x\to\pm\infty}\frac{1}{e^{x^2}} = 0$; $\lim_{x\to\pm\infty} f(x) - 0\cdot x = 0$.

asymptotou je $y = 0$.