$f(x,y) = x^2y + y^2 - xy$

$f_x(x,y) = 2xy - y = 0 \Rightarrow y(2x-1) = 0 \Rightarrow y = 0 \lor x = \frac{1}{2}$

$f_y(x,y) = x^2 + 2y - x = 0 \Rightarrow$

i) $y = 0$ $\quad x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0 \lor x = 1$

$S_1 := [0,0]$, $S_2 := [1, 0]$

ii) $x = \frac{1}{2}$, $\quad \frac{1}{4} + 2y - \frac{1}{2} = 0 \Rightarrow y = \frac{1}{8}$

$S_3 = [\frac{1}{2}, \frac{1}{8}]$

$$H_f(x,y) = \begin{pmatrix} 2y & 2x-1 \\ 2x-1 & 2 \end{pmatrix}$$

$$H_f(0,0) = \begin{pmatrix} 0 & -1 \\ -1 & 2 \end{pmatrix} \qquad |H_f(0,0)| = -1 < 0$$

$$H_f(1,0) = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \qquad |H_f(0,0)| = -1 < 0$$

H_f je v obou těchto bodech indefinitní, v těchto bodech nenastává extrém, jedná se o tzv. sedlové body.

$$H_f\left(\tfrac{1}{2}, \tfrac{1}{8}\right) = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & 2 \end{pmatrix}, \qquad \left|H_f\left(\tfrac{1}{2}, \tfrac{1}{2}\right)\right| = \frac{1}{2} > 0$$

$f(x,y) = x^3+y^2$

stac. bod je bod $(0,0)$

$H_f = \begin{pmatrix} 6x & 0 \\ 0 & 2 \end{pmatrix}$, $H_f(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$

semidefinitní

$f(x,y) = x^4+y^2$, st. bod $(0,0)$

$H_f = \begin{pmatrix} 12x^2 & 0 \\ 0 & 2 \end{pmatrix}$, $H_f(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$

10 4-10:11

$(1,1,2)$ .... směrový vektor kolmice

$p: \quad x = t, \quad y = t, \quad z = 2t$

$p \cap \rho: \quad t + t + 4t = 1 \Rightarrow t = \frac{1}{6}$

$\left[\frac{1}{6}, \frac{1}{6}, \frac{1}{3}\right]$

$x = t \quad = \frac{1}{6}$

$y = 1 - 2s - t \quad = \frac{1}{6}$

$z = s \quad = \frac{1}{3}$

Čtverec vzdálenosti lib. bodu roviny ρ od $[0,0,0]$

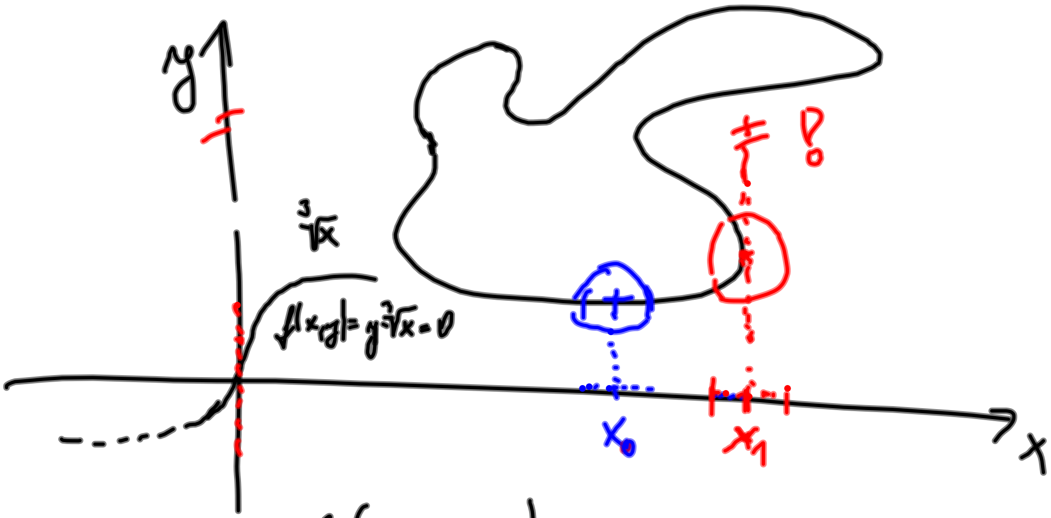
$$d(s,t) = t^2 + (1 - 2s - t)^2 + s^2 = 2t^2 + 5s^2 + 4st - 4s - 2t + 1$$

