$\mathbb{N} \ni n \longmapsto x_n = f(n)$

$x_{n+1} = F(n, \underbrace{x_n, x_{n-1}, \dots, x_{n-k+1}}_{k})$

dif. rovnice k-tého řádu

1. řád: $x_{n+1} = F(n, x_n)$

$x_n = n!$ $\quad \boxed{(n+1)! = (n+1)\cdot n!}$

$x_{n+1} \qquad F(n, x_n)$

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$$\boxed{\begin{aligned} &a_0(n)\, x_n + a_1(n)\, x_{n-1} + \dots + a_k(n)\, x_{n-k} = 0 \\ &\ne 0 \qquad\qquad\qquad\qquad \ne 0 \end{aligned}}$$

$\Rightarrow \; x_n = \frac{1}{a_0(n)} \cdot \left(-a_1(n)\, x_{n-1} - \dots - a_k(n)\, x_{n-k}\right)$ PLATÍ

řešení: $x_0, x_1, \dots, x_n, x_{n+1}, \dots$

$y_0, y_1, \dots, y_n, y_{n+1}, \dots$

součet: $z_n = x_n + y_n$ splňuje: $\quad (x_{n-k} + y_{n-k})$

$a_0(n)\, z_n + \dots + a_k(n)\, z_{n-k} = a_0(n)(x_n + y_n) + \dots + a_k(n)$

$= \underbrace{\boxed{a_0(n)\, x_n + a_1(n)\, x_{n-1} + \dots}}_{=0} + \underbrace{\boxed{a_0(n)\, y_n + \dots}}_{=0}$

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násobek: $\alpha x_0, \alpha x_1, \dots, \alpha x_n, \dots$

$a_0(n)(\alpha \cdot x_n) + a_1(n)(\alpha \cdot x_{n-1}) + \dots$

$= \alpha \cdot \underbrace{\left(a_0(n)\, x_n + \dots\right)}_{=0} = 0$

✓

Množina řešení je vekt. prostor $\subset \mathbb{K}^\infty$

dimenze je $\underline{k}$.

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$$\boxed{\begin{aligned} &n\, x_n + 3\, x_{n-1} + (n-1)\, x_{n-2} = 0 \\ &x_n = \tfrac{1}{n}\left(-3x_{n-1} - (n-1)\, x_{n-2}\right) \end{aligned}}$$

$\boxed{x_0 = 0, \quad x_1 = 1}, \quad x_2 = -\frac{3}{2}$

$x_3 = \frac{1}{3}\left(-3 \cdot \left(-\frac{3}{2}\right) - 2 \cdot 1\right) = \frac{1}{3} \cdot \frac{5}{2}$

$\vdots$

$\boxed{x_0 = 1, \quad x_1 = 0}, \quad x_2 = \frac{1}{2}(-1) = -\frac{1}{2}$

$x_3 = \dots$

$\Rightarrow$ dvě řešení, která generují všechna a jsou nezávislá

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$x_{n+1} = a\, x_n \quad \Rightarrow \quad x_n = \boxed{a^n}\, x_0$

$\boxed{x_n = \lambda^n} \qquad \lambda \in \mathbb{K}$

$\boxed{a_0 \lambda^n + a_1 \lambda^{n-1} + \dots + a_k \lambda^{n-k} = 0} \qquad n \ge k$

$\lambda^{n-k} \underbrace{\left(a_0 \lambda^k + a_1 \lambda^{k-1} + \dots + a_k \lambda^0\right)}_{=0 \text{ nebo } \lambda = 0} = 0$

charakt. polyn.

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Příklad: $\boxed{\begin{aligned} &p_{n+2} = p_n + p_{n+1} \\ &n = 2, 3, 4, 5, \dots \end{aligned}}$ $\quad p_0 = 1$, $p_1 = 1$

$\boxed{\lambda^2 - \lambda - 1 = 0}$ $\quad \boxed{\lambda^{n+2} - \lambda^{n+1} - \lambda^n = 0}$

$\lambda_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

$$\boxed{\begin{aligned} x_n &= \left(\frac{1+\sqrt{5}}{2}\right)^n \\ y_n &= \left(\frac{1-\sqrt{5}}{2}\right)^n \end{aligned}}$$

Řešení je $A \cdot x_n + B \cdot y_n$ $\quad n=0$: $\boxed{A \cdot 1 + B \cdot 1 = 1}$

