$$\mathrm{grad}_f = \left(\sqrt[n]{x_2 \cdots x_n}\ \frac{1}{n} x_1^{\frac{1-n}{n}}, \ \ldots, \ \sqrt[n]{x_1 \cdots x_{n-1}} \cdot \frac{1}{n} x_n^{\frac{1-n}{n}} \right)$$

$$\lambda \qquad\qquad\qquad\qquad \lambda$$

$$\text{normala} = (1, \ \ldots, \ 1)$$

$$\frac{x_n^{\frac{1}{n}}\, x_1^{\frac{1-n}{n}}}{x_1^{\frac{1}{n}}\, x_n^{\frac{1-n}{n}}} = 1 \Rightarrow x_1 = x_n$$

$$\Rightarrow \quad x_i = \frac{c}{n}$$

$$-y^2\lambda = x$$

$$-2xy\lambda = y$$

$$1\cdot\lambda = 2z$$

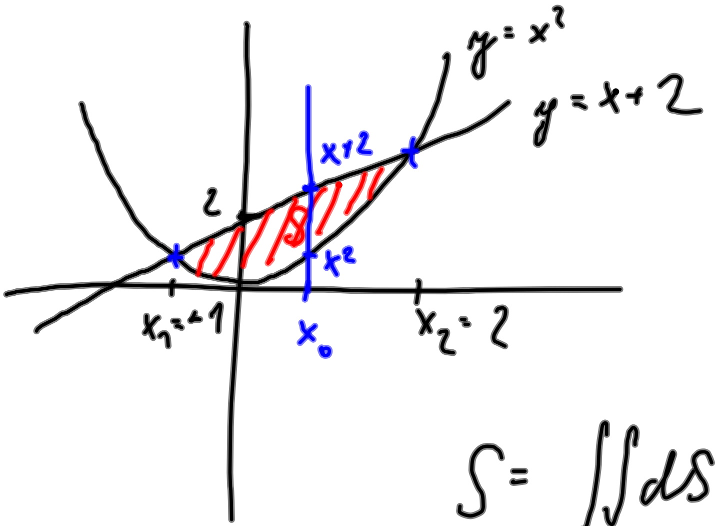
Zne: $y(1+2x\lambda)=0 \Rightarrow (1)\ y=0 \Rightarrow x=0 \Rightarrow z=\pm\frac{1}{\sqrt{2}}$

$(2)\ x=-\frac{1}{2\lambda} \Rightarrow -y^2=-\frac{1}{2\lambda^2} \Rightarrow y=\pm\frac{1}{\sqrt{2}\lambda},\ z=\frac{\lambda}{2}$

$$\frac{1}{4\lambda^2}+\frac{1}{2\lambda^2}+\frac{\lambda^2}{2}=1$$

$$1+2+2\lambda^4=4\lambda^2 \qquad s=\lambda^2$$

$$2s^2-4s+3=0 \qquad D=16-24=-8<0$$



$$x^2 = x+2$$

$$x^2 - x - 2 = 0$$

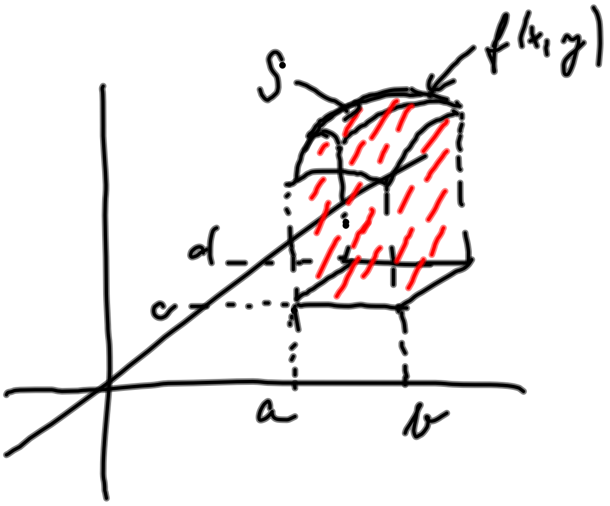
$$(x-2)(x+1) = 0$$

$$S = \iint_S dS = \int_{-1}^{2}\int_{x^2}^{x+2} 1\, dy\, dx =$$

$$= \int_{-1}^{2} (x+2) - x^2\, dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} =$$

$$= \left(2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}\right) =$$

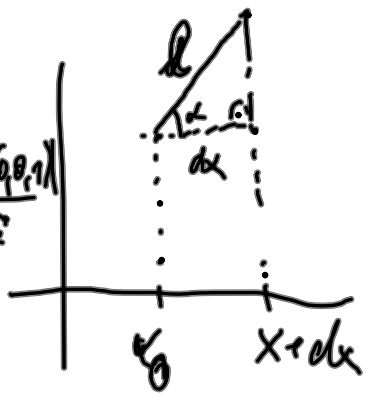
$$= 8 - \frac{7}{2} = \frac{9}{2}$$



$$V = \int_a^b \int_c^d f(x,y)\, dy\, dx = \int_a^b \int_c^d \int_0^{f(x,y)} 1\, dz\, dy\, dx$$