$$d_t(s,t) = 4t + 4s - 2 \Rightarrow t = \tfrac{1}{2} - s, \quad d_s(s,t) = 10s + 4t - 4 = 0$$

$$\boxed{\begin{array}{c} 10s + 2 - 4s - 4 = 0 \\ 6s = 2,\ s = \frac{1}{3} \\ \Rightarrow t = \frac{1}{6} \end{array}}$$

$$\log_2 x = \frac{\ln x}{\ln 2}$$

$$f(x,y) = \frac{1}{\ln 2} \cdot \ln\left(x^2 y + y^2 + 2\right)$$



$$f(x, g(x)) = 0$$

$$F(x, g(x)) = 0$$

$$F_x + F_y \cdot g'(x) = 0$$

$$g'(x) = -\frac{F_x}{F_y}$$

$$f_y(x, y) = e^x \cos(y) + 1$$

$$f_y\left(0, \frac{\pi}{2}\right) = 1 \neq 0$$

$$g'(0) = -\frac{f_x\left(0, \frac{\pi}{2}\right)}{f_y\left(0, \frac{\pi}{2}\right)} = -1$$

$$f_x(x, y) = e^x \sin y$$

$$f_x\left(0, \frac{\pi}{2}\right) = 1$$

10 4-10:38

$$F_z(x,y,z) = y\cos(yz) + x\cos(xz)$$

$$F_z\left(\sqrt{\tfrac{\pi}{2}}, \sqrt{\tfrac{\pi}{2}}, 0\right) = \sqrt{\tfrac{\pi}{2}} + \sqrt{\tfrac{\pi}{2}} = 2\sqrt{\tfrac{\pi}{2}} \neq 0 \Rightarrow$$

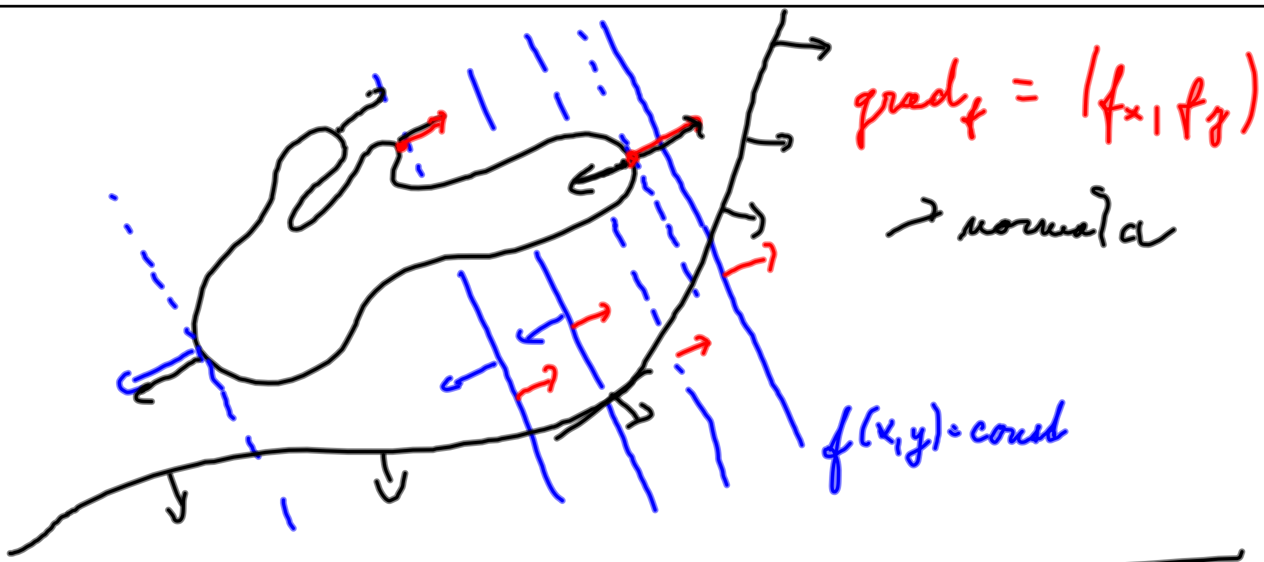
$\Rightarrow$ předpis $F(x,y,z)=0$ zadává funkci f splňující

