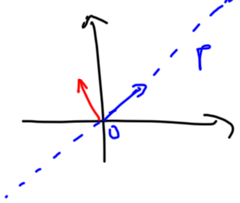
rovnice: $A\cdot x = b$

matice m/n — vektor n — vektor m

$A\cdot(ax) = a\cdot(A\cdot x) = 0$

podprostory v $\mathbb{R}^2$



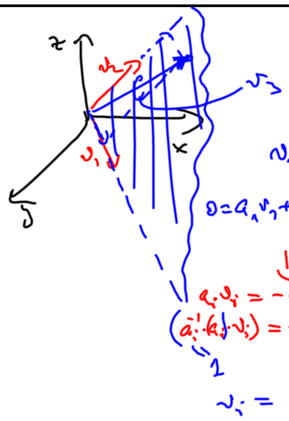
$V = \mathbb{R}^2$

$V =$ přímka p, $0 \in p$

$V = \{0\}$

10 22-16:07

$\mathbb{R}^3$



$v_3 = v_1 + v_2$

$v_1, \dots, v_k$ závislé:

$0 = a_1v_1 + \dots + a_iv_i + \dots + a_kv_k$, $a_i \neq 0$

$\Downarrow$

$a_iv_i = -a_1v_1 - \dots - a_kv_k$

$\underbrace{(a_i^{-1}a_i)}_{=1}v_i = -\frac{a_1}{a_i}v_1 - \dots - \frac{a_k}{a_i}v_k$

$v_i = \dots$

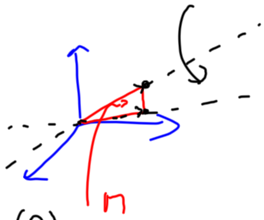
10 22-16:17

$\mathbb{R}_3[x] = \{a_3x^3 + a_2x^2 + a_1x + a_0\} \cong \mathbb{R}^4$

$\langle M \rangle$

$M \subset V$

$\bigcap_{i\in I} W_i \subset V$, $W_i \supset M$



báze $\mathbb{R}^n$: $\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix} \begin{pmatrix}0\\1\\\vdots\\0\end{pmatrix} \cdots \begin{pmatrix}0\\0\\\vdots\\0\\1\end{pmatrix}$ — standardní

$e_1 \quad e_2 \quad \cdots \quad e_n$

10 22-16:27

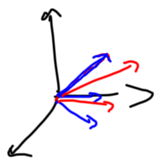
standardní báze $\mathbb{R}_n[x]$: $1, x, x^2, \dots, x^n$

$a_0 = 1$, $a_i = 0$, $i > 0$

$v = x_1e_1 + x_2e_2 + \dots + x_ne_n$

$\begin{pmatrix}x_1\\\vdots\\x_n\end{pmatrix} = x_1\begin{pmatrix}1\\0\\\vdots\\0\\0\end{pmatrix} + x_2\begin{pmatrix}0\\1\\\vdots\\0\\0\end{pmatrix} + \dots$

10 22-16:42



$v_1 = \begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}$ $v_2 = \begin{pmatrix}1\\1\\0\\\vdots\\0\end{pmatrix}$

$v_{n-1} = \begin{pmatrix}1\\1\\\vdots\\1\\0\end{pmatrix}$ $v_n = \begin{pmatrix}1\\1\\\vdots\\1\end{pmatrix}$

$\boxed{v_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad v_2 = \begin{pmatrix}1\\1\\0\end{pmatrix} \quad v_3 = \begin{pmatrix}1\\1\\1\end{pmatrix}}$

$v: \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = (x_1 - x_2)v_1 + (x_2 - x_3)v_2 + x_3v_3$

$\left.\begin{aligned} u &= a_1v_1 + \dots + a_nv_n \\ u' &= a_1'v_1 + \dots + a_n'v_n \end{aligned}\right\}$ $(a_1 + a_1')v_1 + \dots$

10 22-16:48

$u = \begin{pmatrix}a_1\\a_2\\\vdots\\a_n\end{pmatrix}$ $e_1 = \begin{pmatrix}a_{11}\\a_{21}\\\vdots\\a_{n1}\end{pmatrix}$ $e_2 = \begin{pmatrix}a_{12}\\a_{22}\\\vdots\\a_{n2}\end{pmatrix}$ ...