$$\vec{n} = (f_x, f_y, -1)$$

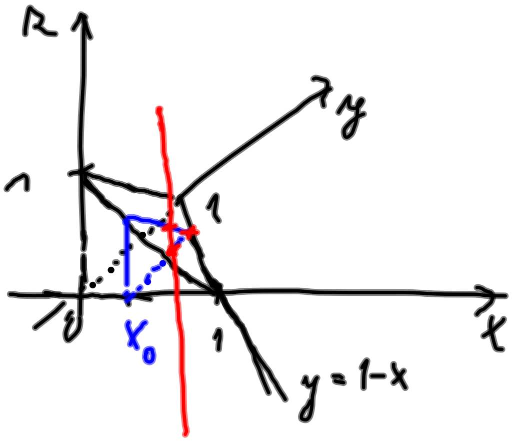
$$\cos\gamma = \frac{|(f_x, f_y, -1)\cdot(0,0,1)|}{\sqrt{1+f_x^2+f_y^2}}$$



$$\frac{dx}{l} = \cos\alpha$$

$$l = \frac{dx}{\cos\alpha}$$

$$S = \int_a^b \int_c^d \sqrt{1+f_x^2+f_y^2}\, dy\, dx$$



$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} 1 \, dz \, dy \, dx =$$

$$x + y + z \le 1$$
$$y \le 1 - x - z$$

$$= \int_0^1 \int_0^{1-x} (1-x-y) \, dy \, dx = \int_0^1 \left[(1-x)y - \frac{y^2}{2} \right]_0^{1-x} dx$$

$$= \int_0^1 \left[(1-x)^2 - \frac{(1-x)^2}{2} \right] dx = \frac{1}{2} \int_0^1 (1-x)^2 = \frac{1}{2} \left[-\frac{(1-x)^3}{3} \right]_0^1 =$$

$$= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

10 11-10:38

Těžiště čtyřstěnu:

$$z_T = 6 \int_0^1 \int_0^{1-z} \int_0^{1-y-z} z \, dx \, dy \, dz =$$

$$= 6 \int_0^1 \int_0^{1-z} z(1-y-z) \, dy \, dz =$$

$$= 6 \int_0^1 \left[y \cdot z(1-z) - z\frac{y^2}{2} \right]_0^{1-z} dz = 6 \int_0^1 \left[z(1-z)^2 - \tfrac{1}{2} z (1-z)^2 \right] dz =$$

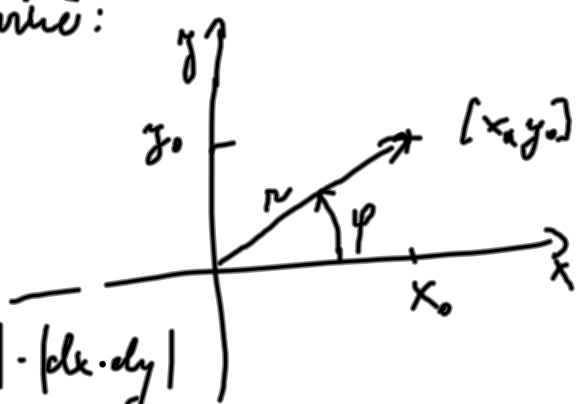
$$= 3 \int_0^1 z(1-z)^2 \, dz = 3 \int_0^1 (z^3 - 2z^2 + z) \, dz = 3 \left[\frac{z^4}{4} - 2\frac{z^3}{3} + \frac{z^2}{2} \right]_0^1 =$$

$$= 3 \left(\frac{1}{4} - \frac{2}{3} + \frac{1}{2} \right) = \frac{1}{4}$$

$\Rightarrow$ těžiště čtyřstěnu má souřadnice $\left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right]$

Polární souřadnice v rovině:

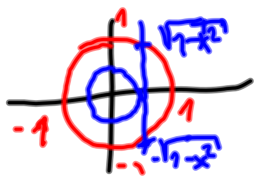
$$\Phi: \quad \begin{aligned} x &= r\cdot\cos\varphi \\ y &= r\cdot\sin\varphi \end{aligned}$$



$$|d\varphi\cdot dr| = |J\Phi|\cdot|dx\cdot dy|$$

$$|J\Phi| = \begin{vmatrix} \cos\varphi & \sin\varphi \\ -r\cdot\sin\varphi & r\cos\varphi \end{vmatrix} = r\cos^2\varphi + r\sin^2\varphi = r(\cos^2\varphi+\sin^2\varphi) = r$$

