$$a \cdot (v + w) = a \cdot v + a \cdot w$$

$$(a + b) \cdot \vec{v} = a \cdot v + b \cdot v$$

$$A = (a_{ij}) \quad , \; 1 \le i \le m \;\; 1 \le j \le n$$

$$a_{ij}$$

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$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$\vdots$$

$$a_{m1}x_1 + \dots + a_{mn}x_n = b_m$$

$$\begin{pmatrix} a_{11} & \dots & a_{1n} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{m1} & & a_{mn} & b_m \end{pmatrix}$$

matice soustavy

$$\boxed{A \cdot x = b}$$

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

$$(a_{ij}) \qquad (b_{ij}) \qquad (c_{ij})$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 29 & 40 & 51 \end{pmatrix}$$

$$c_{ij} = \sum_{k=1}^{2} a_{ik} b_{kj}$$

$$A^{-1} \cdot A \cdot x = A^{-1} b$$

$$x = A^{-1} b$$

1

$$E_n = \begin{pmatrix} 1 & \cdots & 0 \\ 0 & 1 \cdots & 0 \\ \vdots & & \\ 0 & \cdots & 1 \end{pmatrix}$$

$$E \cdot A = A \cdot E = A$$

Ex. A^{-1} tak, že

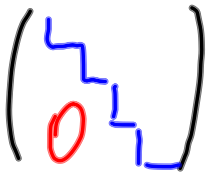
$$A \cdot A^{-1} = A^{-1} \cdot A = E$$

$$(A \cdot B) \cdot C = A(B \cdot C)$$

$$A \cdot (B + C) = AB + AC$$

Obecně $AB \neq BA$

Výpočet inverzní matice



$$\begin{pmatrix}1&2&3\\4&5&6\\3&3&6\end{pmatrix}\left|\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right.$$

-4, -3

E_1

$$\begin{pmatrix}1&2&3\\0&-3&-6\\0&-3&-3\end{pmatrix}\left|\begin{pmatrix}1&0&0\\-4&1&0\\-3&0&1\end{pmatrix}\right.$$

-1

E_2

$$\begin{pmatrix}1&2&3\\0&-3&-6\\0&0&3\end{pmatrix}\left|\begin{pmatrix}1&0&0\\-4&1&0\\1&-1&1\end{pmatrix}\right.$$

$-\frac{1}{3}$, $\frac{1}{3}$

$$\begin{pmatrix}1&2&3\\0&1&2\\0&0&1\end{pmatrix}\left|\begin{pmatrix}1&0&0\\\frac{4}{3}&-\frac{1}{3}&0\\\frac{1}{3}&-\frac{1}{3}&\frac{1}{3}\end{pmatrix}\right.$$

-3, -2

$$\begin{pmatrix}1&2&0\\0&1&0\\0&0&1\end{pmatrix}\left|\begin{pmatrix}0&1&-1\\\frac{2}{3}&\frac{1}{3}&-\frac{2}{3}\\\frac{1}{3}&-\frac{1}{3}&\frac{1}{3}\end{pmatrix}\right.$$

$|A^{-1}| = \frac{1}{9}$

$|A| = 9$

$$\begin{pmatrix}1&2&0\\0&1&0\\0&0&1\end{pmatrix}\left|\begin{pmatrix}0&1&-1\\\frac{2}{3}&\frac{1}{3}&-\frac{2}{3}\\\frac{1}{3}&-\frac{1}{3}&\frac{1}{3}\end{pmatrix}\right.$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\left|\underbrace{\begin{pmatrix}-\frac{4}{3}&\frac{1}{3}&\frac{1}{3}\\\frac{2}{3}&\frac{1}{3}&-\frac{2}{3}\\\frac{1}{3}&-\frac{1}{3}&\frac{1}{3}\end{pmatrix}}_{A^{-1}}\right.$$

$$E \cdot E_1 \dots E_s$$

$$A \cdot \underbrace{E_1 \cdot E_2 \cdots E_s}_{A^{-1}} = E$$

$$\left.\begin{aligned} x + 2y + z &= 1 \\ x - y - z &= 0 \\ 2x + y &= 1\,2 \end{aligned}\right\} \Rightarrow -3y - 2z = -1 \Rightarrow y = \tfrac{1}{3}(1-2t)$$

$$\Rightarrow x = \tfrac{1}{3}(1+t)$$

$$z = t, \quad t \in \mathbb{R}$$

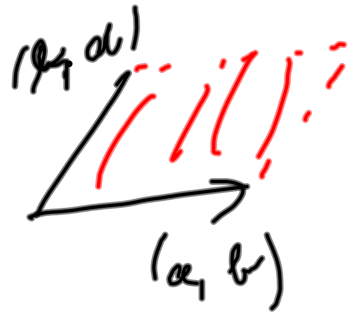
matice soustavy $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & -1 \\ 2 & 1 & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & -3 & -2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow h(A) = 2$

rozšířená matice $\bar{A} = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & -1 & -1 & 0 \\ 2 & 1 & 0 & 2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & -3 & -2 & -1 \\ 0 & -3 & -2 & -0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & -3 & -2 & -1 \\ 0 & 0 & 0 & 0\;1 \end{pmatrix}$

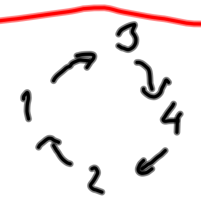
$h(\bar{A}) = 2$

$h(\bar{A}) = 3$



$$S = \mathrm{abs}\begin{vmatrix} a & b \\ c & d \end{vmatrix} =$$
$$= \mathrm{abs}(ad - bc)$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 2 & 6 & 5 \end{pmatrix} = (1,3,4,2)\circ(5,6)$$
$$= (1,3)\circ(3,4)(4,2)(5,6)$$



$$(1,5)(5,1)(1,3)(3,4)(4,2)\underbrace{(5,6)\,[2]}_{} =$$
$$(1,3)(3,4)\underbrace{(4,2)\,[2]}_{} =$$
$$(1,3)\underbrace{(3,4)\,[4]}_{} = (1,3)[3] = 1$$

Počet inverzí $2 + 0 + 1 + 0 + 1 + 0 = 4$

$$|A| = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma) \cdot a_{1\sigma(1)} \cdot a_{2\sigma(2)} \cdot \ldots \cdot a_{n\sigma(n)}$$

$$\begin{vmatrix} \overset{a_{11}}{a} & \overset{a_{12}}{b} \\ \underset{a_{21}}{c} & \underset{a_{22}}{d} \end{vmatrix} = \sum_{\sigma \in S_2} \operatorname{sign}(\sigma) \cdot a_{1\sigma(1)} a_{2(\sigma(2))} =$$

$$= a_{11} a_{22} - 1 \cdot a_{12} a_{21}$$

$$= ad - bc$$

$$\begin{vmatrix} 1 & 2 & 5 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 1 + 4 + 5 - 10 - 2 - 1 = -3$$

$$\begin{vmatrix} 1 & 2 & 3 & 5 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix} = 3 \cdot \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} + 0 \cdot \begin{vmatrix} \cdots \end{vmatrix} + 0 \cdot \begin{vmatrix} \cdots \end{vmatrix}$$

$$- 1 \begin{vmatrix} 1 & 2 & 5 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 6 + 3 = 9$$