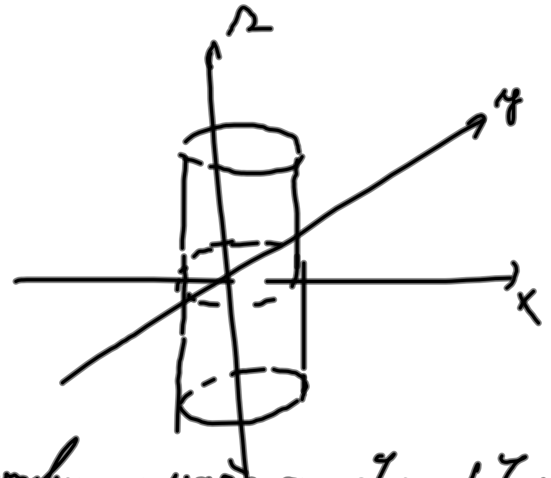
$x^2+y^2=4 \Leftrightarrow r=2$

$x^2+0^2=4$



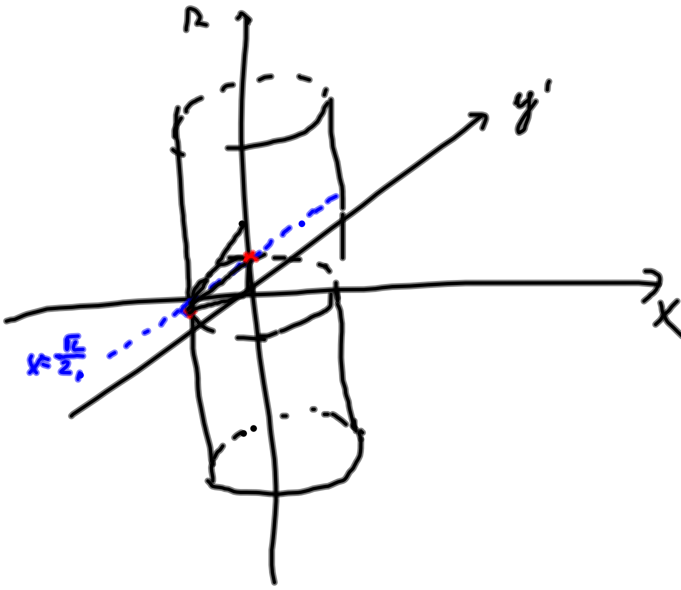
Nad kruhem $x^2+y^2=4$ uvážím graf funkce $z=\sqrt{4-x^2}$. Povrch grafu této funkce je polovinou povrchu uvažovaného tělesa.

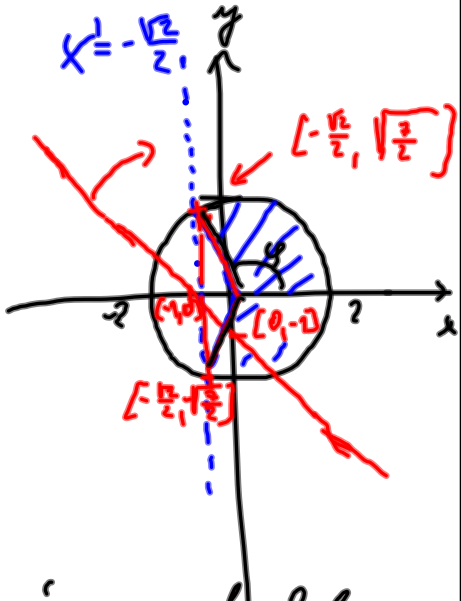
$$\iint_{\Omega} \sqrt{1+f_x^2+f_y^2}\,dx\,dy = \iint_{\Omega} \sqrt{1+\frac{x^2}{4-x^2}}\,dx\,dy =$$

$$f_x = -\frac{x}{\sqrt{4-x^2}}$$

$$f_y = 0$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \frac{2}{\sqrt{4-x^2}}\,dx\,dy = 16$$





Uvažujme rotaci tělesa o 45° v záporném smyslu kolem osy z. Matice této rotace (v rovině z = 0)

$$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$$

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Nové souřadnice:

$$x' = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \qquad x = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'$$

$$y' = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \qquad y = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$$

Rovnice roviny $z = x + y + 1 =$

$$= \left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right) + \left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) + 1$$

$$= \sqrt{2}x' + 1$$

Nyní si zadané těleso rozdělím na 2 části:

1, těleso pod grafem funkce $z = \sqrt{2}x + 1$ nad kruhovou výsečí v kruhu se středem v počátku a ohraničenou úhly $-\arccos\left(-\frac{1}{2\sqrt{2}}\right)$ a $\arccos\left(-\frac{1}{2\sqrt{2}}\right)$

2, čtyřstěn s vrcholy $\left[-\frac{\sqrt{2}}{2}, \sqrt{\frac{3}{2}}, 0\right]$, $\left[-\frac{\sqrt{2}}{2}, -\sqrt{\frac{3}{2}}, 0\right]$, $[0,0,0]$, $[0,0,1]$

s objemem

$$\frac{1}{2}\left|\begin{matrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{7}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{7}}{2} \end{matrix}\right| \cdot \frac{1}{3} \cdot 1 = \frac{\sqrt{7}}{6}$$

Co se týče 1. části:

$$2 \int_0^{\arccos\left(\frac{1}{2\sqrt{2}}\right)} \int_0^2 r\left(\sqrt{2}\, r \cdot \cos\varphi + 1\right) dr\, d\varphi =$$