$x_1e_1 + x_2e_2 + \dots = \begin{pmatrix}a_1\\a_2\\\vdots\\a_n\end{pmatrix}$

$u_1 \sim \begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix} \longmapsto f(u_1) \sim \begin{pmatrix}a_{11}\\a_{21}\\\vdots\\a_{m1}\end{pmatrix}$

$u_2 \sim \begin{pmatrix}0\\1\\\vdots\\0\end{pmatrix} \longmapsto f(u_2) \sim \begin{pmatrix}a_{12}\\a_{22}\\\vdots\\a_{m2}\end{pmatrix}$

$x_1\cdot u_1 + \dots + x_n\cdot u_n \longmapsto x_1\cdot\begin{pmatrix}a_{11}\\\vdots\\a_{m1}\end{pmatrix} + x_2\cdot\begin{pmatrix}\vdots\end{pmatrix} + \dots$

10 22-17:08

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

obrazy bázových vektorů

matice zobr. f v bázích $\underline{u}$, $\underline{v}$

$x' = T \cdot x \quad | \cdot T^{-1}$

$T^{-1} \cdot x' = x$

10 22-17:17

$e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}$, ...

$e_i \cdot e_j = \langle e_i, e_j \rangle = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$ ortonormální báze $\mathbb{R}^n$

skalární součin

$(u, v) \longmapsto \langle u, v \rangle$

lineární v u ✓

lineární v v ✓

symetrické ✓

$(x_1, \dots, x_n) \cdot (y_1, \dots, y_n)$

$= x_1 y_1 + \dots + x_n y_n$

$\|v\|^2 = x_1^2 + \dots + x_n^2$

10 22-17:23

lineární forma $\alpha = (y_1, \dots, y_n)$, $u = (x_1, \dots, x_n)$

$\alpha(u) = (y_1\, y_2 \cdots y_n) \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

$(V, \langle\,,\rangle)$ prostor se skal. součinem

např. $(\mathbb{R}^n, \cdot)$ ✓

10 22-17:30

$(e_1, \dots, e_n) = \underline{e}$ ortonormální

$\langle x_1 e_1 + \dots + x_n e_n, y_1 e_1 + \dots + y_n e_n \rangle =$

$x_1 \langle e_1, y_1 e_1 + \dots \rangle + x_2 \langle e_2, y_1 e_1 + \dots \rangle + \dots =$

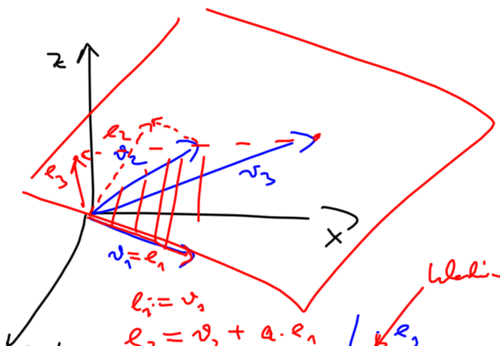
$x_1 y_1 \langle e_1, e_1 \rangle + x_1 y_2 \langle e_1, e_2 \rangle + \dots = \sum_{i,j=1}^{n} x_i y_j \langle e_i, e_j \rangle$

$= \sum_{i=1}^{n} x_i y_i = x^T \cdot y$

$S = (\langle e_i, e_j \rangle)$

$= x^T \cdot S \cdot y$

10 22-17:39



$e_1 = v_1$

$e_2 = v_2 + a \cdot e_1 \quad | \cdot e_1$

$0 = e_1 \cdot e_2 = e_1 \cdot v_2 + a \|e_1\|^2$

$\Rightarrow a = -\dfrac{e_1 \cdot v_2}{\|e_1\|^2}$

10 22-17:43

10 22-17:50