Př. Obsah kruhu $x^2+y^2 \leq 1$



$$S: \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\,dx = 2\int_{-1}^{1}\sqrt{1-x^2}\,dx = \dots$$

$$\int_0^1 \int_0^{2\pi} r \, d\varphi \, dr = \int_0^1 [2\pi r] \, dr = [\pi r^2]_0^1 = \pi$$

$x^2 + (y-1)^2 = 1$

$r^2\cos^2\varphi + r^2\sin^2\varphi = 2r\sin\varphi$

$r^2 = 2r\sin\varphi$

$r = 2\sin\varphi$

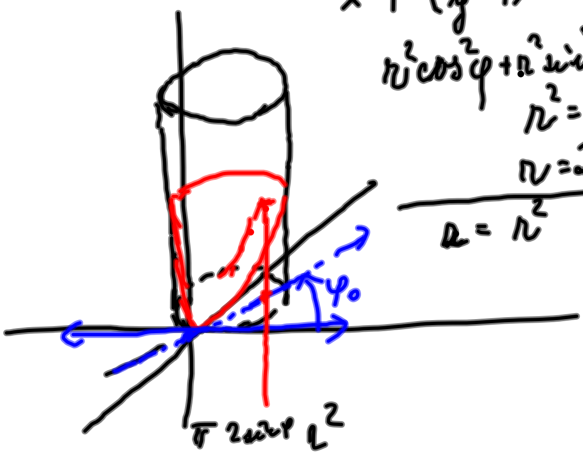
$z = r^2$

Válcové souřadnice:

$$\Psi: \begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ z &= z \end{aligned}$$

$|J\varphi| = r$

$$J\varphi = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -r\sin\varphi & r\cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$V = \int_0^{\pi} \int_0^{2\sin\varphi} \int_0^{r^2} r \, dz \, dr \, d\varphi = \int_0^{\pi} \int_0^{2\sin\varphi} r^3 \, dr \, d\varphi = \int_0^{\pi} 4\sin^4\varphi \, d\varphi =$$

10 11-11:17

$$\int \sin^4 x\,dx = \int \sin^2 x(1-\cos^2 x)\,dx = \int \sin^2 x\,dx -$$

$$-\int \sin^2 x\cos^2 x\,dx = \int \sin^2 x\,dx - \frac{1}{4}\int \sin^2 2x\,dx$$

$$\int \sin^2 x\,dx = -\sin x\cos x + \int \cos^2 x\,dx = \sin x\cos x + \int (1-\sin^2 x)\,dx$$

$$\begin{cases} u = \sin x & u' = \cos x \\ v' = \sin x & v = -\cos x \end{cases} \qquad = \sin x\cos x + x - \int \sin^2 x\,dx$$

$$\Rightarrow \int \sin^2 x\,dx = \frac{1}{2}(x - \sin x\cos x)$$

$$\int \sin^2 2x = \frac{1}{2}\int \sin^2 t\,dt = \frac{1}{2}\left[\frac{1}{2}\left[2x - \sin 2x\cos 2x\right]\right] =$$

$$\begin{cases} t = 2x \\ dt = 2dx \end{cases} \qquad = \frac{x}{2} - \frac{1}{4}\sin 2x\cos 2x$$

$$= 4\left[\frac{1}{2}(x - \sin x\cos x) - \frac{1}{4}\left(\frac{x}{2} - \frac{1}{4}\sin 2x\cos 2x\right)\right]_0^{\pi} =$$

$$= \frac{3}{2}\pi$$

$$\iint_S \sqrt{1+f_x^2+f_y^2}\,dx\,dy$$

$$x^2+(y-2)^2=4$$

$$r^2=4r\sin\varphi$$

$$r=4\sin\varphi$$

$$z=\frac{1}{\sqrt{3}}\sqrt{x^2+y^2}$$

$$f_x=\frac{1}{2\sqrt{3}}\,\frac{2x}{\sqrt{x^2+y^2}}$$

$$f_y=\frac{1}{\sqrt{3}}\,\frac{y}{\sqrt{x^2+y^2}}$$

$$=\iint_S \sqrt{1+\frac{x^2}{3(x^2+y^2)}+\frac{y^2}{3(x^2+y^2)}}\,dx\,dy=$$

$$=\iint_S \frac{2}{\sqrt{3}}\,dx\,dy=$$

$$=\frac{2}{\sqrt{3}}\int_0^{\pi}\int_0^{4\sin\varphi} r\,dr\,d\varphi=\frac{2}{\sqrt{3}}\int_0^{\pi} 8\sin^2\varphi\,d\varphi=$$

$$=\frac{16}{\sqrt{3}}\left[\frac{1}{2}(\varphi-\sin\varphi\cos\varphi)\right]_0^{\pi}=\frac{8\pi}{\sqrt{3}}$$