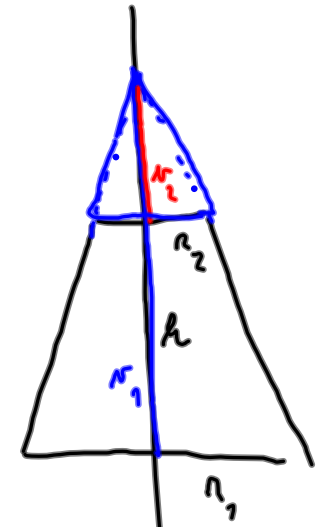
$$= 2 \int_0 \left[\frac{r^3}{3}\sqrt{2} \cdot \cos\varphi + \frac{r^2}{2}\right]_0^2 d\varphi =$$

$$= 2 \int_0^{\arccos\left(-\frac{1}{2\sqrt{2}}\right)} \left(\frac{8}{3}\sqrt{2} \cdot \cos\varphi + 2\right) d\varphi =$$

$$2\left[\frac{8}{3}\sqrt{2}\sin\varphi + 2\varphi\right]_0^{\arccos\left(-\frac{1}{2\sqrt{2}}\right)} =$$

$$= 2\left[\frac{8}{3}\sqrt{2}\cdot\frac{\sqrt{7}}{2\sqrt{2}} + 2\cdot\arccos\left(-\frac{1}{2\sqrt{2}}\right)\right] \doteq 15{,}22$$

Celkem cca 15,22...



$$\frac{v_2}{r_2} = \frac{v_1}{r_1} = \frac{v_2 + h}{r_2} \Rightarrow$$

$$\Rightarrow v_2 = \frac{r_2 h}{r_1 - r_2}$$

$$v_1 = v_2 + h = \frac{r_1 h}{r_1 - r_2}$$

$$V_2 \cdot r_2 + \overset{(V_1 - V_2)}{V_K}\, r_K = V_1 \cdot r_1$$

$$\frac{1}{3}\pi r_2^2 \frac{r_2 h}{r_1 - r_2} \cdot \left(h + \frac{1}{4}\frac{r_2 h}{r_1 - r_2}\right) + r_K\left(\frac{1}{3}\pi r_1^2 \cdot \frac{r_1 h}{r_1 - r_2} - \frac{1}{3}\pi r_2^2 \frac{r_2 h}{r_1 - r_2}\right) =$$

$$= \frac{1}{4}\frac{r_1 h}{r_1 - r_2}\left(\frac{1}{3}\pi \frac{r_1^3}{r_1 - r_2} h\right)$$

$$R_k = \frac{1}{4} h \left(\frac{(r_1^2 + r_2^2)(r_1 - r_2) - 3 r_2^2}{r_1^3 - r_2^3} \right)$$

$$f(y)\cdot y' = g(x)$$
$$F(y) = G(x)$$

$$\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Cx+B}{1+x^2}$$
$$1 = A(1+x^2) + (Cx+B)x$$
$$A=1,\ C=-1,\ B=0$$

$$y' = -\frac{1+y^2}{xy^2(1+x^2)}$$
$$y'\cdot\frac{y^2}{1+y^2} = -\frac{1}{x(1+x^2)}$$
$$\frac{dy}{dx}\cdot\frac{y^2}{1+y^2} = -\frac{1}{x(1+x^2)}$$
$$\frac{y^2}{1+y^2}\cdot dy = -\frac{1}{x(1+x^2)}\,dx$$

$$\int\frac{y^2}{1+y^2}\,dy = -\int\frac{1}{x(1+x^2)}\,dx$$
$$y - \operatorname{arctg}(y) = \frac{1}{2}\ln|1+x^2| - \ln|x|$$
y dána implicitně

$$\int\frac{dx}{x(1+x^2)} = \int\frac{dx}{x} - \int\frac{x}{1+x^2}$$
$$= \ln|x| - \frac{1}{2}\ln|1+x^2|$$

T_0 ... teplota okolí

$T(t)$ ... teplota čaje v závislosti na čase t.

$$k(T_0 - T) = T' = \frac{dT}{dt} \quad (k \in \mathbb{R}^+)$$

$$k\,dt = \frac{dT}{T_0 - T}$$

$$kt + C = -\ln(T - T_0) \qquad (K = e^{-C})$$

$$K \cdot e^{-kt} = T - T_0 \Rightarrow T = T_0 + K e^{-kt}$$

$$T(0) = 80, \quad 80 = 20 + K \Rightarrow K = 60$$

$$T(5) = 60 = 20 + 60 \cdot e^{-k5} \Rightarrow \frac{2}{3} = e^{-k5} \Rightarrow$$

$$\Rightarrow -5k = \ln\left(\frac{2}{3}\right) \Rightarrow k = \frac{1}{5} \ln\left(\frac{3}{2}\right)$$

$$T(t) = 20 + 60 \cdot e^{-\frac{1}{5}\ln\left(\frac{3}{2}\right)\cdot t}$$

$$40 = 20 + 60 \cdot e^{-\frac{1}{5}\ln\left(\frac{3}{2}\right)\cdot t}$$

$$\frac{1}{3} = e^{-\frac{1}{5}\ln\left(\frac{3}{2}\right)\cdot t}$$

$$\ln\left(\frac{1}{3}\right) = -\frac{1}{5}\ln\left(\frac{3}{2}\right)\cdot t \Rightarrow t = \frac{5\ln(3)}{\ln\left(\frac{3}{2}\right)} \doteq 13.548$$