DDÚ Relace

1) a) $\sigma^{-1} = \{(d,a),(c,a)\} \subseteq Y \times X$

$\rho^{-1} = \{(b,d),(b,e),(a,c)\}$

$\sigma \circ \rho = \{(c,d),(c,c)\}$

$\rho \circ \sigma = \{(a,b),(a,a) \subseteq X \times X$

b) $\sigma \cup ((\rho\circ\sigma)^{-1}\circ\rho)^{-1} = \{(a,d),(a,e),(a,c)\} \subseteq X \times Y$

c) $\sigma \cap ((\rho\circ\sigma)^{-1}\rho)\cap\rho = \emptyset$ není relace, nejde $\sigma\cap\rho$ jako relace.

2) a) $\{(0,0),(1,1)\}, \{(0,0),(1,1),(1,0)\}, \{(0,0),(1,1),(0,1)\}, \{(0,0),(1,1),(0,1),(1,0)\}$.

b) $\emptyset, \{(0,0)\}, \{(1,1)\}, \{(0,0),(1,1)\}, \{(1,0),(0,1),(1,1),(0,0)\}$

c) $\emptyset, \{(0,0)\}, \{(1,1)\}, \{(0,0),(1,1)\}$

3) a) $\emptyset$

b) $\{(3,1),(3,2),(2,3)\}$

c) $\{(n,n),(1,2),(4,5),(5,4) \mid n\in\mathbb{N}\}$

d) $\{(1,2),(2,3)\}$

e) $\{(1,2),(2,3),(3,5),(5,4)\}$

4) je reflexivní, symetrická

není tranzitivní ani antisymetrická, není relací ekvivalence

5) a) $a \sim b \Leftrightarrow |a| \leq b$

R: $a \sim a \Leftrightarrow |a| \leq a$ NE: $a=-5$

S: $a \sim b \Rightarrow |a| \leq b \overset{?}{\Rightarrow} |b| \leq a \Rightarrow b \sim a$

$\Downarrow$

NE: $a=-5$
$b=6$

T: $a \sim b \,\&\, b \sim c \Rightarrow |a| \leq b \,\&\, |b| \leq c \overset{?}{\Rightarrow} |a| \leq c \Rightarrow a \sim c$

$\Downarrow$

ANO: $|a| \leq b \leq |b| \leq c$

A: $a \sim b \,\&\, b \sim a \Rightarrow |a| \leq b \,\&\, |b| \leq a \overset{?}{\Rightarrow} a=b$

$\Downarrow$

ANO: $a \leq |a| \leq b \leq |b| \leq a$

proto všechny nerovnosti jsou rovnosti.

b) $a \sim b \Leftrightarrow a \cdot b \leq 0$

R: $a \sim a \Leftrightarrow a \cdot a \leq 0$ NE: $a=1$

S: $a \sim b \Rightarrow a \cdot b \leq 0 \Rightarrow b \cdot a \leq 0 \Rightarrow b \sim a$ ANO

T: $a \sim b \,\&\, b \sim c \Rightarrow a \cdot b \leq 0 \,\&\, b \cdot c \leq 0 \overset{?}{\Rightarrow} a \cdot c \leq 0$

$\Downarrow$

NE: $b=0$
$a=c=1$

A: $a \sim b \,\&\, b \sim a \Rightarrow a \cdot b \leq 0 \,\&\, b \cdot a \leq 0 \overset{?}{\Rightarrow} a=b$

$\Downarrow$

NE: $a=-3, b=1$

c) $a\sim b \Leftrightarrow (a\le b \ \&\ a\cdot b\ge 0)$

R: $a\sim a \Leftrightarrow a\le a \ \&\ a\cdot a\ge 0$ ANO $(a=a \ \&\ a\cdot a\ge 0)$

S: $a\sim b \Rightarrow a\le b \ \&\ a\cdot b\ge 0 \overset{?}{\Rightarrow} b\le a \ \&\ b\cdot a\ge 0 \Rightarrow b\sim a$

NE:
$a=1$
$b=2$

T: $a\sim b \ \&\ b\sim c \Rightarrow a\le b \ \&\ a\cdot b\ge 0 \ \&\ b\le c \ \&\ b\cdot c\ge 0 \overset{?}{\Rightarrow}$

$\Rightarrow a\le b\le c \ \&\ a\cdot c\ge 0 \Rightarrow a\sim c$

NE:
$a=-1$
$b=0$
$c=1$

A: $a\sim b \ \&\ b\sim a \Rightarrow a\le b \ \&\ a\cdot b\ge 0 \ \&\ b\le a \ \&\ b\cdot a\ge 0 \overset{?}{\Rightarrow} a=b$

ANO: $a\le b\le a$, tj. $a=b$.

