$$f(x,y) = x^2 \sin(xy)$$

$$f_x(x,y) = 2x \sin(xy) + x^2 y \cos(xy)$$

$$f_y(x,y) = x^3 \cos(xy)$$

$$f_{xy} = 2x^2 \cos(xy) + x^2 \cos(xy) - x^3 y \sin(xy)$$

$$f_{yx}(x,y) = 3x^2 \cos(xy) - x^3 y \sin(xy$$

$$f_{xy} = f_{yx}$$

9 26-14:01

Bew:

$$f_x(x,y) = \lim_{h\to 0} \frac{1}{h}\big(f(x+h,y) - f(x,y)\big)$$

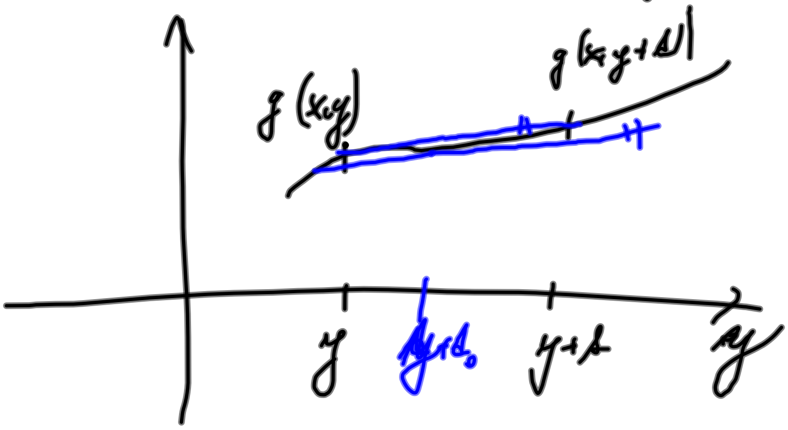
$$f_{xy}(x,y) = \lim_{s\to 0} \frac{1}{s}\Big[\lim_{h\to 0} \frac{1}{h}\big(f(x+h,y+s) - f(x,y+s)\big) - \lim_{h\to 0} \frac{1}{h}\big(f(x+h,y) - f(x,y)\big)\Big]$$

$$\overset{?}{=} \lim_{s\to 0} \lim_{h\to 0} \Big[\frac{1}{sh} \lim_{h\to 0} \big[f(x+h,y+s) - f(x,y+s) - f(x+h,y) + f(x,y)\big]$$

$$= \lim_{h\to 0} \frac{1}{h^2} \underbrace{\big[f(x+h,y+h) - f(x,y+h) - f(x+h,y) + f(x,y)\big]}_{\varphi(x,y,h)}$$

$$g(x,y) := f(x+h,y) - f(x,y).$$

$$g(x, y+\Delta) - g(x,y) = \Delta \cdot g_y(x, y+\Delta_0), \quad \Delta_0 \in (0,\Delta)$$



$$h(x,y) = f(x, y+\Delta) - f(x,y)$$

$$\approx f_{xy}(x+\Delta_1, y+\Delta_0)$$

$$\varphi(x,y,\Delta) = \frac{1}{\Delta^2}\left(g(x,y+\Delta) - g(x,y)\right) = \qquad \Delta_1 \in (0,\Delta)$$

$$= \frac{1}{\Delta^2}\left(\Delta g_y(x, y+\Delta_0)\right) =$$

$$= \frac{1}{\Delta}\left(f_y(x+\Delta, y+\Delta_0) - f_y(x, y+\Delta_0)\right) =$$

$$= \frac{1}{\Delta}\left(\Delta f_{yx}(x+\Delta_1, y+\Delta_0)\right) = f_{yx}(x+\Delta_1, y+\Delta_0) \overset{?}{=}$$

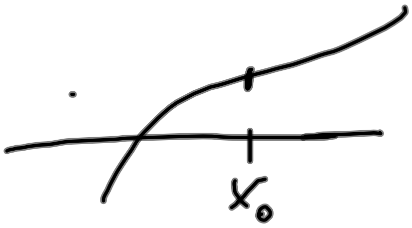
$$Hf(x) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} \qquad f: \mathbb{R}^2 \to \mathbb{R}$$

$f'(x_0 + \Delta v)$

$= f_v(x_0 + \Delta v)$

bilineární forma ~ A

hodnota formy na v. x $\quad x^T A x$



$$f''(x_0 + \Delta v) = f'(f_v(x_0 + \Delta v)) = f_{vv}(x_0)$$

$$= \left(v_1 \cdot f_x(\mathbf{x}_0 + t v) + v_2 f_y(\mathbf{x}_0 + t v) \right)' = \qquad v = (v_1, v_2), \quad \mathbf{x}_0 = (x_0, y_0)$$

$$= v_1 \cdot \left[v_1 \cdot f_{xx}(\mathbf{x}_0 + t v) + v_2 \cdot f_{xy}(\mathbf{x}_0 + t v) \right] +$$

$$+ v_2 \cdot \left(v_1 \cdot f_{yx}(\mathbf{x}_0 + t v) + v_2 \cdot f_{yy}(\mathbf{x}_0 + t v) \right) =$$

$$= v_1^2 f_{xx}(\mathbf{x}_0 + t v) + 2 v_1 v_2 f_{xy}(\mathbf{x}_0 + t v) + v_2^2 f_{yy}(\mathbf{x}_0 + t v)$$

$$f''(\mathbf{x}_0) = v_1^2 f_{xx}(\mathbf{x}_0) + 2 v_1 v_2 f_{xy}(\mathbf{x}_0) + v_2^2 f_{yy}(\mathbf{x}_0)$$

$$= Hf(\mathbf{x}_0)(v) =$$

$$= (v_1, v_2) \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$h^1(x_1,\dots,x_n) = g_1(f_1(x_1,\dots,x_n),\dots,f_m(x_1,\dots,x_n))$$

$$H := G \circ F$$

$$h_1^1 = g_{11} \cdot f_{11}(x_1,\dots,x_n) + g_{12} \cdot f_{21}(x_1,\dots,x_n) + \dots + g_{1m} \cdot f_{m1}(x_1,\dots,x_n)$$

$$JH = \begin{pmatrix} h_1^1 & \dots \\ \vdots & \end{pmatrix}$$

9 26-15:45