$F(x,y,f(x,y)) = 0$. Navíc

$$f_x\left(\sqrt{\tfrac{\pi}{2}}, \sqrt{\tfrac{\pi}{2}}\right) = -\frac{F_x\left(\sqrt{\tfrac{\pi}{2}}, \sqrt{\tfrac{\pi}{2}}, 0\right)}{F_z\left(\sqrt{\tfrac{\pi}{2}}, \sqrt{\tfrac{\pi}{2}}, 0\right)} = -\frac{0}{2\sqrt{\tfrac{\pi}{2}}} = 0$$

$$F_x(x,y,z) = y\cos(xy) + z\cos(zx) =$$
$$= 0$$

10 4-10:45



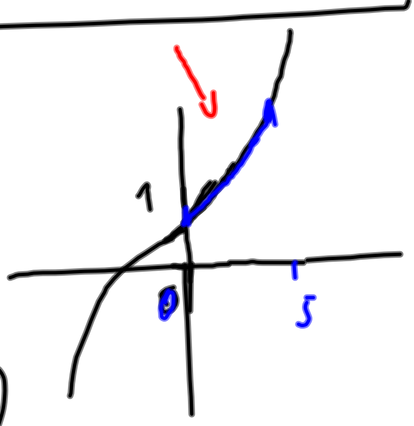
$$y - x^3 - 2x - 1 = 0$$

$$y = x^3 + 2x + 1$$

$$f(x,y) = x - 2y$$

$$\text{grad } f = (1, -2)$$

normála k ploše je $(-3x^2 - 2, 1)$



Podmínka pro vázané extrémy

grad f = k · normála g

$(1, -2) = k(-3x^2 - 2, 1) \iff$ $1 = k(-3x^2 - 2)$
$-2 = k \implies$

$\implies 1 = 6x^2 + 4 \implies 6x^2 = -3$ ↯

$x = 5 \implies y = 125 + 10 + 1 = 136$
$x = 0 \implies y = 1$
$f(5, 136) = 5 - 2 \cdot 136 = -267$ .... maximum
$f(0, 1) = -2$ ... minimum.

$\mathrm{grad}_f = (1, 2, 3)$

normála: $(2x, 2y, -1)$

$(1,2,3) = \lambda(2x, 2xy, -1)$

$1 = 2\lambda x \Rightarrow 1 = -6x \Rightarrow x = -\frac{1}{6}$

$2 = 2\lambda y \Rightarrow 2 = -6y \Rightarrow y = -\frac{1}{3}$

$3 = -\lambda$

$z = x^2 + y^2 \Rightarrow z = \frac{1}{36} + \frac{1}{9} = \frac{5}{36}$

Funkce nabývá v bodě $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{5}{36}\right)$ svého minima.

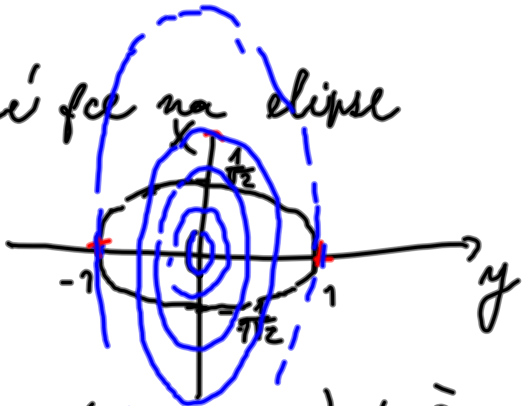
$L(x, y, z, \lambda) = x + 2y + 3z - \lambda(z - x^2 - y^2)$

$$f_x(x,y) = 2x = 0$$
$$f_y(x,y) = 6y = 0$$
$\Rightarrow$ bod $(0,0)$ je to minimum (lok.)

Maximum na okraji:

Hledáme maximum dané fce na elipse

$$2x^2+y^2=1$$



$\text{grad}_f : (2x, 6y)$

normála: $(4x, 2y)$

$$\left.\begin{aligned} 2x &= \lambda\cdot 4x \\ 6y &= \lambda\cdot 2y \end{aligned}\right\} \Rightarrow$$

$(x\neq 0 \wedge y\neq 0)$ N.Ř.

$x=0 \Rightarrow y=\pm 1$

$y=0 \Rightarrow x=\pm\frac{1}{\sqrt{2}}$

$\Rightarrow$ maxima je nabývá v bodech $(0,\pm 1)$