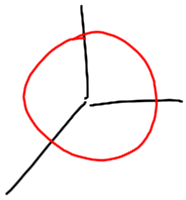
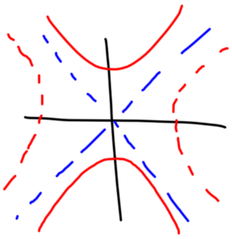
$\boxed{x^2+y^2+z^2=1}$ sféra v $\mathbb{R}^3$

$x^2-y^2=1$

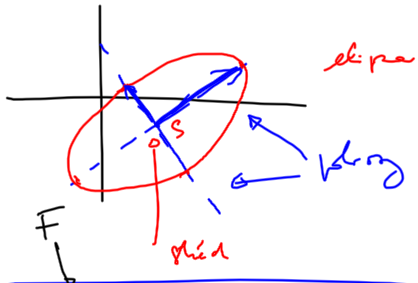




$$\boxed{1\cdot x\cdot x+1\cdot y\cdot y+1\cdot z\cdot z-1=0}$$

$a_{11}=1,\ a_{22}=1,\ a_{33}=1 \quad a=-1$

12 10-16:05



$$\boxed{\boxed{x^2-4xy-5y^2}+2x+4y+3=0}$$

12 10-16:15

$$A=\begin{pmatrix}1&2\\2&-5\end{pmatrix} \qquad \begin{vmatrix}1-\lambda&2\\2&-5-\lambda\end{vmatrix}=-(1-\lambda)(5+\lambda)-4$$

$$=\lambda^2+4\lambda-9 \qquad \lambda_{1,2}=\frac{-4\pm\sqrt{16+36}}{2}=-2\pm\sqrt{13}$$

$$\begin{pmatrix}3-\sqrt{13}&2\\2&-3-\sqrt{13}\end{pmatrix}\sim\begin{pmatrix}3-\sqrt{13}&2\\0&4-(3\cancel{-\sqrt{13}})(-3-\sqrt{13})\end{pmatrix}$$

$13-9=4$

$$u=\begin{pmatrix}2\\\sqrt{13}-3\end{pmatrix} \qquad v=\begin{pmatrix}3-\sqrt{13}\\2\end{pmatrix}$$

12 10-16:20

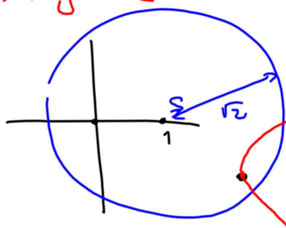
$\boxed{x^2+y^2-2x-1=0}$

$(x-1)^2+y^2-2=0$

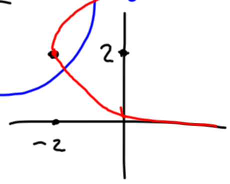
$\boxed{y^2-2x-4y=0}$

$(y-2)^2-\boxed{2x}-4=0$

$(y-2)^2-2(x+2)=0$



$y=x^2$



12 10-16:31

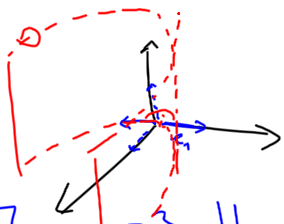
$x^2+\underbrace{2y+2z}=0$

$x=x'$

$\frac{1}{\sqrt2}(y+z)=y'$

$\frac{1}{\sqrt2}(y-z)=z'$

$$\boxed{x'^2+2\sqrt2\,y'=0}$$



v $\mathbb{R}^3$!!

12 10-16:57

$$f(x_1,\dots,x_n)=\sum_{i,j}a_{ij}x_ix_j$$

$$=x^T\cdot A\cdot x$$

$x=S\cdot x'$

$$=(x')^T\cdot\underbrace{S^T A\cdot S}_{A'}\cdot x$$

$$\boxed{3x_1^2-3x_1x_2}=0 \Rightarrow \boxed{(x_1')^2-(x_2')^2=0}$$

$$3\left(x_1-\tfrac12x_2\right)^2-\tfrac34x_2^2=0$$

$$x_1'=\sqrt3\left(x_1-\tfrac12x_2\right) \qquad x_2'=\tfrac{\sqrt3}{2}x_2$$

12 10-17:09

po kr. (1)

$$f(x_1,\dots,x_n) = \pm x_1'^2 + h(x_2',\dots,x_n')$$

$$x_1x_2 = (x_2')^2 - (x_1')^2$$

$$x_1 = x_1' + x_2'$$

$$x_2 = x_2' - x_1'$$

12 10-17:18

Příklad $f(x,y,z) = 3x^2 + 2xy + y^2 + 4yz + 6z^2$

$$= 3(\underbrace{x + \tfrac{1}{3}y}_{x'})^2 + \tfrac{2}{3}y^2 + 4yz + 6z^2$$

$$= 3(x + \tfrac{1}{3}y)^2 + \tfrac{2}{3}(y + 3z)^2$$

$$= +(x')^2 + (y')^2 + \underline{0}(z')^2$$

$x' = \sqrt{3}(x + \frac{1}{3}y)$

$y' = \sqrt{\frac{2}{3}}(y + 3z)$

$z' = z$

12 10-17:22

symetrická A $\longleftrightarrow$ symetrické zobrazení $\langle \varphi(u), v\rangle = \langle u, \varphi(v)\rangle$

$$\langle \varphi(u), u\rangle = x^T \cdot A \cdot x = f(u)$$

$0 \le \langle \varphi(u), u\rangle$ $\forall u$ … pozitivní semidef.

12 10-17:31

$x_1x_2 + x_1x_3 =$

$y_2 = x_2 - x_1$

$y_1 = x_1$

$y_3 = x_3$

$$= y_1(y_2 + y_1) + y_1y_3$$

$$= y_1^2 + y_1y_2 + y_1y_3$$

$$= (\underbrace{y_1 + \tfrac{1}{2}y_2 + \tfrac{1}{2}y_3}) ^2 - \tfrac{1}{4}y_2^2 - \tfrac{1}{2}y_2y_3 - \tfrac{1}{4}y_3^2$$

$$= z_1^2 - \underbrace{\tfrac{1}{4}(y_2 + y_3)^2}_{z_2^2} \quad \checkmark$$

$$z_1 = y_1 + \tfrac{1}{2}y_2 + \tfrac{1}{2}y_3 = \tfrac{1}{2}x_2 + \tfrac{1}{2}x_1 + \tfrac{1}{2}x_3$$

$$z_2 = \tfrac{1}{2}(y_2 + y_3) = \tfrac{1}{2}x_2 - \tfrac{1}{2}x_1 + \tfrac{1}{2}x_3$$

12 10-17:35