6) $a\sim b \Leftrightarrow a, b$ stejný ciferný součet

R: $a\sim a \Leftrightarrow a$ má stejný ciferný součet jako a ANO

S: $a\sim b \Rightarrow a$ stejný součet jako $b \Rightarrow b$ stejný součet jako $a \Rightarrow b\sim a$ ANO

T: $a\sim b \ \&\ b\sim c \Rightarrow a\sim c$ ANO

$M/\sim = \{\{1,10\},\{2,11,20\},\{3,12\},\{4,13\},\{5,14\},\{6,15\},\{7,16\},\{8,17\},\{9,18\},\{19\}\}$

7) $a \sim b \iff 2 \mid (a^2 - b^2)$

R: $a \sim a \iff 2 \mid (a^2 - a^2) \iff 2 \mid 0$ ANO

S: $a \sim b \Rightarrow 2 \mid (a^2 - b^2) \Rightarrow 2 \mid -(b^2 - a^2) \Rightarrow 2 \mid (b^2 - a^2) \Rightarrow b \sim a$

T: $a \sim b \,\&\, b \sim c \Rightarrow 2 \mid (a^2 - b^2) \,\&\, 2 \mid (b^2 - c^2) \Rightarrow 2 \mid (a^2 - b^2 + b^2 - c^2) \Rightarrow$

$\Rightarrow 2 \mid (a^2 - c^2) \Rightarrow a \sim c$

$\mathbb{Z}/\sim = \{\{2n \mid n \in \mathbb{Z}\}, \{2n+1 \mid n \in \mathbb{Z}\}\}$

$(= \{\{\ldots, -4, -2, 0, 2, 4, \ldots\}, \{\ldots, -3, -1, 1, 3, \ldots\}\})$

8) a) $a \sim b \iff a = b + 1$

R: $a \sim a \iff a = a + 1$ NE: $a = 0$

S: $a \sim b \Rightarrow a = b + 1 \Rightarrow b = a - 1 \overset{?}{\Rightarrow} b \sim a$

NE: $a = 0, b = -1$

T: $a \sim b \,\&\, b \sim c \Rightarrow a = b + 1 \,\&\, b = c + 1 \Rightarrow$

$\Rightarrow a = (c + 1) + 1 = c + 2 \overset{?}{\Rightarrow} a \sim c$

NE: $a = 1, b = 0, c = -1$.

A: $a \sim b \,\&\, b \sim a \Rightarrow a = b + 1 \,\&\, b = a + 1 \Rightarrow$

$\Rightarrow b = (b+1) + 1 \overset{?}{\Rightarrow} a = b$

$\boxed{\text{ANO}}$ taková dvě a, b splňující $a = b + 1$ & $b = a + 1$ neexistují!

b) $a \sim b \Leftrightarrow a \le b^2$

R: $a \sim a \not\Leftrightarrow a \le a^2$ NE: $a = 0{,}5$

S: $a \sim b \Rightarrow a \le b^2 \overset{?}{\Rightarrow} b \le a^2 \Rightarrow b \sim a$

$\Downarrow$

NE: $a = 1,\ b = 2$

T: $a \sim b \ \&\ b \sim c \Rightarrow a \le b^2 \ \&\ b \le c^2 \overset{?}{\Rightarrow} a \le c^2 \Rightarrow a \sim c$

$\Downarrow$

NE:
$a = 1$
$b = -1$
$c = 0$

A: $a \sim b \ \&\ b \sim a \Rightarrow a \le b^2 \ \&\ b \le a^2 \overset{?}{\Rightarrow} a = b$

$\Downarrow$

NE: $a = -5$
$b = -6$

c) $a \sim b \Leftrightarrow a \cdot b \le 1$

R: $a \sim a \Leftrightarrow a \cdot a \le 1$ NE: $a = 10$

S: $a \sim b \Rightarrow a \cdot b \le 1 \Rightarrow b \cdot a \le 1 \Rightarrow b \sim a$ ANO

T: $a \sim b \ \&\ b \sim c \Rightarrow a \cdot b \le 1 \ \&\ b \cdot c \le 1 \overset{?}{\Rightarrow} a \cdot c \le 1 \Rightarrow a \sim c$

$\Downarrow$

NE: $b = 0$
$a = 10$
$c = 20$

A: $a \sim b \ \&\ b \sim a \Rightarrow a \cdot b \le 1 \ \&\ b \cdot a \le 1 \overset{?}{\Rightarrow} a = b$

$\Downarrow$

NE: $a = 1$
$b = 0$