$\boxed{A = \frac{1+\sqrt{5}}{2\sqrt{5}}}$ $\quad n=1$: $A \cdot \frac{1+\sqrt{5}}{2} + B \cdot \frac{1-\sqrt{5}}{2} = 1$

$(1-B)\frac{1+\sqrt{5}}{2} + B \cdot \frac{1-\sqrt{5}}{2} = 1$

$(1+\sqrt{5}) + B(-2\sqrt{5}) = 2 \Rightarrow \boxed{B = \frac{1-\sqrt{5}}{-2\sqrt{5}}}$

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$$\begin{vmatrix} \lambda_1^0 & \lambda_2^0 & \cdots & \lambda_k^0 \\ \lambda_1^1 & \lambda_2^1 & & \lambda_k^1 \\ \lambda_1^2 & \lambda_2^2 & & \lambda_k^2 \\ \vdots & \vdots & & \vdots \\ \lambda_1^k & \lambda_2^k & & \lambda_k^k \end{vmatrix} \neq 0$$

$(\lambda-2)^2 = \lambda^2 - 2\lambda + 4 = 0$

$\lambda_{1,2} = 2$

$\boxed{x_{m+2} = 2x_{n+1} - 4x_n}$

$\boxed{x_n = 2^n}$

$\boxed{y_n = n2^n}$

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$(\lambda-3)^3(\lambda-1) = (\lambda^3 - 9\lambda^2 + 27\lambda - 27)(\lambda - 1)$

$(\lambda-3)(\lambda-3)(\lambda-3) = \lambda^4 - 10\lambda^3 - \dots$

$x_{n+4} = 10x_{n+3} + \dots$

$\lambda_{1,2,3} = 3, \quad \lambda_4 = 1$

$x_n = 3^n$
$x_n = n3^n$
$x_n = n^2 3^n$
$x_n = 1$

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$\boxed{y_{t+2} - a(1+b)y_{t+1} + ab\,y_t = 1}$

$a = 3/4 \quad b = 1/3$ $\quad \boxed{y_0 = 1,\ y_1 = 1}$

homogenizovaný vztah:

$y_{t+2} - a(1+b)y_{t+1} + ab\,y_t = 0$

$y_{t+2} - 3/4 \cdot 4/3 \cdot y_{t+1} + \frac{1}{4} y_t = 0$

$\lambda^2 - \lambda + \frac{1}{4} = 0 \qquad \lambda_{1,2} = \frac{1 \pm \sqrt{0}}{2} = \frac{1}{2}$

obecné řešení: $\boxed{A \cdot \left(\frac{1}{2}\right)^n + B \cdot n\left(\frac{1}{2}\right)^n}$

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$y_{t+2} - y_{t+1} + \frac{1}{4} y_t = 1$ $\qquad y_0 = 1,\ y_1 = 1,\ y_2 = \frac{7}{4}$

$y_t = C: \quad C - C + \frac{1}{4} C = 1$

$\Rightarrow C = 4$

$\Rightarrow$ obecné řešení: $y_t = -\frac{3}{2^t} - \frac{3t}{2^t} + 4$

$\boxed{y_t = A \cdot \frac{1}{2^t} + B \cdot t \cdot \frac{1}{2^t} + 4}$

$y_0 = A + 4 = 1 \Rightarrow A = -3$

$y_1 = \frac{1}{2}A + \frac{1}{2}B + 4 = -\frac{3}{2} + 4 + \frac{1}{2}B = 1$
$\Rightarrow B = -3$

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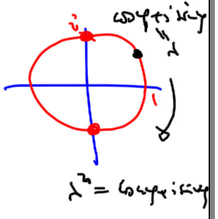
$\boxed{y_{t+2} + y_t = 0}$ $\qquad \boxed{\begin{aligned} y_0 &= -1 \\ y_1 &= 1 \end{aligned}}$

$\left.\begin{aligned} &\lambda^2 + 1 = 0 \\ &\lambda_{1,2} = \pm i \in \mathbb{C} \end{aligned}\right\} \Rightarrow$ obecné řešení

$y_t = a \cdot i^n + b(-i)^n$

$= a\boxed{\left(\cos t\pi/2 + i \sin t\pi/2\right)}$
$+ b\boxed{\left(\cos t\pi/2 - i \sin t\pi/2\right)}$



$\boxed{\begin{aligned} y_t &= \cos t\pi/2 \\ y_t &= \sin t\pi/2 \end{aligned}}$

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