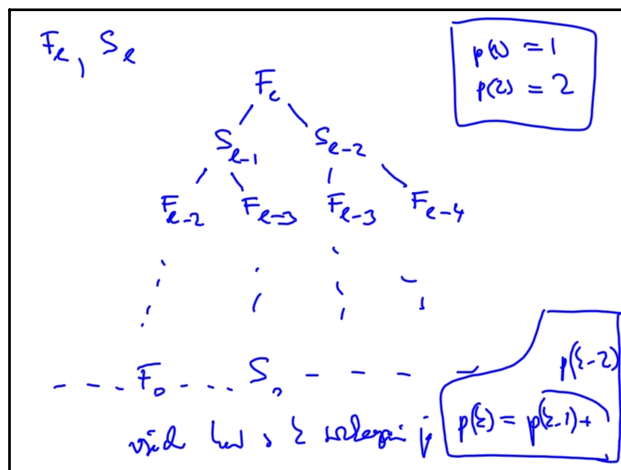
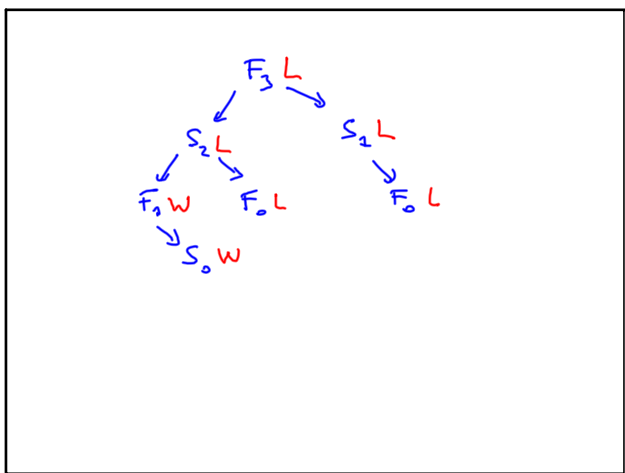


Strong her: (nepravni kombinatori (log))  
Primer: the Nim! (rijed od 1 do n)  
 • 2 k igrača dva kugle koje se mogu podijeliti  
 • njihova kugla, što je podijeljeno  
 star log = početak igrača (što je u k)

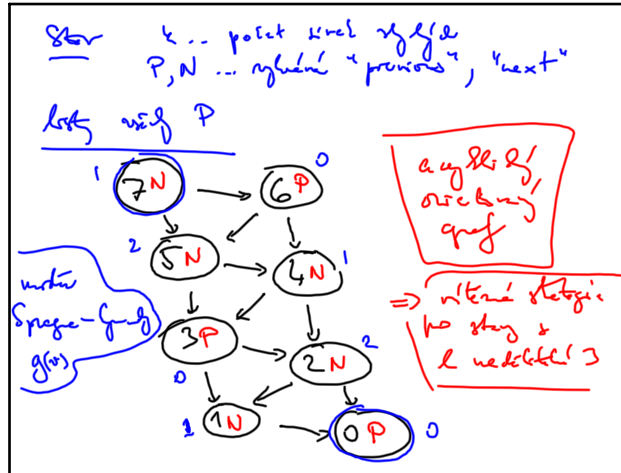
12 5-14:04



12 5-14:11



12 5-14:17



12 5-14:22

Savjet her:  
Primer: 3 kugle, koje se mogu podijeliti  
Primer 1: a igrači, a igrači, a igrači, a igrači  
Primer 2: a igrači, a igrači, a igrači, a igrači  
 savjet  $G_1 + G_2 = (V, E)$   
 $V = V_1 \times V_2$   
 $E = \{ (v_1, v_2, w_1, w_2), (v_1, w_1) \in E_1, v_2 \in V_2 \}$   
 $\cup \{ (v_1, v_2, w_1, w_2), (v_1, w_2) \in E_2, w_1 \in V_1 \}$

12 5-14:30

Sprague, Grundy: f-ica  $g: V \rightarrow \mathbb{N}$   
 $g(v) = \text{mex} \{ g(w) \mid (v,w) \in E \}$   
 Sde  $\text{mex } S = \text{mex}(\mathbb{N} \setminus S)$   
Primer: koliko v je P- pozicija  $\Leftrightarrow g(v) = 0$   
 $a, b \in \mathbb{N}$  njihova igra koja je  $\in \mathbb{Z}_2^k$   
 (nešto se ne može podijeliti)  $\mathbb{Z}_2^k$   
 $a \oplus b \in \mathbb{Z}_2^k$  je XOR

12 5-14:37

Theorem:  $G_1, G_2$  su aqk-lal qraf,  $g_1, g_2$  ipsh Sprognung-fundony fca.  $\rho$ sh  $g(v_1, v_2) = g_1(v_1) \oplus g_2(v_2)$  ij Sprognung-fundony fca  $G_1 + G_2$ .

12 5-14:49

$v_1, v_2$  vasa ve  $V_1 \times V_2$   
 $a \in N_0, a < g_1(v_1) \oplus g_2(v_2)$   $(g_1 \oplus g_2)(x, y) = a$   
 $\Rightarrow$  1)  $x, y \in V_1 \times V_2$  tal,  $\exists (v_1, v_2, x, y) \in E$   
 2)  $\text{max } \{g_1(v_1), g_2(v_2)\} \in E$  tal,  $\exists$   
 $g_1(v_1) \oplus g_2(v_2) = g_1(v_1) \oplus g_2(v_2)$   
 $b := g_1(v_1) \oplus g_2(v_2) \oplus a$ .  $\text{tal } b \in \text{cifer}$   
 $\Rightarrow$  na  $\exists$ -ku  $\text{wite } a$  tal  $\rightarrow g_1(v_1) \oplus g_2(v_2)$   
 $\text{wite } b \in 1 \Rightarrow$   $\text{wite } g_1(v_1) \in g_2(v_2)$   
 $\text{na } \exists$ -ku  $\text{wite } 1$ .  
 $\text{wite } c := g_1(v_1) \oplus b$ ,  $\text{nej } g_1 \in 1$  cifer  
 $\Rightarrow$   $\text{tal } v_1 \in V_1, (w_1, w_2) \in E, g_1(w_1) = c$

12 5-14:55

$\text{tal } (v_1, v_2, v_1, v_2) \in E, g_1(v_1) \oplus g_2(v_2) = c \oplus g_2(v_2) = g_1(v_1) \oplus b \oplus g_2(v_2) = g_1(v_1) \oplus a \oplus g_1(v_1) \oplus g_2(v_2) \oplus g_2(v_2) = a$   
 2):  $(v_1, v_2, v_1, v_2) \in E, v_1 = v_2$   
 $\text{tal } g_1(v_1) \oplus g_2(v_2) = g_1(v_1) \oplus g_2(v_2)$   
 $\Rightarrow g_1(v_1) \oplus g_2(v_2) = g_1(v_1) + g_2(v_2)$   
 $\Rightarrow g_1(v_1) = g_1(v_1)$  qraf.  
 fca  $g_1 \oplus g_2$  qraf  $\text{wite } a$  tal,  $\exists$   $\text{tal } v_1, v_2 \in V_1 \times V_2$  tal,  $\exists (v_1, v_2, x, y) \in E$

12 5-15:06

12 5-15:14